Computer algebra independent integration tests

1_Algebraic_functions/1.2_Trinomial_products/1.2.4Improper/1.2.4.2(dx)^m(ax^q+bx^n+c
q))^p

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

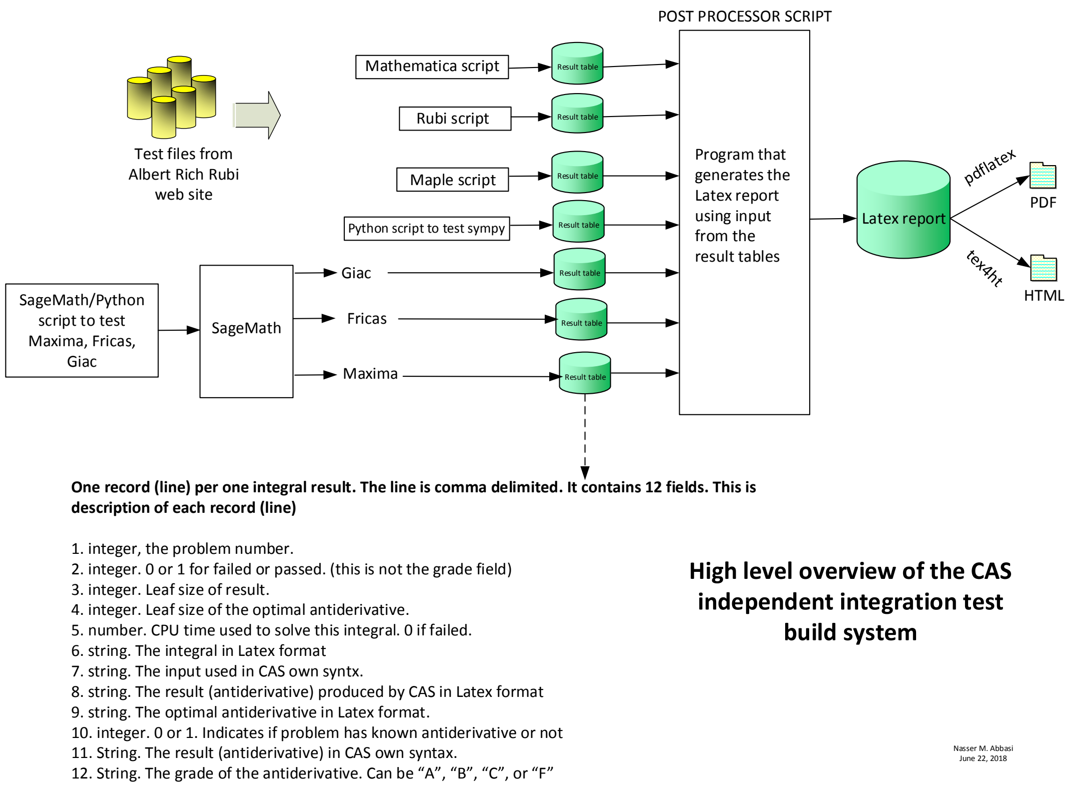
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expressi

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (140)	% 0. (0)
Rubi in Sympy	% 85.71 (120)	% 14.29 (20)
Mathematica	% 99.29 (139)	% 0.71 (1)
Maple	% 97.14 (136)	% 2.86 (4)
Maxima	% 21.43 (30)	% 78.57 (110)
Fricas	% 92.14 (129)	% 7.86 (11)
Sympy	% 46.43 (65)	% 53.57 (75)
Giac	% 57.14 (80)	% 42.86 (60)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

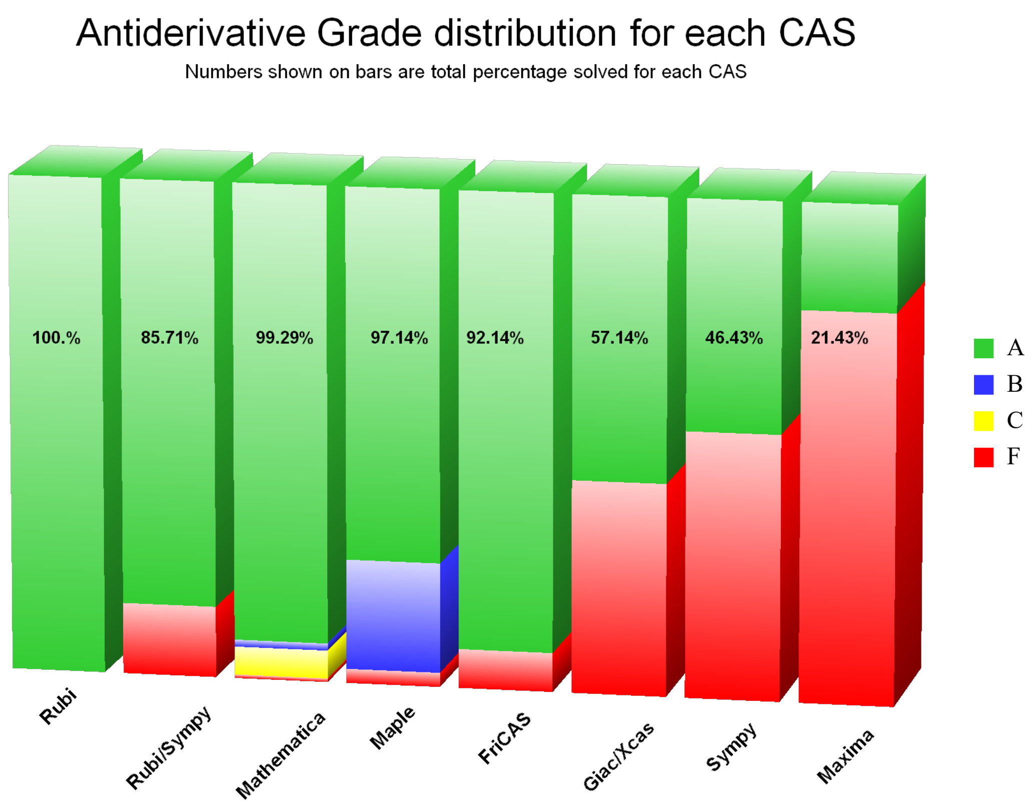
Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented.

For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

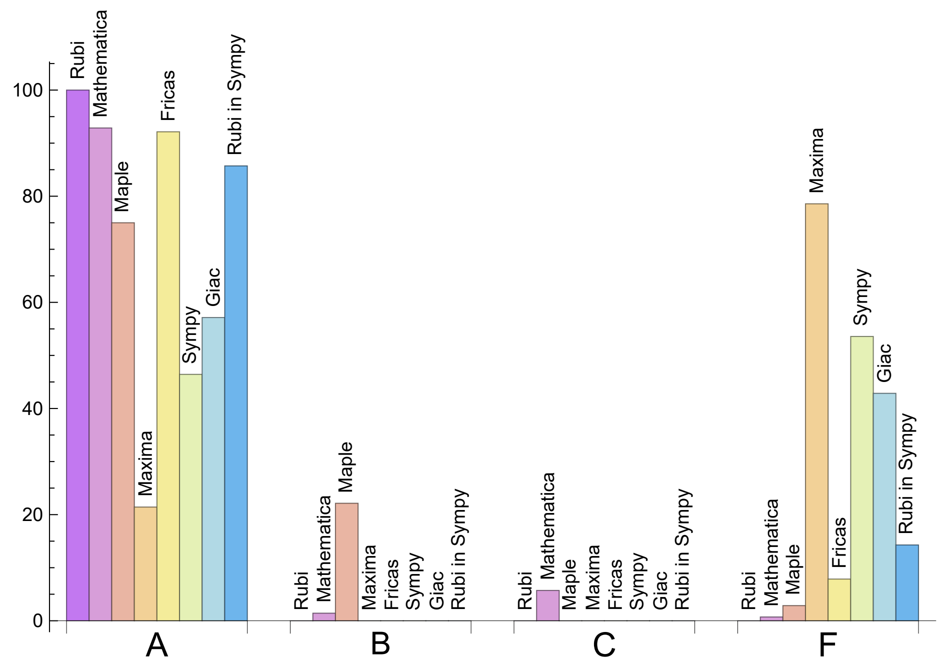
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	85.71	0.	0.	14.29
Mathematica	92.86	1.43	5.71	0.71
Maple	75.	22.14	0.	2.86
Maxima	21.43	0.	0.	78.57
Fricas	92.14	0.	0.	7.86
Sympy	46.43	0.	0.	53.57
Giac	57.14	0.	0.	42.86

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.37	139.59	1.	103.5	1.
Rubi in Sympy	44.08	140.95	1.01	114.	0.94
Mathematica	0.35	145.65	1.08	109.	1.
Maple	0.01	305.37	1.7	143.	1.33
Maxima	0.8	40.9	0.97	27.	1.04
Fricas	0.32	247.09	1.26	1.	0.01
Sympy	12.9	601.45	4.53	265.	1.57
Giac	0.3	139.48	1.37	80.	1.29

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {4, 5, 11, 19, 20, 38, 65, 66, 68, 69, 70, 71, 74, 76, 77, 78, 89, 90, 91, 102}

Not solved by Mathematica {140}

Not solved by Maple {104, 121, 122, 140}

Not solved by Maxima {11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 72, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 136, 140}

Not solved by Fricas {104, 105, 107, 110, 112, 114, 116, 117, 119, 122, 140}

Not solved by Sympy {29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140}

Not solved by Giac {33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 60, 61, 63, 64, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 140}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {110}

Mathematica {104, 122}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	19	26	19
normalized size	1	1.	1.	0.8	1.04	0.04	0.76	1.04	0.76
time (sec)	N/A	0.02	0.004	0.001	0.744	0.238	0.066	0.261	6.172

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	19	26	19
normalized size	1	1.	1.	0.8	1.04	0.04	0.76	1.04	0.76
time (sec)	N/A	0.017	0.002	0.001	0.748	0.257	0.068	0.261	6.014

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	19	26	19
normalized size	1	1.	1.	0.8	1.04	0.04	0.76	1.04	0.76
time (sec)	N/A	0.011	0.	0.001	0.754	0.24	0.062	0.261	1.906

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	26	19	26	0
normalized size	1	1.	1.	0.8	1.04	1.04	0.76	1.04	0.
time (sec)	N/A	0.015	0.002	0.001	0.743	0.26	0.07	0.262	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	22	15	22	0
normalized size	1	1.	1.	0.85	1.1	1.1	0.75	1.1	0.
time (sec)	N/A	0.015	0.002	0.001	0.75	0.255	0.068	0.264	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	1	48	62	48
normalized size	1	1.	1.	0.83	1.09	0.02	0.89	1.15	0.89
time (sec)	N/A	0.102	0.011	0.001	0.759	0.231	0.113	0.262	13.014

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	1	49	62	49
normalized size	1	1.	1.	0.83	1.09	0.02	0.91	1.15	0.91
time (sec)	N/A	0.068	0.011	0.002	0.757	0.243	0.107	0.26	12.839

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	65	1	48	62	48
normalized size	1	1.	1.	0.83	1.2	0.02	0.89	1.15	0.89
time (sec)	N/A	0.057	0.009	0.001	0.748	0.233	0.103	0.262	21.895

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	59	49	62	49
normalized size	1	1.	1.	0.83	1.09	1.09	0.91	1.15	0.91
time (sec)	N/A	0.061	0.009	0.002	0.748	0.265	0.11	0.262	12.991

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	59	48	62	48
normalized size	1	1.	1.	0.83	1.09	1.09	0.89	1.15	0.89
time (sec)	N/A	0.061	0.01	0.001	0.752	0.268	0.108	0.26	13.187

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	132	0	1	381	116	0
normalized size	1	1.	0.94	1.48	0.	0.01	4.28	1.3	0.
time (sec)	N/A	0.175	0.197	0.005	0.	0.303	2.846	0.265	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	101	0	1	306	90	65
normalized size	1	1.	1.04	1.44	0.	0.01	4.37	1.29	0.93
time (sec)	N/A	0.105	0.106	0.004	0.	0.278	2.33	0.264	23.442

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	1	216	74	49
normalized size	1	1.	1.02	1.	0.	0.02	3.86	1.32	0.88
time (sec)	N/A	0.07	0.057	0.003	0.	0.287	0.894	0.264	13.425

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	1	124	46	34
normalized size	1	1.	1.12	1.03	0.	0.03	3.65	1.35	1.
time (sec)	N/A	0.04	0.011	0.002	0.	0.278	0.532	0.264	6.866

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	62	0	1	564	84	54
normalized size	1	1.	0.98	1.	0.	0.02	9.1	1.35	0.87
time (sec)	N/A	0.091	0.112	0.006	0.	0.305	6.554	0.265	20.946

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	112	0	1	862	107	75
normalized size	1	1.	0.95	1.38	0.	0.01	10.64	1.32	0.93
time (sec)	N/A	0.204	0.142	0.006	0.	0.291	12.141	0.264	45.617

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	150	0	1	1525	142	97
normalized size	1	1.	0.98	1.44	0.	0.01	14.66	1.37	0.93
time (sec)	N/A	0.309	0.255	0.007	0.	0.32	17.267	0.264	44.282

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	214	0	1	2105	184	129
normalized size	1	1.	0.96	1.56	0.	0.01	15.36	1.34	0.94
time (sec)	N/A	0.391	0.173	0.008	0.	0.367	28.052	0.264	57.4

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	132	569	0	1	842	217	0
normalized size	1	1.	0.88	3.79	0.	0.01	5.61	1.45	0.
time (sec)	N/A	0.331	0.311	0.015	0.	0.306	5.991	0.269	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	109	330	0	1	729	169	0
normalized size	1	1.	0.96	2.89	0.	0.01	6.39	1.48	0.
time (sec)	N/A	0.209	0.232	0.013	0.	0.322	4.44	0.27	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	97	0	1	280	119	61
normalized size	1	1.	1.21	1.45	0.	0.01	4.18	1.78	0.91
time (sec)	N/A	0.074	0.152	0.009	0.	0.292	2.752	0.266	13.888

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	1	252	103	60
normalized size	1	1.	1.05	1.06	0.	0.02	3.82	1.56	0.91
time (sec)	N/A	0.065	0.106	0.004	0.	0.289	2.625	0.268	11.811

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	1	265	103	60
normalized size	1	1.	1.06	1.03	0.	0.02	4.02	1.56	0.91
time (sec)	N/A	0.06	0.123	0.003	0.	0.282	2.611	0.267	9.858

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	389	0	1	2236	170	102
normalized size	1	1.	0.99	3.6	0.	0.01	20.7	1.57	0.94
time (sec)	N/A	0.311	0.319	0.011	0.	0.346	25.111	0.27	43.784

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	131	545	0	1	2672	231	143
normalized size	1	1.	0.89	3.68	0.	0.01	18.05	1.56	0.97
time (sec)	N/A	0.395	0.467	0.013	0.	0.436	38.041	0.271	73.096

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	175	646	0	1	4083	309	192
normalized size	1	1.	0.87	3.2	0.	0.	20.21	1.53	0.95
time (sec)	N/A	0.511	0.778	0.021	0.	0.546	58.744	0.274	82.886

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	218	808	0	1	4774	381	245
normalized size	1	1.	0.87	3.21	0.	0.	18.94	1.51	0.97
time (sec)	N/A	0.647	0.516	0.024	0.	0.751	90.578	0.271	124.5

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	272	923	0	1	6181	468	308
normalized size	1	1.	0.86	2.9	0.	0.	19.44	1.47	0.97
time (sec)	N/A	0.804	0.643	0.027	0.	1.041	137.52	0.275	127.985

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	180	310	0	1	0	382	241
normalized size	1	1.	0.7	1.21	0.	0.	0.	1.49	0.94
time (sec)	N/A	0.945	0.438	0.013	0.	0.298	0.	0.302	91.259

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	150	265	0	1	0	311	190
normalized size	1	1.	0.73	1.29	0.	0.	0.	1.52	0.93
time (sec)	N/A	0.59	0.333	0.011	0.	0.292	0.	0.304	59.514

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	117	167	0	1	0	224	146
normalized size	1	1.	0.72	1.02	0.	0.01	0.	1.37	0.9
time (sec)	N/A	0.125	0.333	0.008	0.	0.28	0.	0.3	24.555

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	100	146	0	1	0	169	107
normalized size	1	1.	0.84	1.23	0.	0.01	0.	1.42	0.9
time (sec)	N/A	0.132	0.236	0.007	0.	0.286	0.	0.294	19.825

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	144	126	0	1	0	0	158
normalized size	1	1.	0.83	0.73	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.261	0.165	0.007	0.	0.312	0.	0.	37.026

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	133	173	0	1	0	0	160
normalized size	1	1.	0.77	1.	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.261	0.223	0.009	0.	0.332	0.	0.	36.796

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	125	206	0	1	0	0	128
normalized size	1	1.	1.1	1.81	0.	0.01	0.	0.	1.12
time (sec)	N/A	0.238	0.172	0.009	0.	0.293	0.	0.	41.954

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	145	234	0	1	0	0	167
normalized size	1	1.	0.94	1.51	0.	0.01	0.	0.	1.08
time (sec)	N/A	0.413	0.39	0.009	0.	0.293	0.	0.	64.522

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	186	387	0	1	0	0	214
normalized size	1	1.	0.91	1.89	0.	0.	0.	0.	1.04
time (sec)	N/A	0.619	0.392	0.011	0.	0.308	0.	0.	84.796

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	304	649	0	1	0	703	0
normalized size	1	1.	0.72	1.54	0.	0.	0.	1.67	0.
time (sec)	N/A	1.871	0.647	0.014	0.	0.374	0.	0.34	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	239	479	0	1	0	579	348
normalized size	1	1.	0.66	1.32	0.	0.	0.	1.59	0.96
time (sec)	N/A	1.598	0.489	0.012	0.	0.338	0.	0.323	171.091

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	211	431	0	1	0	493	274
normalized size	1	1.	0.73	1.5	0.	0.	0.	1.71	0.95
time (sec)	N/A	0.829	0.387	0.012	0.	0.311	0.	0.319	107.947

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	161	289	0	1	0	383	187
normalized size	1	1.	0.81	1.46	0.	0.01	0.	1.93	0.94
time (sec)	N/A	0.297	0.286	0.008	0.	0.338	0.	0.315	40.016

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	130	265	0	1	0	313	155
normalized size	1	1.	0.79	1.61	0.	0.01	0.	1.9	0.94
time (sec)	N/A	0.2	0.211	0.007	0.	0.298	0.	0.319	30.543

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	248	222	0	1	0	0	209
normalized size	1	1.	1.09	0.98	0.	0.	0.	0.	0.92
time (sec)	N/A	0.467	0.326	0.009	0.	0.409	0.	0.	59.549

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	229	254	0	1	0	0	206
normalized size	1	1.	1.05	1.16	0.	0.	0.	0.	0.94
time (sec)	N/A	0.46	0.258	0.009	0.	0.359	0.	0.	57.1

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	173	338	0	1	0	0	207
normalized size	1	1.	0.79	1.54	0.	0.	0.	0.	0.95
time (sec)	N/A	0.453	0.356	0.01	0.	0.345	0.	0.	60.809

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	187	435	0	1	0	0	238
normalized size	1	1.	0.73	1.69	0.	0.	0.	0.	0.93
time (sec)	N/A	0.644	0.752	0.013	0.	0.368	0.	0.	79.319

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	156	501	0	1	0	0	207
normalized size	1	1.	0.79	2.54	0.	0.01	0.	0.	1.05
time (sec)	N/A	0.591	0.316	0.012	0.	0.328	0.	0.	83.266

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	193	534	0	1	0	0	262
normalized size	1	1.	0.78	2.14	0.	0.	0.	0.	1.05
time (sec)	N/A	0.808	0.409	0.014	0.	0.316	0.	0.	118.298

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	103	144	0	1	0	0	129
normalized size	1	1.	0.72	1.01	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.292	0.17	0.011	0.	0.299	0.	0.	31.519

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	87	88	0	1	0	146	92
normalized size	1	1.	0.84	0.85	0.	0.01	0.	1.42	0.89
time (sec)	N/A	0.129	0.092	0.008	0.	0.294	0.	0.282	17.932

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	65	0	1	0	50	66
normalized size	1	1.	0.93	0.92	0.	0.01	0.	0.7	0.93
time (sec)	N/A	0.063	0.04	0.007	0.	0.286	0.	0.281	10.336

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	66	0	1	0	80	68
normalized size	1	1.	1.56	1.47	0.	0.02	0.	1.78	1.51
time (sec)	N/A	0.033	0.07	0.008	0.	0.278	0.	0.276	19.153

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	106	88	0	1	0	0	94
normalized size	1	1.	1.38	1.14	0.	0.01	0.	0.	1.22
time (sec)	N/A	0.094	0.13	0.009	0.	0.297	0.	0.	28.709

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	138	152	0	1	0	0	134
normalized size	1	1.	1.16	1.28	0.	0.01	0.	0.	1.13
time (sec)	N/A	0.25	0.231	0.01	0.	0.29	0.	0.	42.759

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	181	283	0	1	0	0	241
normalized size	1	1.	0.69	1.08	0.	0.	0.	0.	0.92
time (sec)	N/A	0.785	0.327	0.012	0.	0.348	0.	0.	78.86

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	139	199	0	1	0	263	187
normalized size	1	1.	0.69	0.99	0.	0.	0.	1.31	0.93
time (sec)	N/A	0.482	0.201	0.01	0.	0.329	0.	0.288	53.769

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	110	166	0	1	0	149	139
normalized size	1	1.	0.72	1.08	0.	0.01	0.	0.97	0.91
time (sec)	N/A	0.274	0.18	0.011	0.	0.324	0.	0.293	33.372

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	53	0	99	0	61	36
normalized size	1	1.	0.92	1.32	0.	2.48	0.	1.52	0.9
time (sec)	N/A	0.139	0.041	0.005	0.	0.291	0.	0.286	9.375

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	0	97	0	61	37
normalized size	1	1.	0.92	1.33	0.	2.49	0.	1.56	0.95
time (sec)	N/A	0.056	0.03	0.005	0.	0.285	0.	0.285	9.387

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	132	166	0	1	0	0	112
normalized size	1	1.	1.4	1.77	0.	0.01	0.	0.	1.19
time (sec)	N/A	0.11	0.261	0.009	0.	0.299	0.	0.	29.448

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	163	201	0	1	0	0	162
normalized size	1	1.	1.13	1.4	0.	0.01	0.	0.	1.12
time (sec)	N/A	0.268	0.28	0.01	0.	0.309	0.	0.	57.449

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	220	292	0	1	0	4	221
normalized size	1	1.	1.05	1.4	0.	0.	0.	0.02	1.06
time (sec)	N/A	0.485	0.57	0.013	0.	0.353	0.	0.614	79.898

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	265	340	0	1	0	0	282
normalized size	1	1.	0.98	1.25	0.	0.	0.	0.	1.04
time (sec)	N/A	0.74	0.582	0.015	0.	0.377	0.	0.	119.222

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	322	446	0	1	0	0	352
normalized size	1	1.	0.94	1.3	0.	0.	0.	0.	1.03
time (sec)	N/A	0.975	0.926	0.017	0.	0.451	0.	0.	145.7

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	77	0	96	280	169	0
normalized size	1	1.	0.95	2.08	0.	2.59	7.57	4.57	0.
time (sec)	N/A	0.03	0.037	0.004	0.	0.293	3.36	0.269	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	19	26	0
normalized size	1	1.	1.	0.8	1.04	0.04	0.76	1.04	0.
time (sec)	N/A	0.018	0.003	0.001	0.777	0.238	0.069	0.249	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	19	26	19
normalized size	1	1.	1.	0.8	1.04	0.04	0.76	1.04	0.76
time (sec)	N/A	0.016	0.003	0.001	0.771	0.235	0.067	0.25	6.748

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	19	26	0
normalized size	1	1.	1.	0.8	1.04	0.04	0.76	1.04	0.
time (sec)	N/A	0.012	0.	0.001	0.762	0.235	0.07	0.253	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	22	15	22	0
normalized size	1	1.	1.	0.85	1.1	1.1	0.75	1.1	0.
time (sec)	N/A	0.014	0.001	0.001	0.767	0.251	0.067	0.251	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	17	27	0
normalized size	1	1.	1.	0.86	1.1	1.1	0.81	1.29	0.
time (sec)	N/A	0.016	0.003	0.003	0.766	0.255	0.169	0.279	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	27	12	22	0
normalized size	1	1.	1.	0.94	1.22	1.5	0.67	1.22	0.
time (sec)	N/A	0.015	0.003	0.005	0.772	0.25	0.982	0.253	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	300	0	325	1377	620	66
normalized size	1	1.	0.92	3.95	0.	4.28	18.12	8.16	0.87
time (sec)	N/A	0.09	0.069	0.01	0.	0.28	12.458	0.286	26.186

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	1	51	62	51
normalized size	1	1.	1.	0.83	1.09	0.02	0.94	1.15	0.94
time (sec)	N/A	0.076	0.012	0.001	0.772	0.247	0.112	0.248	14.021

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	59	1	46	62	0
normalized size	1	1.	0.89	0.83	1.09	0.02	0.85	1.15	0.
time (sec)	N/A	0.13	0.013	0.001	0.773	0.248	0.112	0.249	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	65	1	51	62	51
normalized size	1	1.	1.	0.83	1.2	0.02	0.94	1.15	0.94
time (sec)	N/A	0.062	0.01	0.002	0.769	0.248	0.103	0.277	13.772

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	59	59	46	62	0
normalized size	1	1.	0.89	0.83	1.09	1.09	0.85	1.15	0.
time (sec)	N/A	0.081	0.012	0.001	0.77	0.254	0.108	0.261	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	55	55	48	58	0
normalized size	1	1.	1.	0.86	1.12	1.12	0.98	1.18	0.
time (sec)	N/A	0.048	0.008	0.002	0.78	0.272	0.112	0.248	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	93	142	0	1	391	124	0
normalized size	1	1.	0.93	1.42	0.	0.01	3.91	1.24	0.
time (sec)	N/A	0.241	0.171	0.005	0.	0.29	6.992	0.254	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	250	467	0	2111	194	1	212
normalized size	1	1.	1.23	2.3	0.	10.4	0.96	0.	1.04
time (sec)	N/A	1.137	0.284	0.03	0.	0.291	8.063	0.803	77.583

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	1	316	101	73
normalized size	1	1.	0.96	1.37	0.	0.01	3.9	1.25	0.9
time (sec)	N/A	0.167	0.075	0.004	0.	0.268	5.63	0.258	32.595

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	1430	129	1	189
normalized size	1	1.	1.13	1.92	0.	7.99	0.72	0.01	1.06
time (sec)	N/A	0.528	0.189	0.004	0.	0.277	5.896	0.775	55.566

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	1	223	80	54
normalized size	1	1.	0.98	0.95	0.	0.02	3.54	1.27	0.86
time (sec)	N/A	0.123	0.038	0.003	0.	0.279	2.708	0.258	21.638

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	208	0	755	75	1	141
normalized size	1	1.	1.1	1.39	0.	5.03	0.5	0.01	0.94
time (sec)	N/A	0.215	0.165	0.021	0.	0.27	2.563	0.691	27.373

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	1	131	47	34
normalized size	1	1.	1.08	1.	0.	0.03	3.64	1.31	0.94
time (sec)	N/A	0.072	0.016	0.001	0.	0.263	1.41	0.25	12.48

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	828	87	1	138
normalized size	1	1.	0.86	0.77	0.	5.52	0.58	0.01	0.92
time (sec)	N/A	0.183	0.134	0.018	0.	0.284	3.04	0.352	19.837

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	1	253	92	63
normalized size	1	1.	1.64	0.96	0.	0.01	3.67	1.33	0.91
time (sec)	N/A	0.142	0.118	0.006	0.	0.279	9.405	0.251	28.98

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	232	0	1507	148	1	177
normalized size	1	1.	1.1	1.33	0.	8.66	0.85	0.01	1.02
time (sec)	N/A	0.427	0.736	0.023	0.	0.307	6.541	0.777	46.816

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	135	119	0	1	345	127	87
normalized size	1	1.	1.52	1.34	0.	0.01	3.88	1.43	0.98
time (sec)	N/A	0.263	0.235	0.01	0.	0.302	22.522	0.256	45.227

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	151	600	0	1	877	0	0
normalized size	1	1.	0.91	3.61	0.	0.01	5.28	0.	0.
time (sec)	N/A	0.423	0.306	0.018	0.	0.299	17.107	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	327	2280	0	3856	450	0	0
normalized size	1	1.	0.99	6.89	0.	11.65	1.36	0.	0.
time (sec)	N/A	1.42	1.109	0.069	0.	0.395	22.633	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	121	342	0	1	745	0	0
normalized size	1	1.	0.92	2.59	0.	0.01	5.64	0.	0.
time (sec)	N/A	0.297	0.309	0.011	0.	0.304	12.883	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	282	2158	0	3047	379	0	267
normalized size	1	1.	1.04	7.96	0.	11.24	1.4	0.	0.99
time (sec)	N/A	1.122	0.929	0.069	0.	0.325	16.15	0.	100.33

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	1	282	0	70
normalized size	1	1.	1.19	1.33	0.	0.01	3.62	0.	0.9
time (sec)	N/A	0.129	0.15	0.005	0.	0.322	6.47	0.	20.403

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	235	1641	0	2252	294	0	218
normalized size	1	1.	0.99	6.92	0.	9.5	1.24	0.	0.92
time (sec)	N/A	0.773	0.722	0.052	0.	0.303	11.96	0.	59.369

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	1	267	0	65
normalized size	1	1.	1.05	1.03	0.	0.01	3.56	0.	0.87
time (sec)	N/A	0.12	0.122	0.005	0.	0.307	5.894	0.	18.357

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	222	342	0	2268	298	0	201
normalized size	1	1.	1.	1.55	0.	10.26	1.35	0.	0.91
time (sec)	N/A	0.521	0.803	0.084	0.	0.323	12.467	0.	47.879

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	1	267	0	66
normalized size	1	1.	1.07	1.01	0.	0.01	3.61	0.	0.89
time (sec)	N/A	0.119	0.144	0.005	0.	0.284	5.744	0.	14.874

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	3117	394	0	230
normalized size	1	1.	0.96	2.91	0.	12.37	1.56	0.	0.91
time (sec)	N/A	0.925	0.753	0.073	0.	0.34	15.714	0.	67.413

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	207	405	0	1	772	0	116
normalized size	1	1.	1.7	3.32	0.	0.01	6.33	0.	0.95
time (sec)	N/A	0.371	0.685	0.012	0.	0.361	111.429	0.	52.529

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	302	2012	0	3931	481	0	282
normalized size	1	1.	0.98	6.53	0.	12.76	1.56	0.	0.92
time (sec)	N/A	2.588	1.09	0.059	0.	0.418	24.558	0.	140.539

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	248	569	0	1	0	0	153
normalized size	1	1.	1.53	3.51	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.493	0.466	0.015	0.	0.426	0.	0.	83.363

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	344	2349	0	4637	566	0	0
normalized size	1	1.	0.95	6.51	0.	12.84	1.57	0.	0.
time (sec)	N/A	5.664	1.235	0.07	0.	0.574	43.168	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	328	671	0	1	0	0	209
normalized size	1	1.	1.5	3.06	0.	0.	0.	0.	0.95
time (sec)	N/A	0.627	0.626	0.027	0.	0.528	0.	0.	96.251

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	383	0	0	0	0	0	138
normalized size	1	1.	2.7	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.479	0.31	0.027	0.	0.	0.	0.	42.429

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	486	1042	0	0	0	0	350
normalized size	1	1.	1.28	2.74	0.	0.	0.	0.	0.92
time (sec)	N/A	0.528	2.619	0.048	0.	0.	0.	0.	71.591

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	122	157	0	1	0	171	117
normalized size	1	1.	0.95	1.22	0.	0.01	0.	1.33	0.91
time (sec)	N/A	0.167	0.135	0.014	0.	0.289	0.	0.301	22.449

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	452	508	0	0	0	0	314
normalized size	1	1.	1.3	1.46	0.	0.	0.	0.	0.9
time (sec)	N/A	0.433	1.871	0.025	0.	0.	0.	0.	63.434

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	165	136	0	1	0	0	177
normalized size	1	1.	0.85	0.7	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.445	0.325	0.015	0.	0.324	0.	0.	46.393

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	181	369	0	1	0	0	230
normalized size	1	1.	0.74	1.51	0.	0.	0.	0.	0.94
time (sec)	N/A	0.584	0.246	0.019	0.	0.307	0.	0.	63.789

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	609	1880	0	0	0	0	454
normalized size	1	1.	1.25	3.86	0.	0.	0.	0.	0.93
time (sec)	N/A	0.847	3.989	0.033	0.	0.	0.	0.	103.107

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	295	0	1	0	624	167
normalized size	1	1.	0.85	1.67	0.	0.01	0.	3.53	0.94
time (sec)	N/A	0.246	0.198	0.015	0.	0.323	0.	0.412	32.15

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	540	1394	0	0	0	0	394
normalized size	1	1.	1.27	3.28	0.	0.	0.	0.	0.93
time (sec)	N/A	0.796	3.099	0.029	0.	0.	0.	0.	96.39

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	72	0	1	0	54	75
normalized size	1	1.	0.98	0.88	0.	0.01	0.	0.66	0.91
time (sec)	N/A	0.11	0.058	0.013	0.	0.297	0.	0.337	14.876

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	193	177	0	0	0	0	109
normalized size	1	1.	1.6	1.46	0.	0.	0.	0.	0.9
time (sec)	N/A	0.098	0.236	0.024	0.	0.	0.	0.	17.618

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	89	72	0	1	0	0	76
normalized size	1	1.	1.75	1.41	0.	0.02	0.	0.	1.49
time (sec)	N/A	0.052	0.152	0.016	0.	0.288	0.	0.	20.18

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	303	508	0	0	0	0	296
normalized size	1	1.	0.92	1.54	0.	0.	0.	0.	0.9
time (sec)	N/A	0.38	0.953	0.028	0.	0.	0.	0.	63.671

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	463	533	0	0	0	0	362
normalized size	1	1.	1.18	1.36	0.	0.	0.	0.	0.93
time (sec)	N/A	0.513	1.854	0.028	0.	0.	0.	0.	75.427

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	154	179	0	1	0	0	122
normalized size	1	1.	1.5	1.74	0.	0.01	0.	0.	1.18
time (sec)	N/A	0.128	0.369	0.018	0.	0.31	0.	0.	28.549

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	519	1136	0	0	0	0	434
normalized size	1	1.	1.11	2.43	0.	0.	0.	0.	0.93
time (sec)	N/A	0.789	2.407	0.032	0.	0.	0.	0.	108.185

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	192	220	0	1	0	0	172
normalized size	1	1.	1.25	1.43	0.	0.01	0.	0.	1.12
time (sec)	N/A	0.295	0.393	0.019	0.	0.323	0.	0.	54.254

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	0	0	112	0	0	48
normalized size	1	1.	0.9	0.	0.	2.2	0.	0.	0.94
time (sec)	N/A	0.071	0.128	0.112	0.	0.354	0.	0.	8.144

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	639	0	0	0	0	0	280
normalized size	1	1.	2.23	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	1.087	0.755	0.032	0.	0.	0.	0.	91.436

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	78	58	27	124	0	0	68
normalized size	1	1.	1.73	1.29	0.6	2.76	0.	0.	1.51
time (sec)	N/A	0.021	0.074	0.017	0.879	0.288	0.	0.	17.946

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	78	58	27	124	0	0	68
normalized size	1	1.	1.73	1.29	0.6	2.76	0.	0.	1.51
time (sec)	N/A	0.026	0.03	0.011	0.866	0.3	0.	0.	18.191

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	78	58	27	124	0	0	68
normalized size	1	1.	1.73	1.29	0.6	2.76	0.	0.	1.51
time (sec)	N/A	0.027	0.03	0.007	0.862	0.295	0.	0.	18.371

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	63	95	0	93	85
normalized size	1	1.	0.81	0.94	0.73	1.1	0.	1.08	0.99
time (sec)	N/A	0.079	0.055	0.013	0.881	0.286	0.	0.265	14.934

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	63	95	0	93	85
normalized size	1	1.	0.81	0.94	0.73	1.1	0.	1.08	0.99
time (sec)	N/A	0.081	0.009	0.006	0.891	0.287	0.	0.266	15.17

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	70	81	63	95	0	93	85
normalized size	1	1.	0.81	0.94	0.73	1.1	0.	1.08	0.99
time (sec)	N/A	0.08	0.009	0.006	0.866	0.276	0.	0.264	15.361

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	35	0	1	0	47	34
normalized size	1	1.	1.03	0.92	0.	0.03	0.	1.24	0.89
time (sec)	N/A	0.043	0.056	0.005	0.	0.29	0.	0.271	6.565

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	64	0	1	0	80	68
normalized size	1	1.	1.56	1.42	0.	0.02	0.	1.78	1.51
time (sec)	N/A	0.043	0.056	0.012	0.	0.284	0.	0.274	19.497

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	72	64	0	1	0	47	70
normalized size	1	1.	1.53	1.36	0.	0.02	0.	1.	1.49
time (sec)	N/A	0.118	0.068	0.016	0.	0.348	0.	0.278	19.188

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	74	66	0	1	0	47	71
normalized size	1	1.	1.51	1.35	0.	0.02	0.	0.96	1.45
time (sec)	N/A	0.147	0.061	0.012	0.	0.332	0.	0.284	17.595

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	50	39	0	1	0	0	39
normalized size	1	1.	1.14	0.89	0.	0.02	0.	0.	0.89
time (sec)	N/A	0.087	0.082	0.006	0.	0.309	0.	0.	12.532

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	87	72	0	1	0	0	75
normalized size	1	1.	1.78	1.47	0.	0.02	0.	0.	1.53
time (sec)	N/A	0.032	0.08	0.011	0.	0.3	0.	0.	26.121

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	89	72	0	1	0	0	76
normalized size	1	1.	1.75	1.41	0.	0.02	0.	0.	1.49
time (sec)	N/A	0.106	0.086	0.015	0.	0.296	0.	0.	22.739

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	91	74	0	1	0	0	78
normalized size	1	1.	1.72	1.4	0.	0.02	0.	0.	1.47
time (sec)	N/A	0.12	0.085	0.012	0.	0.282	0.	0.	22.717

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	45	31	27	111	0	0	36
normalized size	1	1.	1.12	0.78	0.68	2.78	0.	0.	0.9
time (sec)	N/A	0.058	0.051	0.005	0.852	0.288	0.	0.	8.575

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	78	58	27	124	0	0	68
normalized size	1	1.	1.73	1.29	0.6	2.76	0.	0.	1.51
time (sec)	N/A	0.025	0.042	0.	0.873	0.29	0.	0.	18.018

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	67	50	34	69	0	69	65
normalized size	1	1.	1.56	1.16	0.79	1.6	0.	1.6	1.51
time (sec)	N/A	0.073	0.034	0.017	0.901	0.305	0.	0.294	14.181

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F(-2)	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0	99
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.41
time (sec)	N/A	0.104	0.347	0.2	0.	0.	0.	0.	24.481

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [86] had the largest ratio of [0.5]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	20	0.05
2	A	2	1	1.	18	0.056
3	A	1	0	1.	16	0.
4	A	2	1	1.	20	0.05
5	A	2	1	1.	20	0.05
6	A	3	2	1.	22	0.091
7	A	3	2	1.	20	0.1
8	A	3	2	1.	18	0.111
9	A	3	2	1.	22	0.091
10	A	3	2	1.	22	0.091
11	A	7	6	1.	22	0.273
12	A	6	6	1.	22	0.273
13	A	5	5	1.	22	0.227
14	A	3	3	1.	22	0.136
15	A	7	7	1.	20	0.35
16	A	8	7	1.	18	0.389
17	A	8	7	1.	22	0.318
18	A	8	7	1.	22	0.318

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
19	A	8	7	1.	22	0.318
20	A	7	7	1.	22	0.318
21	A	4	4	1.	22	0.182
22	A	4	4	1.	22	0.182
23	A	4	4	1.	22	0.182
24	A	8	7	1.	22	0.318
25	A	8	7	1.	22	0.318
26	A	8	7	1.	20	0.35
27	A	8	7	1.	18	0.389
28	A	8	7	1.	22	0.318
29	A	8	6	1.	24	0.25
30	A	7	6	1.	22	0.273
31	A	5	5	1.	20	0.25
32	A	4	4	1.	24	0.167
33	A	7	6	1.	24	0.25
34	A	7	6	1.	24	0.25
35	A	5	5	1.	24	0.208
36	A	6	5	1.	24	0.208
37	A	7	5	1.	24	0.208
38	A	10	7	1.	22	0.318
39	A	10	7	1.	20	0.35
40	A	8	7	1.	24	0.292
41	A	6	5	1.	24	0.208
42	A	5	4	1.	24	0.167
43	A	8	7	1.	24	0.292
44	A	8	7	1.	24	0.292
45	A	8	7	1.	24	0.292
46	A	9	8	1.	24	0.333
47	A	7	6	1.	24	0.25
48	A	8	6	1.	24	0.25
49	A	6	6	1.	24	0.25
50	A	4	4	1.	24	0.167
51	A	3	3	1.	22	0.136
52	A	2	2	1.	20	0.1

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	3	3	1.	24	0.125
54	A	5	5	1.	24	0.208
55	A	8	6	1.	24	0.25
56	A	7	6	1.	24	0.25
57	A	6	6	1.	24	0.25
58	A	1	1	1.	24	0.042
59	A	1	1	1.	24	0.042
60	A	3	3	1.	24	0.125
61	A	5	5	1.	22	0.227
62	A	6	5	1.	20	0.25
63	A	7	5	1.	24	0.208
64	A	8	5	1.	24	0.208
65	A	2	1	1.	18	0.056
66	A	2	1	1.	18	0.056
67	A	2	1	1.	16	0.062
68	A	1	0	1.	14	0.
69	A	2	1	1.	18	0.056
70	A	2	1	1.	18	0.056
71	A	2	1	1.	18	0.056
72	A	3	2	1.	20	0.1
73	A	3	2	1.	20	0.1
74	A	4	3	1.	18	0.167
75	A	3	2	1.	16	0.125
76	A	4	3	1.	20	0.15
77	A	3	2	1.	20	0.1
78	A	8	7	1.	20	0.35
79	A	6	5	1.	20	0.25
80	A	7	7	1.	20	0.35
81	A	5	4	1.	20	0.2
82	A	6	6	1.	20	0.3
83	A	4	3	1.	20	0.15
84	A	4	4	1.	20	0.2
85	A	4	3	1.	18	0.167
86	A	8	8	1.	16	0.5

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
87	A	5	4	1.	20	0.2
88	A	9	8	1.	20	0.4
89	A	9	8	1.	20	0.4
90	A	7	5	1.	20	0.25
91	A	8	8	1.	20	0.4
92	A	6	5	1.	20	0.25
93	A	5	5	1.	20	0.25
94	A	5	4	1.	20	0.2
95	A	5	5	1.	20	0.25
96	A	5	4	1.	20	0.2
97	A	5	5	1.	20	0.25
98	A	5	4	1.	20	0.2
99	A	9	8	1.	18	0.444
100	A	6	5	1.	16	0.312
101	A	9	8	1.	20	0.4
102	A	7	5	1.	20	0.25
103	A	9	8	1.	20	0.4
104	A	3	3	1.	20	0.15
105	A	5	5	1.	24	0.208
106	A	5	5	1.	24	0.208
107	A	5	5	1.	24	0.208
108	A	8	7	1.	24	0.292
109	A	8	8	1.	24	0.333
110	A	6	6	1.	24	0.25
111	A	6	5	1.	24	0.208
112	A	6	6	1.	24	0.25
113	A	4	4	1.	24	0.167
114	A	2	2	1.	24	0.083
115	A	2	2	1.	24	0.083
116	A	6	6	1.	24	0.25
117	A	5	5	1.	24	0.208
118	A	3	3	1.	24	0.125
119	A	6	6	1.	24	0.25
120	A	5	5	1.	24	0.208

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
121	A	1	1	1.	34	0.029
122	A	7	4	1.	27	0.148
123	A	2	2	1.	18	0.111
124	A	3	3	1.	18	0.167
125	A	3	3	1.	17	0.176
126	A	5	5	1.	18	0.278
127	A	6	6	1.	18	0.333
128	A	6	6	1.	17	0.353
129	A	2	2	1.	18	0.111
130	A	3	3	1.	18	0.167
131	A	3	3	1.	22	0.136
132	A	3	3	1.	24	0.125
133	A	3	3	1.	20	0.15
134	A	3	3	1.	20	0.15
135	A	3	3	1.	24	0.125
136	A	3	3	1.	26	0.115
137	A	3	3	1.	18	0.167
138	A	3	3	1.	18	0.167
139	A	3	3	1.	20	0.15
140	A	2	2	1.	36	0.056

3 Listing of integrals

3.1 $\int x^2 \left(ax^2+bx^3+cx^4\right) \, dx$

Optimal. Leaf size=25

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

```
[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Rubi [A] time = 0.0200076, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Rubi in Sympy [A] time = 6.17171, size = 19, normalized size = 0.76

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**3+a*x**2),x)

[Out] a*x**5/5 + b*x**6/6 + c*x**7/7
```

Mathematica [A] time = 0.00403179, size = 25, normalized size = 1.

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

Maxima [A] time = 0.743829, size = 26, normalized size = 1.04

$$\frac{1}{7}\,cx^7 + \frac{1}{6}\,bx^6 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Fricas [A] time = 0.238269, size = 1, normalized size = 0.04

$$\frac{1}{7}x^7c + \frac{1}{6}x^6b + \frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/6*x^6*b + 1/5*x^5*a
```

Sympy [A] time = 0.065646, size = 19, normalized size = 0.76

$$\frac{ax^5}{5} + \frac{bx^6}{6} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] a*x**5/5 + b*x**6/6 + c*x**7/7
```

GIAC/XCAS [A] time = 0.260683, size = 26, normalized size = 1.04

$$\frac{1}{7}cx^7 + \frac{1}{6}bx^6 + \frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

3.2 $\int x\left(ax^2+bx^3+cx^4\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rubi [A] time = 0.0167831, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4),x]

[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rubi in Sympy [A] time = 6.01357, size = 19, normalized size = 0.76

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x*(c*x**4+b*x**3+a*x**2),x)

[Out] a*x**4/4 + b*x**5/5 + c*x**6/6
```

Mathematica [A] time = 0.00215061, size = 25, normalized size = 1.

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Maxima [A] time = 0.74847, size = 26, normalized size = 1.04

$$\frac{1}{6}\,cx^6 + \frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^3 + a*x^2)*x,x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Fricas [A] time = 0.256558, size = 1, normalized size = 0.04

$$\frac{1}{6}x^6c + \frac{1}{5}x^5b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^3 + a*x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/5*x^5*b + 1/4*x^4*a
```

Sympy [A] time = 0.068408, size = 19, normalized size = 0.76

$$\frac{ax^4}{4} + \frac{bx^5}{5} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] a*x**4/4 + b*x**5/5 + c*x**6/6
```

GIAC/XCAS [A] time = 0.261343, size = 26, normalized size = 1.04

$$\frac{1}{6}cx^6 + \frac{1}{5}bx^5 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)*x,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

3.3 $\int \left(ax^2+bx^3+cx^4\right) \, dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi [A] time = 0.0111658, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a*x^2 + b*x^3 + c*x^4,x]

[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi in Sympy [A] time = 1.90647, size = 19, normalized size = 0.76

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(c*x**4+b*x**3+a*x**2,x)

[Out] a*x**3/3 + b*x**4/4 + c*x**5/5
```

Mathematica [A] time = 0.0000735961, size = 25, normalized size = 1.

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x^2 + b*x^3 + c*x^4,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(c*x^4+b*x^3+a*x^2,x)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Maxima [A] time = 0.754257, size = 26, normalized size = 1.04

$$\frac{1}{5}\,cx^5 + \frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^3 + a*x^2,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A] time = 0.240485, size = 1, normalized size = 0.04

$$\frac{1}{5}x^5c + \frac{1}{4}x^4b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^3 + a*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/4*x^4*b + 1/3*x^3*a
```

Sympy [A] time = 0.062307, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x**4+b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4 + c*x**5/5
```

GIAC/XCAS [A] time = 0.261066, size = 26, normalized size = 1.04

$$\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^3 + a*x^2,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

3.4 $\int \frac{ax^2+bx^3+cx^4}{x} \, dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rubi [A] time = 0.0149669, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\int x\,dx+\frac{bx^3}{3}+\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)/x,x)

[Out] a*Integral(x, x) + b*x**3/3 + c*x**4/4
```

Mathematica [A] time = 0.0018943, size = 25, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]

[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)/x,x)

[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Maxima [A] time = 0.74278, size = 26, normalized size = 1.04

$$\frac{1}{4}\,cx^4 + \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)/x,x, algorithm="maxima")

[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A] time = 0.260403, size = 26, normalized size = 1.04

$$\frac{1}{4}\,cx^4 + \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)/x,x, algorithm="fricas")

[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A] time = 0.069922, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)/x,x)
```

```
[Out] a*x**2/2 + b*x**3/3 + c*x**4/4
```

GIAC/XCAS [A] time = 0.26214, size = 26, normalized size = 1.04

$$\frac{1}{4}\,cx^4 + \frac{1}{3}\,bx^3 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

3.5 $\int \frac{ax^2+bx^3+cx^4}{x^2}\,dx$

Optimal. Leaf size=20

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Rubi [A] time = 0.0146872, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]

[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$b\int x\,dx+\frac{cx^3}{3}+\int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)/x**2,x)

[Out] b*Integral(x, x) + c*x**3/3 + Integral(a, x)
```

Mathematica [A] time = 0.00161879, size = 20, normalized size = 1.

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)/x^2,x)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3
```

Maxima [A] time = 0.749836, size = 22, normalized size = 1.1

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [A] time = 0.254699, size = 22, normalized size = 1.1

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A] time = 0.067583, size = 15, normalized size = 0.75

$$ax + \frac{bx^2}{2} + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)/x**2,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3
```

GIAC/XCAS [A] time = 0.263675, size = 22, normalized size = 1.1

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

3.6 $\int x^2 \left(ax^2+bx^3+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5
+ (c^2*x^11)/11
```

Rubi [A] time = 0.101585, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5
+ (c^2*x^11)/11
```

Rubi in Sympy [A] time = 13.0142, size = 48, normalized size = 0.89

$$\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{bcx^{10}}{5}+\frac{c^2x^{11}}{11}+x^9\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*
a*c/9 + b**2/9)
```

Mathematica [A] time = 0.0113613, size = 54, normalized size = 1.

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5
+ (c^2*x^11)/11
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{\left(2\,ac + b^2\right)x^9}{9} + \frac{bcx^{10}}{5} + \frac{c^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2
*x^11
```

Maxima [A] time = 0.758578, size = 59, normalized size = 1.09

$$\frac{1}{11}\,c^2x^{11} + \frac{1}{5}\,bcx^{10} + \frac{1}{4}\,abx^8 + \frac{1}{9}\left(b^2 + 2\,ac\right)x^9 + \frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^
9 + 1/7*a^2*x^7
```

Fricas [A] time = 0.231491, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}c^2 + \frac{1}{5}x^{10}cb + \frac{1}{9}x^9b^2 + \frac{2}{9}x^9ca + \frac{1}{4}x^8ba + \frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 1/5*x^10*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 1/4*x^
8*b*a + 1/7*x^7*a^2
```

Sympy [A] time = 0.112542, size = 48, normalized size = 0.89

$$\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{bcx^{10}}{5}+\frac{c^2x^{11}}{11}+x^9\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*
a*c/9 + b**2/9)
```

GIAC/XCAS [A] time = 0.26187, size = 62, normalized size = 1.15

$$\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{9}\,b^2x^9+\frac{2}{9}\,acx^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*
b*x^8 + 1/7*a^2*x^7
```

3.7 $\int x \left(a x^2+b x^3+c x^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)
/9 + (c^2*x^10)/10
```

Rubi [A] time = 0.0684306, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)
/9 + (c^2*x^10)/10
```

Rubi in Sympy [A] time = 12.8389, size = 49, normalized size = 0.91

$$\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{2bcx^9}{9}+\frac{c^2x^{10}}{10}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*
(a*c/4 + b**2/8)
```

Mathematica [A] time = 0.0113031, size = 54, normalized size = 1.

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)
/9 + (c^2*x^10)/10
```

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^6}{6}+\frac{2\,abx^7}{7}+\frac{\left(2\,ac+b^2\right)x^8}{8}+\frac{2\,bcx^9}{9}+\frac{c^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*
x^10
```

Maxima [A] time = 0.757404, size = 59, normalized size = 1.09

$$\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{2}{7}\,abx^7+\frac{1}{8}\left(b^2+2\,ac\right)x^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2*x,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8
 + 1/6*a^2*x^6
```

Fricas [A] time = 0.242693, size = 1, normalized size = 0.02

$$\frac{1}{10}x^{10}c^2+\frac{2}{9}x^9cb+\frac{1}{8}x^8b^2+\frac{1}{4}x^8ca+\frac{2}{7}x^7ba+\frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2*x,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^2 + 2/9*x^9*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 2/7*x^7
*b*a + 1/6*x^6*a^2
```

Sympy [A] time = 0.106732, size = 49, normalized size = 0.91

$$\frac{a^2x^6}{6} + \frac{2abx^7}{7} + \frac{2bcx^9}{9} + \frac{c^2x^{10}}{10} + x^8\left(\frac{ac}{4} + \frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*
(a*c/4 + b**2/8)
```

GIAC/XCAS [A] time = 0.260314, size = 62, normalized size = 1.15

$$\frac{1}{10}\,c^2x^{10} + \frac{2}{9}\,bcx^9 + \frac{1}{8}\,b^2x^8 + \frac{1}{4}\,acx^8 + \frac{2}{7}\,abx^7 + \frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^3 + a*x^2)^2*x,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b
*x^7 + 1/6*a^2*x^6
```

3.8 $\int \left(ax^2+bx^3+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 +
 (c^2*x^9)/9
```

Rubi [A] time = 0.0566453, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 +
 (c^2*x^9)/9
```

Rubi in Sympy [A] time = 21.8952, size = 48, normalized size = 0.89

$$\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{bcx^8}{4}+\frac{c^2x^9}{9}+x^7\left(\frac{2ac}{7}+\frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c
/7 + b**2/7)
```

Mathematica [A] time = 0.00937774, size = 54, normalized size = 1.

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 +
 (c^2*x^9)/9
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{\left(2\,ac+b^2\right)x^7}{7}+\frac{bcx^8}{4}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x
^9
```

Maxima [A] time = 0.747767, size = 65, normalized size = 1.2

$$\frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{7}\,b^2x^7+\frac{1}{5}\,a^2x^5+\frac{1}{21}\left(6\,cx^7+7\,bx^6\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 1/5*a^2*x^5 + 1/21*(6*c
*x^7 + 7*b*x^6)*a
```

Fricas [A] time = 0.233127, size = 1, normalized size = 0.02

$$\frac{1}{9}x^9c^2+\frac{1}{4}x^8cb+\frac{1}{7}x^7b^2+\frac{2}{7}x^7ca+\frac{1}{3}x^6ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 1/4*x^8*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 1/3*x^6*b
*a + 1/5*x^5*a^2
```

Sympy [A] time = 0.10332, size = 48, normalized size = 0.89

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{bcx^8}{4} + \frac{c^2x^9}{9} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c
/7 + b**2/7)
```

GIAC/XCAS [A] time = 0.262102, size = 62, normalized size = 1.15

$$\frac{1}{9}\,c^2x^9 + \frac{1}{4}\,bcx^8 + \frac{1}{7}\,b^2x^7 + \frac{2}{7}\,acx^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x
^6 + 1/5*a^2*x^5
```

3.9 $\int \frac{(ax^2+bx^3+cx^4)^2}{x}\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)
/7 + (c^2*x^8)/8
```

Rubi [A] time = 0.0607494, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)
/7 + (c^2*x^8)/8
```

Rubi in Sympy [A] time = 12.9906, size = 49, normalized size = 0.91

$$\frac{a^2x^4}{4}+\frac{2abx^5}{5}+\frac{2bcx^7}{7}+\frac{c^2x^8}{8}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**2/x,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a
*c/3 + b**2/6)
```

Mathematica [A] time = 0.0093435, size = 54, normalized size = 1.

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)
/7 + (c^2*x^8)/8
```

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{2\,abx^5}{5} + \frac{\left(2\,ac + b^2\right)x^6}{6} + \frac{2\,bcx^7}{7} + \frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^2/x,x)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x
^8
```

Maxima [A] time = 0.747992, size = 59, normalized size = 1.09

$$\frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{2}{5}\,abx^5 + \frac{1}{6}\left(b^2 + 2\,ac\right)x^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 +
 1/4*a^2*x^4
```

Fricas [A] time = 0.265088, size = 59, normalized size = 1.09

$$\frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{2}{5}\,abx^5 + \frac{1}{6}\left(b^2 + 2\,ac\right)x^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 +
 1/4*a^2*x^4
```

Sympy [A] time = 0.109953, size = 49, normalized size = 0.91

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^8}{8} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**2/x,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a
*c/3 + b**2/6)
```

GIAC/XCAS [A] time = 0.262427, size = 62, normalized size = 1.15

$$\frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 2/5*a*b*x
^5 + 1/4*a^2*x^4
```

3.10 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^2} dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5\left(2ac+b^2\right) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 +
 (c^2*x^7)/7
```

Rubi [A] time = 0.0606733, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5\left(2ac+b^2\right) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 +
 (c^2*x^7)/7
```

Rubi in Sympy [A] time = 13.1869, size = 48, normalized size = 0.89

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c
/5 + b**2/5)
```

Mathematica [A] time = 0.0101182, size = 54, normalized size = 1.

$$\frac{a^2x^3}{3} + \frac{1}{5}x^5\left(2ac+b^2\right) + \frac{1}{2}abx^4 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 +
 (c^2*x^7)/7
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{(2\,ac+b^2)\,x^5}{5}+\frac{bcx^6}{3}+\frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^2/x^2,x)
```

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x
^7
```

Maxima [A] time = 0.752253, size = 59, normalized size = 1.09

$$\frac{1}{7}\,c^2x^7+\frac{1}{3}\,bcx^6+\frac{1}{2}\,abx^4+\frac{1}{5}\,(b^2+2\,ac)\,x^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 +
 1/3*a^2*x^3
```

Fricas [A] time = 0.267614, size = 59, normalized size = 1.09

$$\frac{1}{7}\,c^2x^7+\frac{1}{3}\,bcx^6+\frac{1}{2}\,abx^4+\frac{1}{5}\,(b^2+2\,ac)\,x^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 +
 1/3*a^2*x^3
```

Sympy [A] time = 0.107811, size = 48, normalized size = 0.89

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c
/5 + b**2/5)
```

GIAC/XCAS [A] time = 0.260175, size = 62, normalized size = 1.15

$$\frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x
^4 + 1/3*a^2*x^3
```

3.11 $\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=89

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.175082, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a +
 b*x + c*x^2])/(2*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b(-3ac+b^2)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^3\sqrt{-4ac+b^2}}+\frac{\int x\,dx}{c}-\frac{\int b\,dx}{c^2}+\frac{(-ac+b^2)\log(a+bx+cx^2)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] b*(-3*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**3*sq
rt(-4*a*c + b**2)) + Integral(x, x)/c - Integral(b, x)/c**2 + (-a
*c + b**2)*log(a + b*x + c*x**2)/(2*c**3)
```

Mathematica [A] time = 0.196826, size = 84, normalized size = 0.94

$$\frac{\left(b^2-ac\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+cx(cx-2b)}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x
)])/(2*c^3)
```

Maple [A] time = 0.005, size = 132, normalized size = 1.5

$$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{\ln\left(cx^2+bx+a\right)a}{2c^2}+\frac{\ln\left(cx^2+bx+a\right)b^2}{2c^3}+3\frac{ab}{c^2\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)-\frac{b^3}{c^3}\arctan\left((2cx+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/2*x^2/c-b*x/c^2-1/2/c^2*ln(c*x^2+b*x+a)*a+1/2/c^3*ln(c*x^2+b*x+
a)*b^2+3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)
)*a*b-1/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302829, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^{3}-3\,abc\right)\log\left(-\frac{b^{3}-4\,abc+2\,\left(b^{2}c-4\,ac^{2}\right)x-\left(2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{2}+bx+a}\right)-\left(c^{2}x^{2}-2\,bcx+\left(b^{2}-ac\right)\log\left(cx^{2}+bx+a\right)\right)\sqrt{b^{2}-4\,ac}}{2\,\sqrt{b^{2}-4\,ac}c^{3}}, -\frac{2\,\left(b^{3}-3\,abc\right)\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{b^{2}-4\,ac}\right)-\left(c^{2}x^{2}-2\,bcx+\left(b^{2}-ac\right)\log\left(cx^{2}+bx+a\right)\right)\sqrt{-b^{2}+4\,ac}}{2\,\sqrt{-b^{2}+4\,ac}c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*
x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2
 + b*x + a)) - (c^2*x^2 - 2*b*c*x + (b^2 - a*c)*log(c*x^2 + b*x +
 a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^3), -1/2*(2*(b^3 - 3
*a*b*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (
c^2*x^2 - 2*b*c*x + (b^2 - a*c)*log(c*x^2 + b*x + a))*sqrt(-b^2 +
 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)]
```

Sympy [A] time = 2.84633, size = 381, normalized size = 4.28

$$-\frac{bx}{c^{2}}+\left(-\frac{b\sqrt{-4ac+b^{2}}\left(3ac-b^{2}\right)}{2c^{3}\left(4ac-b^{2}\right)}-\frac{ac-b^{2}}{2c^{3}}\right)\log\left(x+\frac{2a^{2}c-ab^{2}+4ac^{3}\left(-\frac{b\sqrt{-4ac+b^{2}}(3ac-b^{2})}{2c^{3}(4ac-b^{2})}-\frac{ac-b^{2}}{2c^{3}}\right)-b^{2}c^{2}\left(-\frac{b\sqrt{-4ac+b^{2}}(3ac-b^{2})}{2c^{3}(4ac-b^{2})}-\frac{ac-b^{2}}{2c^{3}}\right)}{3abc-b^{3}}\right)+\left(\frac{b\sqrt{-4ac+b^{2}}\left(3ac-b^{2}\right)}{2c^{3}\left(4ac-b^{2}\right)}-\frac{ac-b^{2}}{2c^{3}}\right)\log\left(x+\frac{2a^{2}c-ab^{2}+4ac^{3}\left(\frac{b\sqrt{-4ac+b^{2}}(3ac-b^{2})}{2c^{3}(4ac-b^{2})}-\frac{ac-b^{2}}{2c^{3}}\right)-b^{2}c^{2}\left(\frac{b\sqrt{-4ac+b^{2}}(3ac-b^{2})}{2c^{3}(4ac-b^{2})}-\frac{ac-b^{2}}{2c^{3}}\right)}{3abc-b^{3}}\right)+\frac{x^{2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
 - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4
*a*c**3*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b
**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(-b*sqrt(-4*a*c + b**2
)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
)/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c*
*3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a
*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4
*a*c - b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c
 + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2
*c**3)))/(3*a*b*c - b**3)) + x**2/(2*c)
```

GIAC/XCAS [A] time = 0.265261, size = 116, normalized size = 1.3

$$\frac{cx^2 - 2\,bx}{2\,c^2} + \frac{\left(b^2 - ac\right)\ln\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*ln(c*x^2 + b*x + a)/c^3
 - (b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-
b^2 + 4*a*c)*c^3)
```

3.12 $\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=70

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.105264, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rubi in Sympy [A] time = 23.4422, size = 65, normalized size = 0.93

$$-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}-\frac{(-2ac+b^2)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -b*log(a + b*x + c*x**2)/(2*c**2) + x/c - (-2*a*c + b**2)*atanh((
b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.106363, size = 73, normalized size = 1.04

$$\frac{(b^2-2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2
*Sqrt[-b^2 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A] time = 0.004, size = 101, normalized size = 1.4

$$\frac{x}{c}-\frac{b\ln\left(cx^2+bx+a\right)}{2\,c^2}-2\,\frac{a}{c\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^2}{c^2}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x
+b)/(4*a*c-b^2)^(1/2))*a+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278062, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\sqrt{b^2-4\,ac}\left(2\,cx-b\log\left(cx^2+bx+a\right)\right)}{2\,\sqrt{b^2-4\,ac}c^2},\ \frac{2\,\left(b^2-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x +
 (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 +
b*x + a)) - sqrt(b^2 - 4*a*c)*(2*c*x - b*log(c*x^2 + b*x + a)))/(
sqrt(b^2 - 4*a*c)*c^2), 1/2*(2*(b^2 - 2*a*c)*arctan(-sqrt(-b^2 +
4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(2*c*x - b
*log(c*x^2 + b*x + a)))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 2.32972, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\left(-\frac{b}{2c^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{2c^2\left(4ac-b^2\right)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{2c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c
- b**2)))*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b
**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2
) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2))))/
```

```
(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c**2*(-b/(2*c**2)
 + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2))) +
b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*
(4*a*c - b**2))))/(2*a*c - b**2)) + x/c
```

GIAC/XCAS [A] time = 0.264203, size = 90, normalized size = 1.29

$$\frac{x}{c} - \frac{b\ln\left(cx^2 + bx + a\right)}{2\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*ln(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x
 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.13 $\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])
+ Log[a + b*x + c*x^2]/(2*c)
```

Rubi [A] time = 0.0704529, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c])
+ Log[a + b*x + c*x^2]/(2*c)
```

Rubi in Sympy [A] time = 13.425, size = 49, normalized size = 0.88

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c\sqrt{-4ac+b^2}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] b*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c*sqrt(-4*a*c + b**2))
+ log(a + b*x + c*x**2)/(2*c)
```

Mathematica [A] time = 0.0565976, size = 57, normalized size = 1.02

$$\frac{\log(a+x(b+cx))-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 + Log[a + x*(b + c*x)])/(2*c)
```

Maple [A] time = 0.003, size = 56, normalized size = 1.

$$\frac{\ln\left(cx^2+bx+a\right)}{2\,c}-\frac{b}{c}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286598, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x+(2\,c^2x^2+2\,bcx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)+\sqrt{b^2-4\,ac}\log\left(cx^2+bx+a\right)}{2\,\sqrt{b^2-4\,ac}c}, -\frac{2\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}\log\left(cx^2+bx+a\right)}{2\,\sqrt{-b^2+4\,ac}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 +
 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + s
qrt(b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(sqrt(b^2 - 4*a*c)*c), -1/
2*(2*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - sq
rt(-b^2 + 4*a*c)*log(c*x^2 + b*x + a))/(sqrt(-b^2 + 4*a*c)*c)]
```

Sympy [A] time = 0.894197, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (
-4*a*c*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) +
2*a + b**2*(-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)
))/b) + (b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*lo
g(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*
c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/
(2*c)))/b)
```

GIAC/XCAS [A] time = 0.263978, size = 74, normalized size = 1.32

$$-\frac{b \arctan\left(\frac{2 \, c x + b}{\sqrt{-b^{2} + 4 \, a c}}\right)}{\sqrt{-b^{2} + 4 \, a c} c} + \frac{\ln\left(c x^{2} + b x + a\right)}{2 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
+ 1/2*ln(c*x^2 + b*x + a)/c
```

3.14 $\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0403207, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a*x^2 + b*x^3 + c*x^4),x]

[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 6.86556, size = 34, normalized size = 1.

$$-\frac{2\,\mathrm{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**3+a*x**2),x)

[Out] -2*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
```

Mathematica [A] time = 0.0108157, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.002, size = 35, normalized size = 1.

$$2\frac{1}{\sqrt{4ac-b^2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277778, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{b^3-4abc+2(b^2c-4ac^2)x-(2c^2x^2+2bcx+b^2-2ac)\sqrt{b^2-4ac}}{cx^2+bx+a}\right)}{\sqrt{b^2-4ac}}, \frac{2\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)}{\sqrt{-b^2+4ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c
*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a))/sqrt(b^2
- 4*a*c), 2*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
/sqrt(-b^2 + 4*a*c)]
```

Sympy [A] time = 0.531573, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2))
+ b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**
2))*log(x + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c
- b**2)) + b)/(2*c))
```

GIAC/XCAS [A] time = 0.263683, size = 46, normalized size = 1.35

$$\frac{2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.15 $\int \frac{x}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c])
+ Log[x]/a - Log[a + b*x + c*x^2]/(2*a)
```

Rubi [A] time = 0.0913724, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c])
+ Log[x]/a - Log[a + b*x + c*x^2]/(2*a)
```

Rubi in Sympy [A] time = 20.9462, size = 54, normalized size = 0.87

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a\sqrt{-4ac+b^2}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] b*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a*sqrt(-4*a*c + b**2))
+ log(x)/a - log(a + b*x + c*x**2)/(2*a)
```

Mathematica [A] time = 0.112108, size = 61, normalized size = 0.98

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log(a+x(b+cx))-2\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - 2*Log[x] + Log[a + x*(b + c*x)])/(2*a)
```

Maple [A] time = 0.006, size = 62, normalized size = 1.

$$\frac{\ln(x)}{a}-\frac{\ln\left(cx^2+bx+a\right)}{2\,a}-\frac{b}{a}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a-1/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305113, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x+(2\,c^2x^2+2\,bcx+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\sqrt{b^2-4\,ac}\left(\log\left(cx^2+bx+a\right)-2\,\log\left(x\right)\right)}{2\,\sqrt{b^2-4\,ac}a},\right.$$
$$\left.-\frac{2\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+\sqrt{-b^2+4\,ac}\left(\log\left(cx^2+bx+a\right)-2\,\log\left(x\right)\right)}{2\,\sqrt{-b^2+4\,ac}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 +
 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) - s
qrt(b^2 - 4*a*c)*(log(c*x^2 + b*x + a) - 2*log(x)))/(sqrt(b^2 - 4
*a*c)*a), -1/2*(2*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 -
 4*a*c)) + sqrt(-b^2 + 4*a*c)*(log(c*x^2 + b*x + a) - 2*log(x)))/
(sqrt(-b^2 + 4*a*c)*a)]
```

Sympy [A] time = 6.55436, size = 564, normalized size = 9.1

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2a\,(4ac-b^2)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}\right.}{9abc^2-2b^3c}\right.$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{2a\,(4ac-b^2)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right.}{9abc^2-2b^3c}\right.$$
$$+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (
24*a**4*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*
```

```
a))**2 - 14*a**3*b**2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**
2)) - 1/(2*a))**2 - 12*a**3*c**2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*
a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-b*sqrt(-4*a*c + b**2)/(2*
a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-b*sqrt(-4*a*c +
 b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2
*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2
*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*
c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*
sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*
c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a
**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**
2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1
/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

GIAC/XCAS [A] time = 0.264624, size = 84, normalized size = 1.35

$$-\frac{b \arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} - \frac{\ln\left(cx^2+bx+a\right)}{2\,a} + \frac{\ln\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
- 1/2*ln(c*x^2 + b*x + a)/a + ln(abs(x))/a
```

3.16 $\int \frac{1}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*
x^2])/(2*a^2)
```

Rubi [A] time = 0.20375, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]
])/(a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*
x^2])/(2*a^2)
```

Rubi in Sympy [A] time = 45.6167, size = 75, normalized size = 0.93

$$-\frac{1}{ax}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -1/(a*x) - b*log(x)/a**2 + b*log(a + b*x + c*x**2)/(2*a**2) - (-2
*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**2*sqrt(-4
*a*c + b**2))
```

Mathematica [A] time = 0.142388, size = 77, normalized size = 0.95

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log(a+x(b+cx))-\frac{2a}{x}-2b\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*
a^2)
```

Maple [A] time = 0.006, size = 112, normalized size = 1.4

$$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(cx^2+bx+a\right)}{2\,a^2}-2\,\frac{c}{a\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^2}{a^2}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-2/a/(4*a*c-b^2)^(1/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c+1/a^2/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291361, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)x\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)-\left(bx\log\left(cx^2+bx+a\right)-2\,bx\log\left(x\right)-2\,a\right)\sqrt{b^2-}}{2\,\sqrt{b^2-4\,ac}a^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*x*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x
 + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2
+ b*x + a)) - (b*x*log(c*x^2 + b*x + a) - 2*b*x*log(x) - 2*a)*sqr
t(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^2*x), 1/2*(2*(b^2 - 2*a*c)*x
*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b*x*log
(c*x^2 + b*x + a) - 2*b*x*log(x) - 2*a)*sqrt(-b^2 + 4*a*c))/(sqrt
(-b^2 + 4*a*c)*a^2*x)]
```

Sympy [A] time = 12.1407, size = 862, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c -
 b**2)))*log(x + (-28*a**6*b*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**
2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b
/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b
**2)))**2 - 4*a**5*c**3*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(2*a**2*(4*a*c - b**2))) - 2*a**4*b**5*(b/(2*a**2) - sqrt
(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 3*a*
*4*b**2*c**2*(b/(2*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*
a**2*(4*a*c - b**2))) + a**3*b**4*c*(b/(2*a**2) - sqrt(-4*a*c + b
**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25
*a**2*b**3*c**2 - 14*a*b**5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b*
*2*c**3 - 12*a*b**4*c**2 + 2*b**6*c)) + (b/(2*a**2) + sqrt(-4*a*c
 + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))*log(x + (-28*a**
6*b*c**2*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2
*(4*a*c - b**2)))**2 + 15*a**5*b**3*c*(b/(2*a**2) + sqrt(-4*a*c +
 b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2)))**2 - 4*a**5*c**3*(
b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c -
b**2))) - 2*a**4*b**5*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c -
b**2)/(2*a**2*(4*a*c - b**2)))**2 - 3*a**4*b**2*c**2*(b/(2*a**2)
+ sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a**2*(4*a*c - b**2))) + a
```

```
**3*b**4*c*(b/(2*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*a*
*2*(4*a*c - b**2))) - 4*a**3*b*c**3 + 25*a**2*b**3*c**2 - 14*a*b*
*5*c + 2*b**7)/(2*a**3*c**4 + 15*a**2*b**2*c**3 - 12*a*b**4*c**2
+ 2*b**6*c)) - 1/(a*x) - b*log(x)/a**2
```

GIAC/XCAS [A] time = 0.263936, size = 107, normalized size = 1.32

$$\frac{b\ln\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\ln\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*b*ln(c*x^2 + b*x + a)/a^2 - b*ln(abs(x))/a^2 + (b^2 - 2*a*c)*
arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) -
 1/(a*x)
```

3.17 $\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx$

Optimal. Leaf size=104

$$-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])
/a^3 - ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)
```

Rubi [A] time = 0.30891, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$

$$-\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])
/a^3 - ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*a^3)
```

Rubi in Sympy [A] time = 44.2823, size = 97, normalized size = 0.93

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b\left(-3ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^3\sqrt{-4ac+b^2}}+\frac{\left(-ac+b^2\right)\log\left(x\right)}{a^3}-\frac{\left(-ac+b^2\right)\log\left(a+bx+cx^2\right)}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -1/(2*a*x**2) + b/(a**2*x) + b*(-3*a*c + b**2)*atanh((b + 2*c*x)/
sqrt(-4*a*c + b**2))/(a**3*sqrt(-4*a*c + b**2)) + (-a*c + b**2)*l
og(x)/a**3 - (-a*c + b**2)*log(a + b*x + c*x**2)/(2*a**3)
```

Mathematica [A] time = 0.254644, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2}+2\log(x)\left(b^2-ac\right)+\left(ac-b^2\right)\log(a+x(b+cx))-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{2ab}{x}}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (
-b^2 + a*c)*Log[a + x*(b + c*x)])/(2*a^3)
```

Maple [A] time = 0.007, size = 150, normalized size = 1.4

$$-\frac{1}{2\,ax^2}-\frac{\ln(x)\,c}{a^2}+\frac{\ln(x)\,b^2}{a^3}+\frac{b}{a^2x}+\frac{c\ln\left(cx^2+bx+a\right)}{2\,a^2}-\frac{\ln\left(cx^2+bx+a\right)\,b^2}{2\,a^3}$$
$$+3\,\frac{bc}{a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-\frac{b^3}{a^3}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/2/a/x^2-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+b/a^2/x+1/2/a^2*c*ln(c*x
^2+b*x+a)-1/2/a^3*ln(c*x^2+b*x+a)*b^2+3/a^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*c-1/a^3/(4*a*c-b^2)^(1/2)*arct
an((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.320145, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^3-3\,abc\right)x^2\log\left(-\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x-\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)+\left(\left(b^2-ac\right)x^2\log\left(cx^2+bx+a\right)-2\left(b^2-ac\right)\right.}{2\sqrt{b^2-4\,ac}a^3x^2}\right.$$

$$-\frac{2\left(b^3-3\,abc\right)x^2\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+\left(\left(b^2-ac\right)x^2\log\left(cx^2+bx+a\right)-2\left(b^2-ac\right)x^2\log\left(x\right)-2\,abx+a^2\right)\sqrt{-b^2}}{2\sqrt{-b^2+4\,ac}a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)*x),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*x^2*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c
*x^2 + b*x + a)) + ((b^2 - a*c)*x^2*log(c*x^2 + b*x + a) - 2*(b^2
 - a*c)*x^2*log(x) - 2*a*b*x + a^2)*sqrt(b^2 - 4*a*c))/(sqrt(b^2
- 4*a*c)*a^3*x^2), -1/2*(2*(b^3 - 3*a*b*c)*x^2*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2 - a*c)*x^2*log(c*x^2
+ b*x + a) - 2*(b^2 - a*c)*x^2*log(x) - 2*a*b*x + a^2)*sqrt(-b^2
+ 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3*x^2)]
```

Sympy [A] time = 17.2672, size = 1525, normalized size = 14.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) +
(a*c - b**2)/(2*a**3))*log(x + (24*a**9*c**3*(-b*sqrt(-4*a*c + b*
*2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3
))**2 - 42*a**8*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 + 17*a**7*b**
4*c*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)
) + (a*c - b**2)/(2*a**3))**2 + 12*a**7*c**4*(-b*sqrt(-4*a*c + b*
*2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3
)) - 2*a**6*b**6*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(
4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 - 15*a**6*b**2*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a
*c - b**2)/(2*a**3)) + 7*a**5*b**4*c**2*(-b*sqrt(-4*a*c + b**2)*(
3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) -
12*a**5*c**5 - a**4*b**6*c*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)
```

```
/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 63*a**4*b**2*
c**4 - 103*a**3*b**4*c**3 + 70*a**2*b**6*c**2 - 20*a*b**8*c + 2*b
**10)/(27*a**4*b*c**5 - 63*a**3*b**3*c**4 + 54*a**2*b**5*c**3 - 1
8*a*b**7*c**2 + 2*b**9*c)) + (b*sqrt(-4*a*c + b**2)*(3*a*c - b**2
)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))*log(x + (24*a*
*9*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b*
*2)) + (a*c - b**2)/(2*a**3))**2 - 42*a**8*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/
(2*a**3))**2 + 17*a**7*b**4*c*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**
2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2 + 12*a**7*
c**4*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)
) + (a*c - b**2)/(2*a**3)) - 2*a**6*b**6*(b*sqrt(-4*a*c + b**2)*(
3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3))**2
 - 15*a**6*b**2*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**
3*(4*a*c - b**2)) + (a*c - b**2)/(2*a**3)) + 7*a**5*b**4*c**2*(b*
sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c
 - b**2)/(2*a**3)) - 12*a**5*c**5 - a**4*b**6*c*(b*sqrt(-4*a*c +
b**2)*(3*a*c - b**2)/(2*a**3*(4*a*c - b**2)) + (a*c - b**2)/(2*a*
*3)) + 63*a**4*b**2*c**4 - 103*a**3*b**4*c**3 + 70*a**2*b**6*c**2
 - 20*a*b**8*c + 2*b**10)/(27*a**4*b*c**5 - 63*a**3*b**3*c**4 + 5
4*a**2*b**5*c**3 - 18*a*b**7*c**2 + 2*b**9*c)) + (-a + 2*b*x)/(2*
a**2*x**2) - (a*c - b**2)*log(x + (-12*a**5*c**5 + 63*a**4*b**2*c
**4 - 12*a**4*c**4*(a*c - b**2) - 103*a**3*b**4*c**3 + 15*a**3*b*
*2*c**3*(a*c - b**2) + 24*a**3*c**3*(a*c - b**2)**2 + 70*a**2*b**
6*c**2 - 7*a**2*b**4*c**2*(a*c - b**2) - 42*a**2*b**2*c**2*(a*c -
 b**2)**2 - 20*a*b**8*c + a*b**6*c*(a*c - b**2) + 17*a*b**4*c*(a*
c - b**2)**2 + 2*b**10 - 2*b**6*(a*c - b**2)**2)/(27*a**4*b*c**5
- 63*a**3*b**3*c**4 + 54*a**2*b**5*c**3 - 18*a*b**7*c**2 + 2*b**9
*c))/a**3
```

GIAC/XCAS [A] time = 0.264168, size = 142, normalized size = 1.37

$$-\frac{(b^2 - ac)\ln(cx^2 + bx + a)}{2a^3} + \frac{(b^2 - ac)\ln(|x|)}{a^3} - \frac{(b^3 - 3abc)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2 + 4ac}a^3} + \frac{2abx - a^2}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)*x),x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*ln(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*ln(abs(x))
/a^3 - (b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sq
rt(-b^2 + 4*a*c)*a^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

3.18 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)} dx$

Optimal. Leaf size=137

$$\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}-\frac{b\log(x)(b^2-2ac)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*
b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*
c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rubi [A] time = 0.390557, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}-\frac{b\log(x)(b^2-2ac)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*
b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*
c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rubi in Sympy [A] time = 57.3996, size = 129, normalized size = 0.94

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{-ac+b^2}{a^3x}-\frac{b(-2ac+b^2)\log(x)}{a^4}+\frac{b(-2ac+b^2)\log(a+bx+cx^2)}{2a^4}-\frac{(2a^2c^2-4ab^2c+b^4)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^4\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -1/(3*a*x**3) + b/(2*a**2*x**2) - (-a*c + b**2)/(a**3*x) - b*(-2*
a*c + b**2)*log(x)/a**4 + b*(-2*a*c + b**2)*log(a + b*x + c*x**2)
/(2*a**4) - (2*a**2*c**2 - 4*a*b**2*c + b**4)*atanh((b + 2*c*x)/s
qrt(-4*a*c + b**2))/(a**4*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.173341, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3}+\frac{6\left(2a^2c^2-4ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{3a^2b}{x^2}-6\log(x)\left(b^3-2abc\right)+3\left(b^3-2abc\right)\log(a+x(b+cx))+\frac{6a\left(ac-b^2\right)}{x}}{6a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 -
4*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sq
rt[-b^2 + 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*L
og[a + x*(b + c*x)])/(6*a^4)
```

Maple [A] time = 0.008, size = 214, normalized size = 1.6

$$-\frac{1}{3\,ax^3}+\frac{c}{a^2x}-\frac{b^2}{a^3x}+2\,\frac{b\ln(x)\,c}{a^3}-\frac{b^3\ln(x)}{a^4}+\frac{b}{2\,a^2x^2}-\frac{c\ln\left(cx^2+bx+a\right)b}{a^3}+\frac{\ln\left(cx^2+bx+a\right)b^3}{2\,a^4}+2\,\frac{c^2}{a^2\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)-4\,\frac{b^2c}{a^3\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)+\frac{b^4}{a^4}\arctan\left((2\,cx+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^3+a*x^2),x)
```

```
[Out] -1/3/a/x^3+1/a^2/x*c-1/a^3/x*b^2+2*b/a^3*ln(x)*c-b^3/a^4*ln(x)+1/
2*b/a^2/x^2-1/a^3*c*ln(c*x^2+b*x+a)*b+1/2/a^4*ln(c*x^2+b*x+a)*b^3
+2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c^2-
4/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^2*c
+1/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.367374, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^4-4\,ab^2c+2\,a^2c^2\right)x^3\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x-\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)+\left(3\left(b^3-2\,abc\right)x^3\log\left(cx^2+bx+a\right)\right.}{6\sqrt{b^2-4\,ac}a^4x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*log(-(b^3 - 4*a*b*c + 2
*(b^2*c - 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b
^2 - 4*a*c))/(c*x^2 + b*x + a)) + (3*(b^3 - 2*a*b*c)*x^3*log(c*x^
2 + b*x + a) - 6*(b^3 - 2*a*b*c)*x^3*log(x) + 3*a^2*b*x - 2*a^3 -
 6*(a*b^2 - a^2*c)*x^2)*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^4
*x^3), 1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^3*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (3*(b^3 - 2*a*b*c)*x^3*log
(c*x^2 + b*x + a) - 6*(b^3 - 2*a*b*c)*x^3*log(x) + 3*a^2*b*x - 2*
a^3 - 6*(a*b^2 - a^2*c)*x^2)*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*a^4*x^3)]
```

Sympy [A] time = 28.0519, size = 2105, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 -
4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))*log(x + (-52*a**11*b*
```

```
c**3*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c*
*2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 57*a**10*b*
*3*c**2*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2
*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 - 19*a**9*
b**5*c*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*
c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 4*a**9*c*
*5*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2
 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 2*a**8*b**7*(-b*
(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*
b**2*c + b**4)/(2*a**4*(4*a*c - b**2)))**2 + 23*a**8*b**2*c**4*(-
b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) - 26*a**7*b**4*c**3*(-b
*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a
*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 9*a**6*b**6*c**2*(-b*(
2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b
**2*c + b**4)/(2*a**4*(4*a*c - b**2))) - 8*a**6*b*c**6 - a**5*b**
8*c*(-b*(2*a*c - b**2)/(2*a**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**
2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))) + 166*a**5*b**3*c
**5 - 361*a**4*b**5*c**4 + 312*a**3*b**7*c**3 - 130*a**2*b**9*c**
2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c**7 + 60*a**5*b**2*c**6 - 20
7*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b**8*c**3 + 24*a
*b**10*c**2 - 2*b**12*c)) + (-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4
*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b
**2)))*log(x + (-52*a**11*b*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sq
rt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*
c - b**2)))**2 + 57*a**10*b**3*c**2*(-b*(2*a*c - b**2)/(2*a**4) +
 sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4
*a*c - b**2)))**2 - 19*a**9*b**5*c*(-b*(2*a*c - b**2)/(2*a**4) +
sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*
a*c - b**2)))**2 + 4*a**9*c**5*(-b*(2*a*c - b**2)/(2*a**4) + sqrt
(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c
- b**2))) + 2*a**8*b**7*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c
 + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2)
))**2 + 23*a**8*b**2*c**4*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a
*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**
2))) - 26*a**7*b**4*c**3*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*
c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2
))) + 9*a**6*b**6*c**2*(-b*(2*a*c - b**2)/(2*a**4) + sqrt(-4*a*c
+ b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c - b**2))
) - 8*a**6*b*c**6 - a**5*b**8*c*(-b*(2*a*c - b**2)/(2*a**4) + sqr
t(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*a**4*(4*a*c
 - b**2))) + 166*a**5*b**3*c**5 - 361*a**4*b**5*c**4 + 312*a**3*b
**7*c**3 - 130*a**2*b**9*c**2 + 26*a*b**11*c - 2*b**13)/(2*a**6*c
**7 + 60*a**5*b**2*c**6 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4
 - 108*a**2*b**8*c**3 + 24*a*b**10*c**2 - 2*b**12*c)) + (-2*a**2
+ 3*a*b*x + x**2*(6*a*c - 6*b**2))/(6*a**3*x**3) + b*(2*a*c - b**
2)*log(x + (-8*a**6*b*c**6 + 166*a**5*b**3*c**5 + 4*a**5*b*c**5*(
2*a*c - b**2) - 361*a**4*b**5*c**4 + 23*a**4*b**3*c**4*(2*a*c - b
**2) + 312*a**3*b**7*c**3 - 26*a**3*b**5*c**3*(2*a*c - b**2) - 52
*a**3*b**3*c**3*(2*a*c - b**2)**2 - 130*a**2*b**9*c**2 + 9*a**2*b
**7*c**2*(2*a*c - b**2) + 57*a**2*b**5*c**2*(2*a*c - b**2)**2 + 2
6*a*b**11*c - a*b**9*c*(2*a*c - b**2) - 19*a*b**7*c*(2*a*c - b**2
)**2 - 2*b**13 + 2*b**9*(2*a*c - b**2)**2)/(2*a**6*c**7 + 60*a**5
*b**2*c**6 - 207*a**4*b**4*c**5 + 224*a**3*b**6*c**4 - 108*a**2*b
```

```
**8*c**3 + 24*a*b**10*c**2 - 2*b**12*c))/a**4
```

GIAC/XCAS [A] time = 0.263691, size = 184, normalized size = 1.34

$$\frac{\left(b^3-2\,abc\right)\ln\left(cx^2+bx+a\right)}{2\,a^4}-\frac{\left(b^3-2\,abc\right)\ln\left(|x|\right)}{a^4}+\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a^4}+\frac{3\,a^2bx-2\,a^3-6\,\left(ab^2-a^2c\right)x^2}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*ln(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*ln(
abs(x))/a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a
^3 - 6*(a*b^2 - a^2*c)*x^2)/(a^4*x^3)
```

3.19 $\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c
)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^
4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3
```

Rubi [A] time = 0.331493, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$

$$-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{3/2}}+\frac{2x\left(b^2-3ac\right)}{c^2\left(b^2-4ac\right)}-\frac{bx^2}{c\left(b^2-4ac\right)}+\frac{x^3(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c
)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^
4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]]
)/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2b\int x\,dx}{c\left(-4ac+b^2\right)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}+\frac{x^3\left(2a+bx\right)}{\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}+\frac{2x\left(-3ac+b^2\right)}{c^2\left(-4ac+b^2\right)}-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^3\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*b*Integral(x, x)/(c*(-4*a*c + b**2)) - b*log(a + b*x + c*x**2)
/c**3 + x**3*(2*a + b*x)/((-4*a*c + b**2)*(a + b*x + c*x**2)) + 2
*x*(-3*a*c + b**2)/(c**2*(-4*a*c + b**2)) - 2*(6*a**2*c**2 - 6*a*
b**2*c + b**4)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**3*(-4*a
*c + b**2)**(3/2))
```

Mathematica [A] time = 0.311148, size = 132, normalized size = 0.88

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a^2c(3b-2cx)-ab^2(b-4cx)+b^4(-x)}{\left(b^2-4ac\right)(a+x(b+cx))}-b\log(a+x(b+cx))+cx}{c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2
 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*A
rcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*L
og[a + x*(b + c*x)])/c^3
```

Maple [B] time = 0.015, size = 569, normalized size = 3.8

$$\frac{x}{c^2}+2\,\frac{a^2x}{c\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-4\,\frac{axb^2}{c^2\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}$$
$$+\frac{xb^4}{c^3\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}-3\,\frac{a^2b}{c^2\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}+\frac{ab^3}{c^3\left(cx^2+bx+a\right)\left(4\,ac-b^2\right)}$$
$$-4\,\frac{\ln\left(\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)ab}{\left(4\,ac-b^2\right)c^2}+\frac{\ln\left(\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)b^3}{c^3\left(4\,ac-b^2\right)}$$
$$-12\,\frac{a^2}{c\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$
$$+12\,\frac{ab^2}{c^2\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$
$$-2\,\frac{b^4}{c^3\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] x/c^2+2/c/(c*x^2+b*x+a)/(4*a*c-b^2)*x*a^2-4/c^2/(c*x^2+b*x+a)/(4*
a*c-b^2)*x*a*b^2+1/c^3/(c*x^2+b*x+a)/(4*a*c-b^2)*x*b^4-3/c^2/(c*x
^2+b*x+a)*a^2*b/(4*a*c-b^2)+1/c^3/(c*x^2+b*x+a)*a*b^3/(4*a*c-b^2)
-4/c^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^2+b*x+a))*a*b+1/c^3/(4*a*c
-b^2)*ln((4*a*c-b^2)*(c*x^2+b*x+a))*b^3-12/c/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2
)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a^2+12/c^2
/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*
c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b
^6)^(1/2))*a*b^2-2/c^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^
2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305713, size = 1, normalized size = 0.01

$$\left[-\frac{\left(ab^4-6\,a^2b^2c+6\,a^3c^2+\left(b^4c-6\,ab^2c^2+6\,a^2c^3\right)x^2+\left(b^5-6\,ab^3c+6\,a^2bc^2\right)x\right)\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x+\left(2\,c^2x^2+2\,bcx+b\right.}{cx^2+bx+a}\right.}{(a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-((a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^
2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x)*log((b^3 - 4*a*b*
c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*s
qrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + (a*b^3 - 3*a^2*b*c - (b^2*
c^2 - 4*a*c^3)*x^3 - (b^3*c - 4*a*b*c^2)*x^2 + (b^4 - 5*a*b^2*c +
 6*a^2*c^2)*x + (a*b^3 - 4*a^2*b*c + (b^3*c - 4*a*b*c^2)*x^2 + (b
^4 - 4*a*b^2*c)*x)*log(c*x^2 + b*x + a))*sqrt(b^2 - 4*a*c))/((a*b
^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c
^4)*x)*sqrt(b^2 - 4*a*c)), (2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 +
(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*
```

```
b*c^2)*x)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 (a*b^3 - 3*a^2*b*c - (b^2*c^2 - 4*a*c^3)*x^3 - (b^3*c - 4*a*b*c^
2)*x^2 + (b^4 - 5*a*b^2*c + 6*a^2*c^2)*x + (a*b^3 - 4*a^2*b*c + (
b^3*c - 4*a*b*c^2)*x^2 + (b^4 - 4*a*b^2*c)*x)*log(c*x^2 + b*x + a
))*sqrt(-b^2 + 4*a*c))/((a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c
^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 5.99094, size = 842, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b
**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c
- b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a*b**
2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**
2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)
) + (-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c
 + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*(-b/c**3 + sqrt(-(4*
a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*
c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6))) - b**4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6
*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b
**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4*a*b**2*c + b**
4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4) + x*
(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

GIAC/XCAS [A] time = 0.268717, size = 217, normalized size = 1.45

$$\frac{2\left(b^4-6\,ab^2c+6\,a^2c^2\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2}-\frac{b\ln\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{\left(b^4-4\,ab^2c+2\,a^2c^2\right)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*ln(c*x
^2 + b*x + a)/c^3 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 -
 3*a^2*b*c)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

3.20 $\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=114

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a
+ b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2
)
```

Rubi [A] time = 0.208615, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a
+ b*x + c*x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2
)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\left(-6ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{c^2\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{x^2\left(2a+bx\right)}{\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}-\frac{\int b\,dx}{c\left(-4ac+b^2\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] b*(-6*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(c**2*(-
4*a*c + b**2)**(3/2)) + x**2*(2*a + b*x)/((-4*a*c + b**2)*(a + b*
x + c*x**2)) - Integral(b, x)/(c*(-4*a*c + b**2)) + log(a + b*x +
```

```
c*x**2)/(2*c**2)
```

Mathematica [A] time = 0.232369, size = 109, normalized size = 0.96

$$\frac{\frac{2\left(-2a^2c+ab(b-3cx)+b^3x\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [B] time = 0.013, size = 330, normalized size = 2.9

$$\frac{1}{cx^2+bx+a}\left(\frac{b\left(3\,ac-b^2\right)x}{\left(4\,ac-b^2\right)c^2}+\frac{a\left(2\,ac-b^2\right)}{\left(4\,ac-b^2\right)c^2}\right)+\frac{\ln\left(c\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)}{2\,c^2}$$
$$-6\,\frac{ab}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\arctan\left(\frac{2\,c^2\left(4\,ac-b^2\right)x+c\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)$$
$$+\frac{b^3}{c}\arctan\left(\left(2\,c^2\left(4\,ac-b^2\right)x+c\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(
c*x^2+b*x+a)+1/2/c^2*ln(c*(4*a*c-b^2)*(c*x^2+b*x+a))-6/(64*a^3*c^
5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c
-b^2)*x+c*(4*a*c-b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-
b^6*c^2)^(1/2))*a*b+1/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6
*c^2)^(1/2)*arctan((2*c^2*(4*a*c-b^2)*x+c*(4*a*c-b^2)*b)/(64*a^3*
c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2))*b^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.32188, size = 1, normalized size = 0.01

$$\left[\frac{\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\log\left(\frac{b^3 - 4\,abc + 2\,\left(b^2c - 4\,ac^2\right)x + \left(2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac\right)\sqrt{b^2 - 4\,ac}}{cx^2 + bx + a}\right) + \left(2\,ab^2 - 4\,a^2\right.}{2\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^2 + (b^3c^2}\right.$$

$$-\frac{2\,\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\arctan\left(-\frac{\sqrt{-b^2 + 4\,ac}(2\,cx + b)}{b^2 - 4\,ac}\right) - \left(2\,ab^2 - 4\,a^2c + 2\,\left(b^3 - 3\,abc\right)x + \left(ab^2\right.\right.}{2\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^2 + (b^3c^2 - 4\,abc^3)x)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b
^2*c)*x)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) +
(2*a*b^2 - 4*a^2*c + 2*(b^3 - 3*a*b*c)*x + (a*b^2 - 4*a^2*c + (b^
2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*log(c*x^2 + b*x + a))*sqr
t(b^2 - 4*a*c))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2
 + (b^3*c^2 - 4*a*b*c^3)*x)*sqrt(b^2 - 4*a*c)), -1/2*(2*(a*b^3 -
6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*arctan
(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*a*b^2 - 4*a^
2*c + 2*(b^3 - 3*a*b*c)*x + (a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*
x^2 + (b^3 - 4*a*b*c)*x)*log(c*x^2 + b*x + a))*sqrt(-b^2 + 4*a*c)
)/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 -
4*a*b*c^3)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 4.43996, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-(4ac-b^2)^3}\,(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2}{6abc-b^3}\right.$$

$$+\left(\frac{b\sqrt{-(4ac-b^2)^3}\,(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{2c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2-b}{6abc-b^3}\right.$$

$$+\frac{2a^2c-ab^2+x(3abc-b^3)}{4a^2c^3-ab^2c^2+x^2(4ac^4-b^2c^3)+x(4abc^3-b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x +
 (-16*a**2*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c*
*2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(
2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*
(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*a*c -
 b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*s
qrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16
*a**2*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2
)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c -
a*b**2 + x*(3*a*b*c - b**3))/(4*a**2*c**3 - a*b**2*c**2 + x**2*(4
*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b**3*c**2))
```

GIAC/XCAS [A] time = 0.270293, size = 169, normalized size = 1.48

$$-\frac{\left(b^{3}-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{(b^{2}c^{2}-4\,ac^{3})\sqrt{-b^{2}+4\,ac}}+\frac{\ln\left(cx^{2}+bx+a\right)}{2\,c^{2}}+\frac{ab^{2}-2\,a^{2}c+\left(b^{3}-3\,abc\right)x}{(cx^{2}+bx+a)(b^{2}-4\,ac)c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2
 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/2*ln(c*x^2 + b*x + a)/c^2 + (
a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*
a*c)*c^2)
```

3.21 $\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=67

$$\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[
(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0743669, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[
(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 13.8881, size = 61, normalized size = 0.91

$$\frac{4a\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{x\,(2a+bx)}{(-4ac+b^2)\,(a+bx+cx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] 4*a*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/2)
 + x*(2*a + b*x)/((-4*a*c + b**2)*(a + b*x + c*x**2))
```

Mathematica [A] time = 0.151806, size = 81, normalized size = 1.21

$$\frac{a(b-2cx)+b^2x}{c\,(4ac-b^2)\,(a+x(b+cx))}+\frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4
*a*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.009, size = 97, normalized size = 1.5

$$\frac{1}{cx^2+bx+a}\left(-\frac{\left(2\,ac-b^2\right)x}{\left(4\,ac-b^2\right)c}+\frac{ab}{\left(4\,ac-b^2\right)c}\right)+4\,\frac{a}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*
a/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292079, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(ac^2x^2+abcx+a^2c\right)\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x-\left(2\,c^2x^2+2\,bcx+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^2+bx+a}\right)+\left(ab+\left(b^2-2\,ac\right)x\right)\sqrt{b^2-4\,ac}}{\left(ab^2c-4\,a^2c^2+\left(b^2c^2-4\,ac^3\right)x^2+\left(b^3c-4\,abc^2\right)x\right)\sqrt{b^2-4\,ac}},\right.$$
$$\left.-\frac{4\left(ac^2x^2+abcx+a^2c\right)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx+b)}{b^2-4\,ac}\right)+\left(ab+\left(b^2-2\,ac\right)x\right)\sqrt{-b^2+4\,ac}}{\left(ab^2c-4\,a^2c^2+\left(b^2c^2-4\,ac^3\right)x^2+\left(b^3c-4\,abc^2\right)x\right)\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(2*(a*c^2*x^2 + a*b*c*x + a^2*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c
 - 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*
a*c))/(c*x^2 + b*x + a)) + (a*b + (b^2 - 2*a*c)*x)*sqrt(b^2 - 4*a
*c))/((a*b^2*c - 4*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4
*a*b*c^2)*x)*sqrt(b^2 - 4*a*c)), -(4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (a*b +
(b^2 - 2*a*c)*x)*sqrt(-b^2 + 4*a*c))/((a*b^2*c - 4*a^2*c^2 + (b^2
*c^2 - 4*a*c^3)*x^2 + (b^3*c - 4*a*b*c^2)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 2.75172, size = 280, normalized size = 4.18

$$-2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)$$
$$+2a\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)$$
$$-\frac{-ab+x\left(2ac-b^2\right)}{4a^2c^2-ab^2c+x^2\left(4ac^3-b^2c^2\right)+x\left(4abc^2-b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4
*a*c - b**2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*
a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1
/(4*a*c - b**2)**3)*log(x + (32*a**3*c**2*sqrt(-1/(4*a*c - b**2)*
*3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b**4*sqrt(-
```

```
1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) - (-a*b + x*(2*a*c - b**2)
)/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*
b*c**2 - b**3*c))
```

GIAC/XCAS [A] time = 0.26607, size = 119, normalized size = 1.78

$$-\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2
 + b*x + a))
```

3.22 $\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0649034, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 11.8107, size = 60, normalized size = 0.91

$$-\frac{2b\,\mathrm{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{2a+bx}{(-4ac+b^2)(a+bx+cx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*b*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/2
) + (2*a + b*x)/((-4*a*c + b**2)*(a + b*x + c*x**2))
```

Mathematica [A] time = 0.10592, size = 69, normalized size = 1.05

$$\frac{2a+bx}{(b^2-4ac)(a+x(b+cx))}-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b +
2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.004, size = 70, normalized size = 1.1

$$\frac{-bx-2a}{(4ac-b^2)(cx^2+bx+a)}-2\frac{b}{(4ac-b^2)^{3/2}}\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289001, size = 1, normalized size = 0.02

$$\left[-\frac{(bcx^2+b^2x+ab)\log\left(\frac{b^3-4abc+2(b^2c-4ac^2)x+(2c^2x^2+2bcx+b^2-2ac)\sqrt{b^2-4ac}}{cx^2+bx+a}\right)-\sqrt{b^2-4ac}(bx+2a)}{(ab^2-4a^2c+(b^2c-4ac^2)x^2+(b^3-4abc)x)\sqrt{b^2-4ac}}, \frac{2(bcx^2+b^2x+ab)\text{ar}}{(ab^2-4a^2c+(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-((b*c*x^2 + b^2*x + a*b)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^
2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*
x^2 + b*x + a)) - sqrt(b^2 - 4*a*c)*(b*x + 2*a))/((a*b^2 - 4*a^2*
c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(b^2 - 4*a*c))
, (2*(b*c*x^2 + b^2*x + a*b)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x +
b)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(b*x + 2*a))/((a*b^2 - 4*a
^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(-b^2 + 4*a
*c))]
```

Sympy [A] time = 2.62484, size = 252, normalized size = 3.82

$$b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$-b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)$$
$$-\frac{2a+bx}{4a^2c-ab^2+x^2\left(4ac^2-b^2c\right)+x\left(4abc-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*
a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sq
rt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b*
*2)**3)*log(x + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*
b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**
3) + b**2)/(2*b*c)) - (2*a + b*x)/(4*a**2*c - a*b**2 + x**2*(4*a*
c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

GIAC/XCAS [A] time = 0.268342, size = 103, normalized size = 1.56

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b
^2 + 4*a*c)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.23 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^2} dx$

Optimal. Leaf size=66

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.0595296, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}} - \frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(
b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 9.85811, size = 60, normalized size = 0.91

$$\frac{4c\,\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}} - \frac{b+2cx}{(-4ac+b^2)(a+bx+cx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] 4*c*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/2)
 - (b + 2*c*x)/((-4*a*c + b**2)*(a + b*x + c*x**2))
```

Mathematica [A] time = 0.123329, size = 70, normalized size = 1.06

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx}{a+x(b+cx)}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-
b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A] time = 0.003, size = 68, normalized size = 1.

$$\frac{2\,cx+b}{(4\,ac-b^2)\,(cx^2+bx+a)}+4\,\frac{c}{(4\,ac-b^2)^{3/2}}\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan(
(2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282119, size = 1, normalized size = 0.02

$$\left[-\frac{2\left(c^{2}x^{2}+bcx+ac\right)\log\left(-\frac{b^{3}-4\,abc+2\left(b^{2}c-4\,ac^{2}\right)x-\left(2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{2}+bx+a}\right)+\sqrt{b^{2}-4\,ac}(2\,cx+b)}{(ab^{2}-4\,a^{2}c+(b^{2}c-4\,ac^{2})x^{2}+(b^{3}-4\,abc)x)\sqrt{b^{2}-4\,ac}},\right.$$
$$\left.-\frac{4\left(c^{2}x^{2}+bcx+ac\right)\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{b^{2}-4\,ac}\right)+\sqrt{-b^{2}+4\,ac}(2\,cx+b)}{(ab^{2}-4\,a^{2}c+(b^{2}c-4\,ac^{2})x^{2}+(b^{3}-4\,abc)x)\sqrt{-b^{2}+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(2*(c^2*x^2 + b*c*x + a*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a
*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/
(c*x^2 + b*x + a)) + sqrt(b^2 - 4*a*c)*(2*c*x + b))/((a*b^2 - 4*a
^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(b^2 - 4*a*
c)), -(4*(c^2*x^2 + b*c*x + a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(2*c*x + b))/((a*b^2 -
 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*sqrt(-b^2 +
 4*a*c))]
```

Sympy [A] time = 2.61054, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}\log\left(x+\frac{-32a^{2}c^{3}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+16ab^{2}c^{2}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}-2b^{4}c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+2bc}{4c^{2}}\right)$$
$$+2c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}\log\left(x+\frac{32a^{2}c^{3}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}-16ab^{2}c^{2}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+2b^{4}c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+2bc}{4c^{2}}\right)$$
$$+\frac{b+2cx}{4a^{2}c-ab^{2}+x^{2}\left(4ac^{2}-b^{2}c\right)+x\left(4abc-b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4
*a*c - b**2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*
b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-
1/(4*a*c - b**2)**3)*log(x + (32*a**2*c**3*sqrt(-1/(4*a*c - b**2)
**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + 2*b**4*c*sqrt(
```

```
-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*c*x)/(4*a**2*c
- a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

GIAC/XCAS [A] time = 0.26679, size = 103, normalized size = 1.56

$$-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-
b^2 + 4*a*c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.24 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b
^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4
*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rubi [A] time = 0.311145, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b
^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4
*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rubi in Sympy [A] time = 43.784, size = 102, normalized size = 0.94

$$\frac{-2ac+b^2+bcx}{a\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}+\frac{b\left(-6ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^2\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(a*(-4*a*c + b**2)*(a + b*x + c*x**2)) +
b*(-6*a*c + b**2)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**2*(-
4*a*c + b**2)**(3/2)) + log(x)/a**2 - log(a + b*x + c*x**2)/(2*a*
*2)
```

Mathematica [A] time = 0.318532, size = 107, normalized size = 0.99

$$\frac{\frac{2a(-2ac+b^2+bcx)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}-\log(a+x(b+cx))+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) +
(2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2
+ 4*a*c)^(3/2) + 2*Log[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [B] time = 0.011, size = 389, normalized size = 3.6

$$\frac{\ln(x)}{a^2}-\frac{bcx}{a(cx^2+bx+a)(4ac-b^2)}+2\frac{c}{(4ac-b^2)(cx^2+bx+a)}-\frac{b^2}{a(cx^2+bx+a)(4ac-b^2)}$$
$$-2\frac{c\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)}{a(4ac-b^2)}+\frac{\ln\left(\left(4ac-b^2\right)\left(cx^2+bx+a\right)\right)b^2}{2(4ac-b^2)a^2}$$
$$-6\frac{bc}{a\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$
$$+\frac{b^3}{a^2}\arctan\left((2\left(4ac-b^2\right)cx+\left(4ac-b^2\right)b)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] ln(x)/a^2-1/a/(c*x^2+b*x+a)*b*c/(4*a*c-b^2)*x+2/(c*x^2+b*x+a)/(4*
a*c-b^2)*c-1/a/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2-2/a/(4*a*c-b^2)*c*ln
((4*a*c-b^2)*(c*x^2+b*x+a))+1/2/a^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c
*x^2+b*x+a))*b^2-6/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(
1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*c+1/a^2/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)
/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.345833, size = 1, normalized size = 0.01

$$\left[\frac{\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\log\left(\frac{b^3 - 4\,abc + 2\left(b^2c - 4\,ac^2\right)x + \left(2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac\right)\sqrt{b^2 - 4\,ac}}{cx^2 + bx + a}\right) + \left(2\,abcx + 2\,a\right.}{2\left(a^3b^2 - 4\,a^4c + \left(\right.\right.}\right.$$

$$-\frac{2\left(ab^3 - 6\,a^2bc + \left(b^3c - 6\,abc^2\right)x^2 + \left(b^4 - 6\,ab^2c\right)x\right)\arctan\left(-\frac{\sqrt{-b^2 + 4\,ac}(2\,cx + b)}{b^2 - 4\,ac}\right) - \left(2\,abcx + 2\,ab^2 - 4\,a^2c - \left(ab^2 - 4\,a^2c\right.\right.}{2\left(a^3b^2 - 4\,a^4c + \left(a^2b^2c - 4\,a^3c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b
^2*c)*x)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) +
(2*a*b*c*x + 2*a*b^2 - 4*a^2*c - (a*b^2 - 4*a^2*c + (b^2*c - 4*a*
c^2)*x^2 + (b^3 - 4*a*b*c)*x)*log(c*x^2 + b*x + a) + 2*(a*b^2 - 4
*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*log(x))*sqrt(
b^2 - 4*a*c))/((a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 +
 (a^2*b^3 - 4*a^3*b*c)*x)*sqrt(b^2 - 4*a*c)), -1/2*(2*(a*b^3 - 6*
a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*arctan(-
sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (2*a*b*c*x + 2*a*
b^2 - 4*a^2*c - (a*b^2 - 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 -
 4*a*b*c)*x)*log(c*x^2 + b*x + a) + 2*(a*b^2 - 4*a^2*c + (b^2*c -
 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*log(x))*sqrt(-b^2 + 4*a*c))/((
a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^2 + (a^2*b^3 - 4*a^
3*b*c)*x)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 25.111, size = 2236, normalized size = 20.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)


[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))*log(x +
 (1536*a**9*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/
(2*a**2))**2 - 2112*a**8*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(
6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) - 1/(2*a**2))**2 + 1136*a**7*b**4*c**3*(-b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 768*a**7*c**5*
(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 300*a
**6*b**6*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*
a**2))**2 + 624*a**6*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*
c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(-b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 184*a**5*b**4*c**3*(-b*sqr
t(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 768*a**5*c**
5 - 2*a**4*b**10*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/
(2*a**2))**2 + 23*a**4*b**6*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*
a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6)) - 1/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**8*c*(-b
*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 -
48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 952*a**3
*b**4*c**3 + 277*a**2*b**6*c**2 - 38*a*b**8*c + 2*b**10)/(864*a**
4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**2*b**5*c**3 - 36*a*b**7*c*
*2 + 2*b**9*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a
**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/
(2*a**2))*log(x + (1536*a**9*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*
a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6)) - 1/(2*a**2))**2 - 2112*a**8*b**2*c**4*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 + 1136*a**7*b**4*c*
*3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c*
*3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 -
768*a**7*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*
(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a
**2)) - 300*a**6*b**6*c**2*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b
**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b*
*6)) - 1/(2*a**2))**2 + 624*a**6*b**2*c**4*(b*sqrt(-(4*a*c - b**2
)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - 1/(2*a**2)) + 39*a**5*b**8*c*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2))**2 - 184*a**5*b**4*c*
*3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c*
*3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 768
*a**5*c**5 - 2*a**4*b**10*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b*
*2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
```

```
6)) - 1/(2*a**2))**2 + 23*a**4*b**6*c**2*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - 1/(2*a**2)) + 1488*a**4*b**2*c**4 - a**3*b**
8*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*a**2*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(2*a**2)) - 95
2*a**3*b**4*c**3 + 277*a**2*b**6*c**2 - 38*a*b**8*c + 2*b**10)/(8
64*a**4*b*c**5 - 738*a**3*b**3*c**4 + 243*a**2*b**5*c**3 - 36*a*b
**7*c**2 + 2*b**9*c)) - (-2*a*c + b**2 + b*c*x)/(4*a**3*c - a**2*
b**2 + x**2*(4*a**2*c**2 - a*b**2*c) + x*(4*a**2*b*c - a*b**3)) +
 log(x)/a**2
```

GIAC/XCAS [A] time = 0.270312, size = 170, normalized size = 1.57

$$-\frac{\left(b^{3}-6\,abc\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{(a^{2}b^{2}-4\,a^{3}c)\sqrt{-b^{2}+4\,ac}}-\frac{\ln\left(cx^{2}+bx+a\right)}{2\,a^{2}}+\frac{\ln\left(|x|\right)}{a^{2}}+\frac{abcx+ab^{2}-2\,a^{2}c}{(cx^{2}+bx+a)(b^{2}-4\,ac)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2
 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/2*ln(c*x^2 + b*x + a)/a^2 + l
n(abs(x))/a^2 + (a*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b
^2 - 4*a*c)*a^2)
```

3.25 $\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\, dx$

Optimal. Leaf size=148

$$\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/
(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c
)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3
```

Rubi [A] time = 0.395174, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{2\left(b^2-3ac\right)}{a^2x\left(b^2-4ac\right)}-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/
(a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c
)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x + c*x^2])/a^3
```

Rubi in Sympy [A] time = 73.0962, size = 143, normalized size = 0.97

$$\frac{-2ac+b^2+bcx}{ax\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}-\frac{2\left(-3ac+b^2\right)}{a^2x\left(-4ac+b^2\right)}-\frac{2b\log\left(x\right)}{a^3}+\frac{b\log\left(a+bx+cx^2\right)}{a^3}-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^3\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(a*x*(-4*a*c + b**2)*(a + b*x + c*x**2))
- 2*(-3*a*c + b**2)/(a**2*x*(-4*a*c + b**2)) - 2*b*log(x)/a**3 +
b*log(a + b*x + c*x**2)/a**3 - 2*(6*a**2*c**2 - 6*a*b**2*c + b**4
)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**3*(-4*a*c + b**2)**(
3/2))
```

Mathematica [A] time = 0.467442, size = 131, normalized size = 0.89

$$-\frac{\frac{2(6a^2c^2-6ab^2c+b^4)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\frac{a(-3abc-2ac^2x+b^3+b^2cx)}{(b^2-4ac)(a+x(b+cx))}-b\log(a+x(b+cx))+\frac{a}{x}+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)
*(a + x*(b + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b
+ 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] -
 b*Log[a + x*(b + c*x)])/a^3)
```

Maple [B] time = 0.013, size = 545, normalized size = 3.7

$$-\frac{1}{a^2x}-2\,\frac{b\ln(x)}{a^3}-2\,\frac{c^2x}{a\,(cx^2+bx+a)\,(4\,ac-b^2)}+\frac{cxb^2}{a^2\,(cx^2+bx+a)\,(4\,ac-b^2)}$$
$$-3\,\frac{bc}{a\,(cx^2+bx+a)\,(4\,ac-b^2)}+\frac{b^3}{a^2\,(cx^2+bx+a)\,(4\,ac-b^2)}$$
$$+4\,\frac{c\ln\left(\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)b}{(4\,ac-b^2)\,a^2}-\frac{\ln\left(\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)b^3}{a^3\,(4\,ac-b^2)}$$
$$-12\,\frac{c^2}{a\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$
$$+12\,\frac{b^2c}{a^2\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$
$$-2\,\frac{b^4}{a^3\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/a^2/x-2*b*ln(x)/a^3-2/a/(c*x^2+b*x+a)*c^2/(4*a*c-b^2)*x+1/a^2/
(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*b^2-3/a/(c*x^2+b*x+a)*b/(4*a*c-b^2)
*c+1/a^2/(c*x^2+b*x+a)*b^3/(4*a*c-b^2)+4/a^2/(4*a*c-b^2)*c*ln((4*
a*c-b^2)*(c*x^2+b*x+a))*b-1/a^3/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^2
+b*x+a))*b^3-12/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2
)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)^(1/2))*c^2+12/a^2/(64*a^3*c^3-48*a^2*b^2*c^2
+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(
64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^2*c-2/a^3/(64*
a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2
)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(
1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.43591, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*
a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*log((b^3 -
4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b*c*x + b^2 - 2*
a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) + (a^2*b^2 - 4*a^3*c +
 2*(a*b^2*c - 3*a^2*c^2)*x^2 + (2*a*b^3 - 7*a^2*b*c)*x - ((b^3*c
- 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b*c)*x)
*log(c*x^2 + b*x + a) + 2*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b
^2*c)*x^2 + (a*b^3 - 4*a^2*b*c)*x)*log(x))*sqrt(b^2 - 4*a*c))/(((
a^3*b^2*c - 4*a^4*c^2)*x^3 + (a^3*b^3 - 4*a^4*b*c)*x^2 + (a^4*b^2
 - 4*a^5*c)*x)*sqrt(b^2 - 4*a*c)), (2*((b^4*c - 6*a*b^2*c^2 + 6*a
^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^
2*b^2*c + 6*a^3*c^2)*x)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b
^2 - 4*a*c)) - (a^2*b^2 - 4*a^3*c + 2*(a*b^2*c - 3*a^2*c^2)*x^2 +
 (2*a*b^3 - 7*a^2*b*c)*x - ((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*
```

```
b^2*c)*x^2 + (a*b^3 - 4*a^2*b*c)*x)*log(c*x^2 + b*x + a) + 2*((b^
3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b*c
)*x)*log(x))*sqrt(-b^2 + 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^3 +
(a^3*b^3 - 4*a^4*b*c)*x^2 + (a^4*b^2 - 4*a^5*c)*x)*sqrt(-b^2 + 4*
a*c))]
```

Sympy [A] time = 38.0411, size = 2672, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b*
*4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
))*log(x + (-1728*a**11*b*c**5*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 2256*a**10*b**3*c**4*(b/a*
*3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(
a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2
 - 1172*a**9*b**5*c**3*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**
2 + 12*a*b**4*c - b**6)))**2 - 288*a**9*c**6*(b/a**3 - sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 303*a**8*b**7*c
**2*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c
+ b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)))**2 - 432*a**8*b**2*c**5*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 - sqrt
(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*
a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 558*a*
*7*b**4*c**4*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*
a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) + 2*a**6*b**11*(b/a**3 - sqrt(-(4*a*c - b**2)**3)
*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3
 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a*
*3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 57
6*a**6*b*c**6 + 34*a**5*b**8*c**2*(b/a**3 - sqrt(-(4*a*c - b**2)*
*3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6))) + 6048*a**5*b**3*c**5 - 2*a*
*4*b**10*c*(b/a**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*
b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6))) - 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*
a**2*b**9*c**2 + 152*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a
**5*b**2*c**6 - 5292*a**4*b**4*c**5 + 3384*a**3*b**6*c**4 - 1008*
a**2*b**8*c**3 + 144*a*b**10*c**2 - 8*b**12*c)) + (b/a**3 + sqrt(
```

```
-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-17
28*a**11*b*c**5*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)))**2 + 2256*a**10*b**3*c**4*(b/a**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 1172*a**9*b*
*5*c**3*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**
2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)))**2 - 288*a**9*c**6*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*
(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6))) + 303*a**8*b**7*c**2*(b/a**3 + s
qrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 - 432
*a**8*b**2*c**5*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6))) - 39*a**7*b**9*c*(b/a**3 + sqrt(-(4*a*c - b**2
)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 558*a**7*b**4*c**4*(b
/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
 + 2*a**6*b**11*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 -
 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)))**2 - 212*a**6*b**6*c**3*(b/a**3 + sqrt(-(4*a*c
 - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 576*a**6*b*c**6 +
 34*a**5*b**8*c**2*(b/a**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6))) + 6048*a**5*b**3*c**5 - 2*a**4*b**10*c*(b/a
**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/
(a**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) -
 7908*a**4*b**5*c**4 + 4264*a**3*b**7*c**3 - 1144*a**2*b**9*c**2
+ 152*a*b**11*c - 8*b**13)/(216*a**6*c**7 + 2808*a**5*b**2*c**6 -
 5292*a**4*b**4*c**5 + 3384*a**3*b**6*c**4 - 1008*a**2*b**8*c**3
+ 144*a*b**10*c**2 - 8*b**12*c)) - (4*a**2*c - a*b**2 + x**2*(6*a
*c**2 - 2*b**2*c) + x*(7*a*b*c - 2*b**3))/(x**3*(4*a**3*c**2 - a*
*2*b**2*c) + x**2*(4*a**3*b*c - a**2*b**3) + x*(4*a**4*c - a**3*b
**2)) - 2*b*log(x)/a**3
```

GIAC/XCAS [A] time = 0.270703, size = 231, normalized size = 1.56

$$\frac{2\left(b^{4}-6\,ab^{2}c+6\,a^{2}c^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{(a^{3}b^{2}-4\,a^{4}c)\sqrt{-b^{2}+4\,ac}}-\frac{2\,b^{2}cx^{2}-6\,ac^{2}x^{2}+2\,b^{3}x-7\,abcx+ab^{2}-4\,a^{2}c}{(a^{2}b^{2}-4\,a^{3}c)(cx^{3}+bx^{2}+ax)}+\frac{b\ln\left(cx^{2}+bx+a\right)}{a^{3}}-\frac{2\,b\ln\left(|x|\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6
*a*c^2*x^2 + 2*b^3*x - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4
*a^3*c)*(c*x^3 + b*x^2 + a*x)) + b*ln(c*x^2 + b*x + a)/a^3 - 2*b*
ln(abs(x))/a^3
```

3.26 $\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}+\frac{\log(x)(3b^2-2ac)}{a^4}+\frac{b(3b^2-11ac)}{a^3x(b^2-4ac)}-\frac{3b^2-8ac}{2a^2x^2(b^2-4ac)}+\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))
/(a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x
^2*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + (
(3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^
2])/(2*a^4)
```

Rubi [A] time = 0.511379, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}+\frac{\log(x)(3b^2-2ac)}{a^4}+\frac{b(3b^2-11ac)}{a^3x(b^2-4ac)}-\frac{3b^2-8ac}{2a^2x^2(b^2-4ac)}+\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -(3*b^2 - 8*a*c)/(2*a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))
/(a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x
^2*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + (
(3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x + c*x^
2])/(2*a^4)
```

Rubi in Sympy [A] time = 82.8861, size = 192, normalized size = 0.95

$$\frac{-2ac+b^2+bcx}{ax^2\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}-\frac{-8ac+3b^2}{2a^2x^2\left(-4ac+b^2\right)}+\frac{b\left(-11ac+3b^2\right)}{a^3x\left(-4ac+b^2\right)}+\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^4\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\left(-2ac+3b^2\right)\log\left(x\right)}{a^4}-\frac{\left(-2ac+3b^2\right)\log\left(a+bx+cx^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(a*x**2*(-4*a*c + b**2)*(a + b*x + c*x**2
)) - (-8*a*c + 3*b**2)/(2*a**2*x**2*(-4*a*c + b**2)) + b*(-11*a*c
 + 3*b**2)/(a**3*x*(-4*a*c + b**2)) + b*(30*a**2*c**2 - 20*a*b**2
*c + 3*b**4)*atanh((b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**4*(-4*a*c
 + b**2)**(3/2)) + (-2*a*c + 3*b**2)*log(x)/a**4 - (-2*a*c + 3*b*
*2)*log(a + b*x + c*x**2)/(2*a**4)
```

Mathematica [A] time = 0.77766, size = 175, normalized size = 0.87

$$\frac{\frac{2b\left(30a^2c^2-20ab^2c+3b^4\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{2a\left(2a^2c^2-4ab^2c-3abc^2x+b^4+b^3cx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)}-\frac{a^2}{x^2}+2\log(x)\left(3b^2-2ac\right)+\left(2ac-3b^2\right)\log(a+x(b+c}{2a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3
*c*x - 3*a*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*
b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a
*c]])/(-b^2 + 4*a*c)^(3/2) + 2*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 +
 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [B] time = 0.021, size = 646, normalized size = 3.2

$$
\begin{aligned}
&-\frac{1}{2\,a^2x^2}-2\,\frac{\ln(x)\,c}{a^3}+3\,\frac{b^2\ln(x)}{a^4}+2\,\frac{b}{a^3x}+3\,\frac{c^2bx}{a^2\,(cx^2+bx+a)\,(4\,ac-b^2)}\\
&-\frac{b^3cx}{a^3\,(cx^2+bx+a)\,(4\,ac-b^2)}-2\,\frac{c^2}{a\,(cx^2+bx+a)\,(4\,ac-b^2)}+4\,\frac{b^2c}{a^2\,(cx^2+bx+a)\,(4\,ac-b^2)}\\
&-\frac{b^4}{a^3\,(cx^2+bx+a)\,(4\,ac-b^2)}+4\,\frac{c^2\ln\left(\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)}{(4\,ac-b^2)\,a^2}\\
&-7\,\frac{c\ln\left(\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)b^2}{a^3\,(4\,ac-b^2)}+\frac{3\,\ln\left(\left(4\,ac-b^2\right)\left(cx^2+bx+a\right)\right)b^4}{2\,a^4\,(4\,ac-b^2)}\\
&+30\,\frac{c^2b}{a^2\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)\\
&-20\,\frac{b^3c}{a^3\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)\\
&+3\,\frac{b^5}{a^4\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/2/a^2/x^2-2/a^3*ln(x)*c+3/a^4*b^2*ln(x)+2/a^3*b/x+3/a^2/(c*x^2
+b*x+a)*b*c^2/(4*a*c-b^2)*x-1/a^3/(c*x^2+b*x+a)*b^3*c/(4*a*c-b^2)
*x-2/a/(c*x^2+b*x+a)/(4*a*c-b^2)*c^2+4/a^2/(c*x^2+b*x+a)/(4*a*c-b
^2)*b^2*c-1/a^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4+4/a^2/(4*a*c-b^2)*c
^2*ln((4*a*c-b^2)*(c*x^2+b*x+a))-7/a^3/(4*a*c-b^2)*c*ln((4*a*c-b^
2)*(c*x^2+b*x+a))*b^2+3/2/a^4/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^2+b
*x+a))*b^4+30/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2
)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2
*c^2+12*a*b^4*c-b^6)^(1/2))*b*c^2-20/a^3/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)
/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3*c+3/a^4/(6
4*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b
^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)
^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.54635, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20
*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3
*b*c^2)*x^2)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x - (2*c^2
*x^2 + 2*b*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a
)) + (a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a
*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*x
+ ((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^4 + (3*b^5 - 14*a*b^3*c
 + 8*a^2*b*c^2)*x^3 + (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*x^2)*l
og(c*x^2 + b*x + a) - 2*((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^4
 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*x^3 + (3*a*b^4 - 14*a^2*b^2
*c + 8*a^3*c^2)*x^2)*log(x))*sqrt(b^2 - 4*a*c))/(((a^4*b^2*c - 4*
a^5*c^2)*x^4 + (a^4*b^3 - 4*a^5*b*c)*x^3 + (a^5*b^2 - 4*a^6*c)*x^
2)*sqrt(b^2 - 4*a*c)), -1/2*(2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*
b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5
 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*arctan(-sqrt(-b^2 + 4*a*c)*(
2*c*x + b)/(b^2 - 4*a*c)) + (a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 1
1*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(
a^2*b^3 - 4*a^3*b*c)*x + ((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^
4 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*x^3 + (3*a*b^4 - 14*a^2*b^
2*c + 8*a^3*c^2)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^4*c - 14*a*b
^2*c^2 + 8*a^2*c^3)*x^4 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*x^3
+ (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*x^2)*log(x))*sqrt(-b^2 + 4
*a*c))/(((a^4*b^2*c - 4*a^5*c^2)*x^4 + (a^4*b^3 - 4*a^5*b*c)*x^3
+ (a^5*b^2 - 4*a^6*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 58.7439, size = 4083, normalized size = 20.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4
)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
```

```
) + (2*a*c - 3*b**2)/(2*a**4))*log(x + (3072*a**14*c**6*(-b*sqrt(
-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*
c - 3*b**2)/(2*a**4))**2 - 9408*a**13*b**2*c**5*(-b*sqrt(-(4*a*c
- b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**
3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b*
*2)/(2*a**4))**2 + 9040*a**12*b**4*c**4*(-b*sqrt(-(4*a*c - b**2)*
*3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 -
 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a
**4))**2 - 4116*a**11*b**6*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(30*
a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2
 + 3072*a**11*c**7*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 2
0*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 987*a**10*b**
8*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 7536*a**10*b**2*c**6*(
-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)
/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
 + (2*a*c - 3*b**2)/(2*a**4)) - 121*a**9*b**10*c*(-b*sqrt(-(4*a*c
 - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a*
*3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b
**2)/(2*a**4))**2 + 8152*a**9*b**4*c**5*(-b*sqrt(-(4*a*c - b**2)*
*3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 -
 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a
**4)) + 6*a**8*b**12*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 -
 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4343*a**
8*b**6*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**
2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 6144*a**8*c**8 + 119
8*a**7*b**8*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*
a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) + 50208*a**7*b**2
*c**7 - 165*a**6*b**10*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2
*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 1377
92*a**6*b**4*c**6 + 9*a**5*b**12*c*(-b*sqrt(-(4*a*c - b**2)**3)*(
30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))
 + 176474*a**5*b**6*c**5 - 119275*a**4*b**8*c**4 + 45448*a**3*b**
10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**14*c - 54*b**16)/(1728
0*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 - 88
605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**3
+ 1080*a*b**13*c**2 - 54*b**15*c)) + (b*sqrt(-(4*a*c - b**2)**3)*
(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)
)*log(x + (3072*a**14*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c
**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 940
8*a**13*b**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*
a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 9040*a**12*b
```

```
**4*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c
+ 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 - 4116*a**11*b**6*c**3*
(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)
/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
 + (2*a*c - 3*b**2)/(2*a**4))**2 + 3072*a**11*c**7*(b*sqrt(-(4*a*
c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*
b**2)/(2*a**4)) + 987*a**10*b**8*c**2*(b*sqrt(-(4*a*c - b**2)**3)
*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4
))**2 - 7536*a**10*b**2*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2
*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4)) - 121*
a**9*b**10*c*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**
2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4))**2 + 8152*a**9*b**4*c*
*5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b*
*4)/(2*a**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6)) + (2*a*c - 3*b**2)/(2*a**4)) + 6*a**8*b**12*(b*sqrt(-(4*a*c -
 b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3
*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**
2)/(2*a**4))**2 - 4343*a**8*b**6*c**4*(b*sqrt(-(4*a*c - b**2)**3)
*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3 - 48
*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*a**4
)) - 6144*a**8*c**8 + 1198*a**7*b**8*c**3*(b*sqrt(-(4*a*c - b**2)
**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*a**3*c**3
- 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3*b**2)/(2*
a**4)) + 50208*a**7*b**2*c**7 - 165*a**6*b**10*c**2*(b*sqrt(-(4*a
*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**4*(64*
a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a*c - 3
*b**2)/(2*a**4)) - 137792*a**6*b**4*c**6 + 9*a**5*b**12*c*(b*sqrt
(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*a**
4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + (2*a
*c - 3*b**2)/(2*a**4)) + 176474*a**5*b**6*c**5 - 119275*a**4*b**8
*c**4 + 45448*a**3*b**10*c**3 - 9846*a**2*b**12*c**2 + 1134*a*b**
14*c - 54*b**16)/(17280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 1124
28*a**5*b**5*c**6 - 88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 -
 8820*a**2*b**11*c**3 + 1080*a*b**13*c**2 - 54*b**15*c)) + (-4*a*
*3*c + a**2*b**2 + x**3*(22*a*b*c**2 - 6*b**3*c) + x**2*(-8*a**2*
c**2 + 25*a*b**2*c - 6*b**4) + x*(12*a**2*b*c - 3*a*b**3))/(x**4*
(8*a**4*c**2 - 2*a**3*b**2*c) + x**3*(8*a**4*b*c - 2*a**3*b**3) +
 x**2*(8*a**5*c - 2*a**4*b**2)) - (2*a*c - 3*b**2)*log(x + (-6144
*a**8*c**8 + 50208*a**7*b**2*c**7 - 3072*a**7*c**7*(2*a*c - 3*b**
2) - 137792*a**6*b**4*c**6 + 7536*a**6*b**2*c**6*(2*a*c - 3*b**2)
 + 3072*a**6*c**6*(2*a*c - 3*b**2)**2 + 176474*a**5*b**6*c**5 - 8
152*a**5*b**4*c**5*(2*a*c - 3*b**2) - 9408*a**5*b**2*c**5*(2*a*c
- 3*b**2)**2 - 119275*a**4*b**8*c**4 + 4343*a**4*b**6*c**4*(2*a*c
 - 3*b**2) + 9040*a**4*b**4*c**4*(2*a*c - 3*b**2)**2 + 45448*a**3
*b**10*c**3 - 1198*a**3*b**8*c**3*(2*a*c - 3*b**2) - 4116*a**3*b*
*6*c**3*(2*a*c - 3*b**2)**2 - 9846*a**2*b**12*c**2 + 165*a**2*b**
10*c**2*(2*a*c - 3*b**2) + 987*a**2*b**8*c**2*(2*a*c - 3*b**2)**2
 + 1134*a*b**14*c - 9*a*b**12*c*(2*a*c - 3*b**2) - 121*a*b**10*c*
(2*a*c - 3*b**2)**2 - 54*b**16 + 6*b**12*(2*a*c - 3*b**2)**2)/(17
```

```
280*a**7*b*c**8 - 69570*a**6*b**3*c**7 + 112428*a**5*b**5*c**6 -
88605*a**4*b**7*c**5 + 37600*a**3*b**9*c**4 - 8820*a**2*b**11*c**
3 + 1080*a*b**13*c**2 - 54*b**15*c))/a**4
```

GIAC/XCAS [A] time = 0.273781, size = 309, normalized size = 1.53

$$-\frac{\left(3\,b^{5}-20\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{4}b^{2}-4\,a^{5}c\right)\sqrt{-b^{2}+4\,ac}}-\frac{\left(3\,b^{2}-2\,ac\right)\ln\left(cx^{2}+bx+a\right)}{2\,a^{4}}+\frac{\left(3\,b^{2}-2\,ac\right)\ln\left(|x|\right)}{a^{4}}-\frac{a^{3}b^{2}-4\,a^{4}c-2\left(3\,ab^{3}c-11\,a^{2}bc^{2}\right)x^{3}-\left(6\,ab^{4}-25\,a^{2}b^{2}c+8\,a^{3}c^{2}\right)x^{2}-3\left(a^{2}b^{3}-4\,a^{3}bc\right)x}{2\left(cx^{2}+bx+a\right)\left(b^{2}-4\,ac\right)a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2
 + 4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2
- 2*a*c)*ln(c*x^2 + b*x + a)/a^4 + (3*b^2 - 2*a*c)*ln(abs(x))/a^4
 - 1/2*(a^3*b^2 - 4*a^4*c - 2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6
*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2 - 3*(a^2*b^3 - 4*a^3*b*c)*
x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

3.27 $\int \frac{1}{(ax^2+bx^3+cx^4)^2} dx$

Optimal. Leaf size=252

$$\frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5}-\frac{2b\log(x)(2b^2-3ac)}{a^5}+\frac{b(2b^2-7ac)}{a^3x^2(b^2-4ac)}-\frac{2(2b^2-5ac)}{3a^2x^3(b^2-4ac)}-\frac{2(5a^2c^2-9ab^2c+2b^4)}{a^4x(b^2-4ac)}-\frac{2(-10a^3c^3+30a^2b^2c^2-15ab^4c+2b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^3(b^2-4ac)(a+bx+cx^2)}$$

```
[Out] (-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*
c))/(a^3*(b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))
/(a^4*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x
^3*(a + b*x + c*x^2)) - (2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 -
 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 -
4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 + (b*(2*b^2 - 3*
a*c)*Log[a + b*x + c*x^2])/a^5
```

Rubi [A] time = 0.646752, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b(2b^2-3ac)\log(a+bx+cx^2)}{a^5}-\frac{2b\log(x)(2b^2-3ac)}{a^5}+\frac{b(2b^2-7ac)}{a^3x^2(b^2-4ac)}-\frac{2(2b^2-5ac)}{3a^2x^3(b^2-4ac)}-\frac{2(5a^2c^2-9ab^2c+2b^4)}{a^4x(b^2-4ac)}-\frac{2(-10a^3c^3+30a^2b^2c^2-15ab^4c+2b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^3(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*
c))/(a^3*(b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))
/(a^4*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x
^3*(a + b*x + c*x^2)) - (2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 -
 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^5*(b^2 -
4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 + (b*(2*b^2 - 3*
a*c)*Log[a + b*x + c*x^2])/a^5
```

Rubi in Sympy [A] time = 124.5, size = 245, normalized size = 0.97

$$\frac{-2ac+b^2+bcx}{ax^3\left(-4ac+b^2\right)\left(a+bx+cx^2\right)}-\frac{2\left(-5ac+2b^2\right)}{3a^2x^3\left(-4ac+b^2\right)}+\frac{b\left(-7ac+2b^2\right)}{a^3x^2\left(-4ac+b^2\right)}-\frac{2\left(5a^2c^2-9ab^2c+2b^4\right)}{a^4x\left(-4ac+b^2\right)}-\frac{2b\left(-3ac+2b^2\right)\log\left(x\right)}{a^5}+\frac{b\left(-3ac+2b^2\right)\log\left(a+bx+cx^2\right)}{a^5}-\frac{2\left(-10a^3c^3+30a^2b^2c^2-15ab^4c+2b^6\right)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^5\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(a*x**3*(-4*a*c + b**2)*(a + b*x + c*x**2
)) - 2*(-5*a*c + 2*b**2)/(3*a**2*x**3*(-4*a*c + b**2)) + b*(-7*a*
c + 2*b**2)/(a**3*x**2*(-4*a*c + b**2)) - 2*(5*a**2*c**2 - 9*a*b*
*2*c + 2*b**4)/(a**4*x*(-4*a*c + b**2)) - 2*b*(-3*a*c + 2*b**2)*l
og(x)/a**5 + b*(-3*a*c + 2*b**2)*log(a + b*x + c*x**2)/a**5 - 2*(
-10*a**3*c**3 + 30*a**2*b**2*c**2 - 15*a*b**4*c + 2*b**6)*atanh((
b + 2*c*x)/sqrt(-4*a*c + b**2))/(a**5*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.516431, size = 218, normalized size = 0.87

$$\frac{-\frac{a^3}{x^3}-\frac{3a\left(5a^2bc^2+2a^2c^3x-5ab^3c-4ab^2c^2x+b^5+b^4cx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)}+\frac{3a^2b}{x^2}-\frac{6\left(-10a^3c^3+30a^2b^2c^2-15ab^4c+2b^6\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+6\log(x)\left(3abc-2b^3\right)+}{3a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-(a^3/x^3) + (3*a^2*b)/x^2 + (3*a*(-3*b^2 + 2*a*c))/x - (3*a*(b^
5 - 5*a*b^3*c + 5*a^2*b*c^2 + b^4*c*x - 4*a*b^2*c^2*x + 2*a^2*c^3
*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (6*(2*b^6 - 15*a*b^4*c +
 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/(-b^2 + 4*a*c)^(3/2) + 6*(-2*b^3 + 3*a*b*c)*Log[x] + 3*(2*b^
3 - 3*a*b*c)*Log[a + x*(b + c*x)])/(3*a^5)
```

Maple [B] time = 0.024, size = 808, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/3/a^2/x^3+2/a^3/x*c-3/a^4*b^2/x+1/a^3*b/x^2+6*b/a^4*ln(x)*c-4*
b^3/a^5*ln(x)+2/a^2/(c*x^2+b*x+a)*c^3/(4*a*c-b^2)*x-4/a^3/(c*x^2+
b*x+a)*c^2/(4*a*c-b^2)*x*b^2+1/a^4/(c*x^2+b*x+a)*c/(4*a*c-b^2)*x*
b^4+5/a^2/(c*x^2+b*x+a)*b/(4*a*c-b^2)*c^2-5/a^3/(c*x^2+b*x+a)*b^3
/(4*a*c-b^2)*c+1/a^4/(c*x^2+b*x+a)*b^5/(4*a*c-b^2)-12/a^3/(4*a*c-
b^2)*c^2*ln((4*a*c-b^2)*(c*x^2+b*x+a))*b+11/a^4/(4*a*c-b^2)*c*ln(
(4*a*c-b^2)*(c*x^2+b*x+a))*b^3-2/a^5/(4*a*c-b^2)*ln((4*a*c-b^2)*(
c*x^2+b*x+a))*b^5+20/a^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^
6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*c^3-60/a^3/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^
2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^2*c^2+3
0/a^4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*
(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^
4*c-b^6)^(1/2))*b^4*c-4/a^5/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.750613, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(-2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x
^5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (
2*a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*log(-(
b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x - (2*c^2*x^2 + 2*b*c*x + b^
```

```
2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) - (a^4*b^2 - 4*a
^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*a*b^5
 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c +
 20*a^4*c^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x - 3*((2*b^5*c - 11*a
*b^3*c^2 + 12*a^2*b*c^3)*x^5 + (2*b^6 - 11*a*b^4*c + 12*a^2*b^2*c
^2)*x^4 + (2*a*b^5 - 11*a^2*b^3*c + 12*a^3*b*c^2)*x^3)*log(c*x^2
+ b*x + a) + 6*((2*b^5*c - 11*a*b^3*c^2 + 12*a^2*b*c^3)*x^5 + (2*
b^6 - 11*a*b^4*c + 12*a^2*b^2*c^2)*x^4 + (2*a*b^5 - 11*a^2*b^3*c
+ 12*a^3*b*c^2)*x^3)*log(x))*sqrt(b^2 - 4*a*c))/(((a^5*b^2*c - 4*
a^6*c^2)*x^5 + (a^5*b^3 - 4*a^6*b*c)*x^4 + (a^6*b^2 - 4*a^7*c)*x^
3)*sqrt(b^2 - 4*a*c)), 1/3*(6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b
^2*c^3 - 10*a^3*c^4)*x^5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 -
 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 1
0*a^4*c^3)*x^3)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - (a^4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*
c^3)*x^4 + 3*(4*a*b^5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2
*b^4 - 29*a^3*b^2*c + 20*a^4*c^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x
 - 3*((2*b^5*c - 11*a*b^3*c^2 + 12*a^2*b*c^3)*x^5 + (2*b^6 - 11*a
*b^4*c + 12*a^2*b^2*c^2)*x^4 + (2*a*b^5 - 11*a^2*b^3*c + 12*a^3*b
*c^2)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^5*c - 11*a*b^3*c^2 + 12
*a^2*b*c^3)*x^5 + (2*b^6 - 11*a*b^4*c + 12*a^2*b^2*c^2)*x^4 + (2*
a*b^5 - 11*a^2*b^3*c + 12*a^3*b*c^2)*x^3)*log(x))*sqrt(-b^2 + 4*a
*c))/(((a^5*b^2*c - 4*a^6*c^2)*x^5 + (a^5*b^3 - 4*a^6*b*c)*x^4 +
(a^6*b^2 - 4*a^7*c)*x^3)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 90.5776, size = 4774, normalized size = 18.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**
3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-4928*a**16
*b*c**6*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*
a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 + 10032*a
**15*b**3*c**5*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**
3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**
5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))**2 -
7980*a**14*b**5*c**4*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b
**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6
)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))
**2 + 3249*a**13*b**7*c**3*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a
*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c -
2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)))**2 + 800*a**13*c**8*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a
```

```
*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c -
2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6))) - 723*a**12*b**9*c**2*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4
*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c
- 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)))**2 + 6704*a**12*b**2*c**7*(-b*(3*a*c - 2*b**2)/a**5 - sq
rt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b
**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6))) + 84*a**11*b**11*c*(-b*(3*a*c - 2*b**2)/a**5 - sqr
t(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b*
*4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)))**2 - 15182*a**11*b**4*c**6*(-b*(3*a*c - 2*b**2)/a**
5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 +
15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6))) - 4*a**10*b**13*(-b*(3*a*c - 2*b**2)/a**5 -
sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a
*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*
b**4*c - b**6)))**2 + 12844*a**10*b**6*c**5*(-b*(3*a*c - 2*b**2)/
a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2
 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6))) - 5546*a**9*b**8*c**4*(-b*(3*a*c - 2*b**2
)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c*
*2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**
2 + 12*a*b**4*c - b**6))) - 4800*a**9*b*c**9 + 1306*a**8*b**10*c*
*3*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*
c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 140384*a**8*b**
3*c**8 - 160*a**7*b**12*c**2*(-b*(3*a*c - 2*b**2)/a**5 - sqrt(-(4
*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c
- 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6))) - 479788*a**7*b**5*c**7 + 8*a**6*b**14*c*(-b*(3*a*c - 2*
b**2)/a**5 - sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**
2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6))) + 709872*a**6*b**7*c**6 - 575864*a*
*5*b**9*c**5 + 279640*a**4*b**11*c**4 - 83528*a**3*b**13*c**3 + 1
5056*a**2*b**15*c**2 - 1504*a*b**17*c + 64*b**19)/(1000*a**9*c**1
0 + 42840*a**8*b**2*c**9 - 232020*a**7*b**4*c**8 + 431760*a**6*b*
*6*c**7 - 406368*a**5*b**8*c**6 + 219600*a**4*b**10*c**5 - 71160*
a**3*b**12*c**4 + 13680*a**2*b**14*c**3 - 1440*a*b**16*c**2 + 64*
b**18*c)) + (-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*
(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x +
(-4928*a**16*b*c**6*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b*
*2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)
/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*
*2 + 10032*a**15*b**3*c**5*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a
*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c -
2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b
**6)))**2 - 7980*a**14*b**5*c**4*(-b*(3*a*c - 2*b**2)/a**5 + sqrt
(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**
4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4
*c - b**6)))**2 + 3249*a**13*b**7*c**3*(-b*(3*a*c - 2*b**2)/a**5
+ sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15
*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
```

```
a*b**4*c - b**6)))**2 + 800*a**13*c**8*(-b*(3*a*c - 2*b**2)/a**5
+ sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15
*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6))) - 723*a**12*b**9*c**2*(-b*(3*a*c - 2*b**2)/a**
5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 +
15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)))**2 + 6704*a**12*b**2*c**7*(-b*(3*a*c - 2*b**
2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c
**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6))) + 84*a**11*b**11*c*(-b*(3*a*c - 2*b**2
)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c*
*2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**
2 + 12*a*b**4*c - b**6)))**2 - 15182*a**11*b**4*c**6*(-b*(3*a*c -
 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*
b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6))) - 4*a**10*b**13*(-b*(3*a*c - 2*b
**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2
*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6)))**2 + 12844*a**10*b**6*c**5*(-b*(3*a*
c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a*
*2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6))) - 5546*a**9*b**8*c**4*(-b*(3*
a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*
a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 4800*a**9*b*c**9 + 1306*a
**8*b**10*c**3*(-b*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**
3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**
5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 140
384*a**8*b**3*c**8 - 160*a**7*b**12*c**2*(-b*(3*a*c - 2*b**2)/a**
5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 - 30*a**2*b**2*c**2 +
15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6))) - 479788*a**7*b**5*c**7 + 8*a**6*b**14*c*(-b
*(3*a*c - 2*b**2)/a**5 + sqrt(-(4*a*c - b**2)**3)*(10*a**3*c**3 -
 30*a**2*b**2*c**2 + 15*a*b**4*c - 2*b**6)/(a**5*(64*a**3*c**3 -
48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 709872*a**6*b**7*c**6
 - 575864*a**5*b**9*c**5 + 279640*a**4*b**11*c**4 - 83528*a**3*b*
*13*c**3 + 15056*a**2*b**15*c**2 - 1504*a*b**17*c + 64*b**19)/(10
00*a**9*c**10 + 42840*a**8*b**2*c**9 - 232020*a**7*b**4*c**8 + 43
1760*a**6*b**6*c**7 - 406368*a**5*b**8*c**6 + 219600*a**4*b**10*c
**5 - 71160*a**3*b**12*c**4 + 13680*a**2*b**14*c**3 - 1440*a*b**1
6*c**2 + 64*b**18*c)) + (-4*a**4*c + a**3*b**2 + x**4*(30*a**2*c*
*3 - 54*a*b**2*c**2 + 12*b**4*c) + x**3*(51*a**2*b*c**2 - 60*a*b*
*3*c + 12*b**5) + x**2*(20*a**3*c**2 - 29*a**2*b**2*c + 6*a*b**4)
 + x*(8*a**3*b*c - 2*a**2*b**3))/(x**5*(12*a**5*c**2 - 3*a**4*b**
2*c) + x**4*(12*a**5*b*c - 3*a**4*b**3) + x**3*(12*a**6*c - 3*a**
5*b**2)) + 2*b*(3*a*c - 2*b**2)*log(x + (-4800*a**9*b*c**9 + 1403
84*a**8*b**3*c**8 + 1600*a**8*b*c**8*(3*a*c - 2*b**2) - 479788*a*
*7*b**5*c**7 + 13408*a**7*b**3*c**7*(3*a*c - 2*b**2) + 709872*a**
6*b**7*c**6 - 30364*a**6*b**5*c**6*(3*a*c - 2*b**2) - 19712*a**6*
b**3*c**6*(3*a*c - 2*b**2)**2 - 575864*a**5*b**9*c**5 + 25688*a**
5*b**7*c**5*(3*a*c - 2*b**2) + 40128*a**5*b**5*c**5*(3*a*c - 2*b*
*2)**2 + 279640*a**4*b**11*c**4 - 11092*a**4*b**9*c**4*(3*a*c - 2
*b**2) - 31920*a**4*b**7*c**4*(3*a*c - 2*b**2)**2 - 83528*a**3*b*
*13*c**3 + 2612*a**3*b**11*c**3*(3*a*c - 2*b**2) + 12996*a**3*b**
```

```
9*c**3*(3*a*c - 2*b**2)**2 + 15056*a**2*b**15*c**2 - 320*a**2*b**
13*c**2*(3*a*c - 2*b**2) - 2892*a**2*b**11*c**2*(3*a*c - 2*b**2)*
*2 - 1504*a*b**17*c + 16*a*b**15*c*(3*a*c - 2*b**2) + 336*a*b**13
*c*(3*a*c - 2*b**2)**2 + 64*b**19 - 16*b**15*(3*a*c - 2*b**2)**2)
/(1000*a**9*c**10 + 42840*a**8*b**2*c**9 - 232020*a**7*b**4*c**8
+ 431760*a**6*b**6*c**7 - 406368*a**5*b**8*c**6 + 219600*a**4*b**
10*c**5 - 71160*a**3*b**12*c**4 + 13680*a**2*b**14*c**3 - 1440*a*
b**16*c**2 + 64*b**18*c))/a**5
```

GIAC/XCAS [A] time = 0.270933, size = 381, normalized size = 1.51

$$\frac{2\left(2\,b^{6}-15\,ab^{4}c+30\,a^{2}b^{2}c^{2}-10\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{(a^{5}b^{2}-4\,a^{6}c)\sqrt{-b^{2}+4\,ac}}+\frac{\left(2\,b^{3}-3\,abc\right)\ln\left(cx^{2}+bx+a\right)}{a^{5}}-\frac{2\left(2\,b^{3}-3\,abc\right)\ln\left(|x|\right)}{a^{5}}-\frac{a^{4}b^{2}-4\,a^{5}c+6\left(2\,ab^{4}c-9\,a^{2}b^{2}c^{2}+5\,a^{3}c^{3}\right)x^{4}+3\left(4\,ab^{5}-20\,a^{2}b^{3}c+17\,a^{3}bc^{2}\right)x^{3}+\left(6\,a^{2}b^{4}-29\,a^{3}b^{2}c+20\,a^{4}c^{2}\right)x^{2}-2\,(}{3\,(cx^{2}+bx+a)(b^{2}-4\,ac)a^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(-2),x, algorithm="giac")
```

```
[Out] 2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*arctan((2*c*
x + b)/sqrt(-b^2 + 4*a*c))/((a^5*b^2 - 4*a^6*c)*sqrt(-b^2 + 4*a*c
)) + (2*b^3 - 3*a*b*c)*ln(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b
*c)*ln(abs(x))/a^5 - 1/3*(a^4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^
2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*a*b^5 - 20*a^2*b^3*c + 17*a^3*b
*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*a^4*c^2)*x^2 - 2*(a^3*
b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^5*x^3)
```

3.28 $\int \frac{1}{x(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=318

$$\frac{b\left(5b^2-17ac\right)}{3a^3x^3\left(b^2-4ac\right)}-\frac{5b^2-12ac}{4a^2x^4\left(b^2-4ac\right)}-\frac{\left(3a^2c^2-12ab^2c+5b^4\right)\log\left(a+bx+cx^2\right)}{2a^6}+\frac{\log(x)\left(3a^2c^2-12ab^2c+5b^4\right)}{a^6}+\frac{b\left(29a^2c^2-27ab^2c+5b^4\right)}{a^5x\left(b^2-4ac\right)}-\frac{12a^2c^2-22ab^2c+5b^4}{2a^4x^2\left(b^2-4ac\right)}+\frac{b\left(-70a^3c^3+105a^2b^2c^2-42ab^4c+5b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

```
[Out] -(5*b^2 - 12*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(5*b^2 - 17*a*c)
)/(3*a^3*(b^2 - 4*a*c)*x^3) - (5*b^4 - 22*a*b^2*c + 12*a^2*c^2)/(
2*a^4*(b^2 - 4*a*c)*x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/
(a^5*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^
4*(a + b*x + c*x^2)) + (b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 -
 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^6*(b^2 -
4*a*c)^(3/2)) + ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/a^6 - (
(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^6)
```

Rubi [A] time = 0.80403, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{b\left(5b^2-17ac\right)}{3a^3x^3\left(b^2-4ac\right)}-\frac{5b^2-12ac}{4a^2x^4\left(b^2-4ac\right)}-\frac{\left(3a^2c^2-12ab^2c+5b^4\right)\log\left(a+bx+cx^2\right)}{2a^6}+\frac{\log(x)\left(3a^2c^2-12ab^2c+5b^4\right)}{a^6}+\frac{b\left(29a^2c^2-27ab^2c+5b^4\right)}{a^5x\left(b^2-4ac\right)}-\frac{12a^2c^2-22ab^2c+5b^4}{2a^4x^2\left(b^2-4ac\right)}+\frac{b\left(-70a^3c^3+105a^2b^2c^2-42ab^4c+5b^6\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] -(5*b^2 - 12*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(5*b^2 - 17*a*c)
)/(3*a^3*(b^2 - 4*a*c)*x^3) - (5*b^4 - 22*a*b^2*c + 12*a^2*c^2)/(
2*a^4*(b^2 - 4*a*c)*x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/
(a^5*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^
4*(a + b*x + c*x^2)) + (b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 -
 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^6*(b^2 -
4*a*c)^(3/2)) + ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/a^6 - (
(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^6)
```

Rubi in Sympy [A] time = 127.985, size = 308, normalized size = 0.97

$$\frac{-2ac+b^2+bcx}{ax^4(-4ac+b^2)(a+bx+cx^2)}-\frac{-12ac+5b^2}{4a^2x^4(-4ac+b^2)}+\frac{b(-17ac+5b^2)}{3a^3x^3(-4ac+b^2)}-\frac{12a^2c^2-22ab^2c+5b^4}{2a^4x^2(-4ac+b^2)}+\frac{b(29a^2c^2-27ab^2c+5b^4)}{a^5x(-4ac+b^2)}+\frac{b(-70a^3c^3+105a^2b^2c^2-42ab^4c+5b^6)\operatorname{atanh}\left(\frac{b+2cx}{\sqrt{-4ac+b^2}}\right)}{a^6(-4ac+b^2)^{\frac{3}{2}}}+\frac{(3a^2c^2-12ab^2c+5b^4)\log(x)}{a^6}-\frac{(3a^2c^2-12ab^2c+5b^4)\log(a+bx+cx^2)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x)/(a*x**4*(-4*a*c + b**2)*(a + b*x + c*x**2
)) - (-12*a*c + 5*b**2)/(4*a**2*x**4*(-4*a*c + b**2)) + b*(-17*a*
c + 5*b**2)/(3*a**3*x**3*(-4*a*c + b**2)) - (12*a**2*c**2 - 22*a*
b**2*c + 5*b**4)/(2*a**4*x**2*(-4*a*c + b**2)) + b*(29*a**2*c**2
- 27*a*b**2*c + 5*b**4)/(a**5*x*(-4*a*c + b**2)) + b*(-70*a**3*c*
*3 + 105*a**2*b**2*c**2 - 42*a*b**4*c + 5*b**6)*atanh((b + 2*c*x)
/sqrt(-4*a*c + b**2))/(a**6*(-4*a*c + b**2)**(3/2)) + (3*a**2*c**
2 - 12*a*b**2*c + 5*b**4)*log(x)/a**6 - (3*a**2*c**2 - 12*a*b**2*
c + 5*b**4)*log(a + b*x + c*x**2)/(2*a**6)
```

Mathematica [A] time = 0.642513, size = 272, normalized size = 0.86

$$\frac{-\frac{3a^4}{x^4}+\frac{8a^3b}{x^3}+\frac{6a^2(2ac-3b^2)}{x^2}+12\log(x)\left(3a^2c^2-12ab^2c+5b^4\right)-6\left(3a^2c^2-12ab^2c+5b^4\right)\log(a+x(b+cx))+\frac{12b(-70a^3c^3+10\cdots}{\cdots}}{12a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] ((-3*a^4)/x^4 + (8*a^3*b)/x^3 + (6*a^2*(-3*b^2 + 2*a*c))/x^2 - (2
4*a*b*(-2*b^2 + 3*a*c))/x - (12*a*(-b^6 + 6*a*b^4*c - 9*a^2*b^2*c
^2 + 2*a^3*c^3 - b^5*c*x + 5*a*b^3*c^2*x - 5*a^2*b*c^3*x))/((b^2
- 4*a*c)*(a + x*(b + c*x))) + (12*b*(5*b^6 - 42*a*b^4*c + 105*a^2
*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-
b^2 + 4*a*c)^(3/2) + 12*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x] -
 6*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + x*(b + c*x)])/(12*a^6
)
```

Maple [B] time = 0.027, size = 923, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^3+a*x^2)^2,x)
```

```
[Out] -1/4/a^2/x^4+1/a^3/x^2*c-3/2/a^4/x^2*b^2+3/a^4*ln(x)*c^2-12/a^5*l
n(x)*b^2*c+5/a^6*ln(x)*b^4+2/3/a^3*b/x^3-6/a^4*b/x*c+4/a^5*b^3/x-
5/a^3/(c*x^2+b*x+a)*b*c^3/(4*a*c-b^2)*x+5/a^4/(c*x^2+b*x+a)*b^3*c
^2/(4*a*c-b^2)*x-1/a^5/(c*x^2+b*x+a)*b^5*c/(4*a*c-b^2)*x+2/a^2/(c
*x^2+b*x+a)/(4*a*c-b^2)*c^3-9/a^3/(c*x^2+b*x+a)/(4*a*c-b^2)*b^2*c
^2+6/a^4/(c*x^2+b*x+a)/(4*a*c-b^2)*b^4*c-1/a^5/(c*x^2+b*x+a)/(4*a
*c-b^2)*b^6-6/a^3/(4*a*c-b^2)*c^3*ln((4*a*c-b^2)*(c*x^2+b*x+a))+5
1/2/a^4/(4*a*c-b^2)*c^2*ln((4*a*c-b^2)*(c*x^2+b*x+a))*b^2-16/a^5/
(4*a*c-b^2)*c*ln((4*a*c-b^2)*(c*x^2+b*x+a))*b^4+5/2/a^6/(4*a*c-b^
2)*ln((4*a*c-b^2)*(c*x^2+b*x+a))*b^6-70/a^3/(64*a^3*c^3-48*a^2*b^
2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)
*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*c^3+105/a
^4/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*
a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)^(1/2))*b^3*c^2-42/a^5/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*c-b^2)*b)/(64*a^3*c^3-
48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^5*c+5/a^6/(64*a^3*c^3-48*
a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x+(4*a*
c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)^2*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.04124, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)^2*x),x, algorithm="fricas")
```

```
[Out] [1/12*(6*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70*a^3*b*c^
4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*
x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x
^4)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x + (2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^2 + b*x + a)) - (3*a^
5*b^2 - 12*a^6*c - 12*(5*a*b^5*c - 27*a^2*b^3*c^2 + 29*a^3*b*c^3)
*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*b^2*c^2 - 12*a^4*c^3)*
x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*x^3 + (10*a^3
*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*c)*x
 + 6*((5*b^6*c - 32*a*b^4*c^2 + 51*a^2*b^2*c^3 - 12*a^3*c^4)*x^6
+ (5*b^7 - 32*a*b^5*c + 51*a^2*b^3*c^2 - 12*a^3*b*c^3)*x^5 + (5*a
*b^6 - 32*a^2*b^4*c + 51*a^3*b^2*c^2 - 12*a^4*c^3)*x^4)*log(c*x^2
 + b*x + a) - 12*((5*b^6*c - 32*a*b^4*c^2 + 51*a^2*b^2*c^3 - 12*a
^3*c^4)*x^6 + (5*b^7 - 32*a*b^5*c + 51*a^2*b^3*c^2 - 12*a^3*b*c^3
)*x^5 + (5*a*b^6 - 32*a^2*b^4*c + 51*a^3*b^2*c^2 - 12*a^4*c^3)*x^
4)*log(x))*sqrt(b^2 - 4*a*c))/(((a^6*b^2*c - 4*a^7*c^2)*x^6 + (a^
6*b^3 - 4*a^7*b*c)*x^5 + (a^7*b^2 - 4*a^8*c)*x^4)*sqrt(b^2 - 4*a*
c)), -1/12*(12*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70*a^
3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2
*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*
c^3)*x^4)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) +
 (3*a^5*b^2 - 12*a^6*c - 12*(5*a*b^5*c - 27*a^2*b^3*c^2 + 29*a^3*
b*c^3)*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*b^2*c^2 - 12*a^4
*c^3)*x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*x^3 + (
10*a^3*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*
b*c)*x + 6*((5*b^6*c - 32*a*b^4*c^2 + 51*a^2*b^2*c^3 - 12*a^3*c^4
)*x^6 + (5*b^7 - 32*a*b^5*c + 51*a^2*b^3*c^2 - 12*a^3*b*c^3)*x^5
+ (5*a*b^6 - 32*a^2*b^4*c + 51*a^3*b^2*c^2 - 12*a^4*c^3)*x^4)*log
(c*x^2 + b*x + a) - 12*((5*b^6*c - 32*a*b^4*c^2 + 51*a^2*b^2*c^3
- 12*a^3*c^4)*x^6 + (5*b^7 - 32*a*b^5*c + 51*a^2*b^3*c^2 - 12*a^3
*b*c^3)*x^5 + (5*a*b^6 - 32*a^2*b^4*c + 51*a^3*b^2*c^2 - 12*a^4*c
^3)*x^4)*log(x))*sqrt(-b^2 + 4*a*c))/(((a^6*b^2*c - 4*a^7*c^2)*x^
6 + (a^6*b^3 - 4*a^7*b*c)*x^5 + (a^7*b^2 - 4*a^8*c)*x^4)*sqrt(-b^
2 + 4*a*c))]
```

Sympy [A] time = 137.52, size = 6181, normalized size = 19.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 +
 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*
a**6))*log(x + (4608*a**19*c**7*(-b*sqrt(-(4*a*c - b**2)**3)*(70*
```

a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(6
4*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*
c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 26432*a**18*b**2*c**6
*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2
+ 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2
*a**6))**2 + 38640*a**17*b**4*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(
70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a*
*2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 26124*a**16*b**6*c
4*(-b*sqrt(-(4*a*c - b2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c*
*2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c
2 + 12*a*b4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)
/(2*a**6))**2 + 9603*a**15*b**8*c**3*(-b*sqrt(-(4*a*c - b**2)**3)
*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a*
6(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*
a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 6912*a**15*c**9*
(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 +
 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*
a**6)) - 1989*a**14*b**10*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a
3*c3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c
2 - 12*a*b2*c + 5*b**4)/(2*a**6))**2 + 37616*a**14*b**2*c**8*
(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 +
 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2
+ 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*
a**6)) + 219*a**13*b**12*c*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*
c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**
3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2
- 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 96472*a**13*b**4*c**7*(-b*
sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*
a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6
)) - 10*a**12*b**14*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 -
105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3
- 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*
b**2*c + 5*b**4)/(2*a**6))**2 + 112063*a**12*b**6*c**6*(-b*sqrt(-
(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4
*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 6
9023*a**11*b**8*c**5*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 -
 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a
*b**2*c + 5*b**4)/(2*a**6)) - 20736*a**11*c**11 + 24355*a**10*b**
10*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**
2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b*
*2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b
4)/(2*a6)) + 373872*a**10*b**2*c**10 - 4964*a**9*b**12*c**3*(
-b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 +
42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a
6)) - 2277288*a9*b**4*c**9 + 545*a**8*b**14*c**2*(-b*sqrt(-(4
*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c

```
 - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) + 648
7391*a**8*b**6*c**8 - 25*a**7*b**16*c*(-b*sqrt(-(4*a*c - b**2)**3
)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a
**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3
*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 9943570*a**7*b**8*
c**7 + 9090837*a**6*b**10*c**6 - 5264714*a**5*b**12*c**5 + 198442
6*a**4*b**14*c**4 - 486146*a**3*b**16*c**3 + 74720*a**2*b**18*c**
2 - 6550*a*b**20*c + 250*b**22)/(90720*a**10*b*c**11 - 844130*a**
9*b**3*c**10 + 3174507*a**8*b**5*c**9 - 5885010*a**7*b**7*c**8 +
6168225*a**6*b**9*c**7 - 3960180*a**5*b**11*c**6 + 1618470*a**4*b
**13*c**5 - 423276*a**3*b**15*c**4 + 68670*a**2*b**17*c**3 - 6300
*a*b**19*c**2 + 250*b**21*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(70*a
**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c
**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))*log(x + (4608*a**19*c**7*(b
*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42
*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**
6))**2 - 26432*a**18*b**2*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(70*a*
*3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*
a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c*
*2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 38640*a**17*b**4*c**5*(
b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 4
2*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a*
*6))**2 - 26124*a**16*b**6*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(70*a
**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64
*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c
**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 9603*a**15*b**8*c**3*(
b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 4
2*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a*
*6))**2 - 6912*a**15*c**9*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c*
*3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*
c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 -
12*a*b**2*c + 5*b**4)/(2*a**6)) - 1989*a**14*b**10*c**2*(b*sqrt(-
(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4
*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2
+ 37616*a**14*b**2*c**8*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3
 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c*
*3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12
*a*b**2*c + 5*b**4)/(2*a**6)) + 219*a**13*b**12*c*(b*sqrt(-(4*a*c
 - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5
*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 - 9647
2*a**13*b**4*c**7*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105
*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 4
8*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**
2*c + 5*b**4)/(2*a**6)) - 10*a**12*b**14*(b*sqrt(-(4*a*c - b**2)*
*3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2
*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6))**2 + 112063*a**12*
```

```
b**6*c**6*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b*
*2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*
b**4)/(2*a**6)) - 69023*a**11*b**8*c**5*(b*sqrt(-(4*a*c - b**2)**
3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*
a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (
3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 20736*a**11*c**11
 + 24355*a**10*b**10*c**4*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c*
*3 - 105*a**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*
c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 -
12*a*b**2*c + 5*b**4)/(2*a**6)) + 373872*a**10*b**2*c**10 - 4964*
a**9*b**12*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a
**2*b**2*c**2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*
a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*
c + 5*b**4)/(2*a**6)) - 2277288*a**9*b**4*c**9 + 545*a**8*b**14*c
**2*(b*sqrt(-(4*a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**
2 + 42*a*b**4*c - 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/
(2*a**6)) + 6487391*a**8*b**6*c**8 - 25*a**7*b**16*c*(b*sqrt(-(4*
a*c - b**2)**3)*(70*a**3*c**3 - 105*a**2*b**2*c**2 + 42*a*b**4*c
- 5*b**6)/(2*a**6*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
 - b**6)) - (3*a**2*c**2 - 12*a*b**2*c + 5*b**4)/(2*a**6)) - 9943
570*a**7*b**8*c**7 + 9090837*a**6*b**10*c**6 - 5264714*a**5*b**12
*c**5 + 1984426*a**4*b**14*c**4 - 486146*a**3*b**16*c**3 + 74720*
a**2*b**18*c**2 - 6550*a*b**20*c + 250*b**22)/(90720*a**10*b*c**1
1 - 844130*a**9*b**3*c**10 + 3174507*a**8*b**5*c**9 - 5885010*a**
7*b**7*c**8 + 6168225*a**6*b**9*c**7 - 3960180*a**5*b**11*c**6 +
1618470*a**4*b**13*c**5 - 423276*a**3*b**15*c**4 + 68670*a**2*b**
17*c**3 - 6300*a*b**19*c**2 + 250*b**21*c)) - (12*a**5*c - 3*a**4
*b**2 + x**5*(348*a**2*b*c**3 - 324*a*b**3*c**2 + 60*b**5*c) + x*
*4*(-72*a**3*c**3 + 480*a**2*b**2*c**2 - 354*a*b**4*c + 60*b**6)
+ x**3*(208*a**3*b*c**2 - 172*a**2*b**3*c + 30*a*b**5) + x**2*(-3
6*a**4*c**2 + 49*a**3*b**2*c - 10*a**2*b**4) + x*(-20*a**4*b*c +
5*a**3*b**3))/(x**6*(48*a**6*c**2 - 12*a**5*b**2*c) + x**5*(48*a*
*6*b*c - 12*a**5*b**3) + x**4*(48*a**7*c - 12*a**6*b**2)) + (3*a*
*2*c**2 - 12*a*b**2*c + 5*b**4)*log(x + (-20736*a**11*c**11 + 373
872*a**10*b**2*c**10 - 2277288*a**9*b**4*c**9 - 6912*a**9*c**9*(3
*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 6487391*a**8*b**6*c**8 + 376
16*a**8*b**2*c**8*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) - 9943570*
a**7*b**8*c**7 - 96472*a**7*b**4*c**7*(3*a**2*c**2 - 12*a*b**2*c
+ 5*b**4) + 4608*a**7*c**7*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**
2 + 9090837*a**6*b**10*c**6 + 112063*a**6*b**6*c**6*(3*a**2*c**2
- 12*a*b**2*c + 5*b**4) - 26432*a**6*b**2*c**6*(3*a**2*c**2 - 12*
a*b**2*c + 5*b**4)**2 - 5264714*a**5*b**12*c**5 - 69023*a**5*b**8
*c**5*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) + 38640*a**5*b**4*c**5
*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 + 1984426*a**4*b**14*c**
4 + 24355*a**4*b**10*c**4*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4) -
26124*a**4*b**6*c**4*(3*a**2*c**2 - 12*a*b**2*c + 5*b**4)**2 - 48
6146*a**3*b**16*c**3 - 4964*a**3*b**12*c**3*(3*a**2*c**2 - 12*a*b
**2*c + 5*b**4) + 9603*a**3*b**8*c**3*(3*a**2*c**2 - 12*a*b**2*c
+ 5*b**4)**2 + 74720*a**2*b**18*c**2 + 545*a**2*b**14*c**2*(3*a**
2*c**2 - 12*a*b**2*c + 5*b**4) - 1989*a**2*b**10*c**2*(3*a**2*c**
2 - 12*a*b**2*c + 5*b**4)**2 - 6550*a*b**20*c - 25*a*b**16*c*(3*a
**2*c**2 - 12*a*b**2*c + 5*b**4) + 219*a*b**12*c*(3*a**2*c**2 - 1
```

```
2*a*b**2*c + 5*b**4)**2 + 250*b**22 - 10*b**14*(3*a**2*c**2 - 12*
a*b**2*c + 5*b**4)**2)/(90720*a**10*b*c**11 - 844130*a**9*b**3*c*
*10 + 3174507*a**8*b**5*c**9 - 5885010*a**7*b**7*c**8 + 6168225*a
**6*b**9*c**7 - 3960180*a**5*b**11*c**6 + 1618470*a**4*b**13*c**5
 - 423276*a**3*b**15*c**4 + 68670*a**2*b**17*c**3 - 6300*a*b**19*
c**2 + 250*b**21*c))/a**6
```

GIAC/XCAS [A] time = 0.274683, size = 468, normalized size = 1.47

$$-\frac{\left(5\,b^7-42\,ab^5c+105\,a^2b^3c^2-70\,a^3bc^3\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^6b^2-4\,a^7c)\sqrt{-b^2+4\,ac}}-\frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\ln\left(cx^2+bx+a\right)}{2\,a^6}+\frac{\left(5\,b^4-12\,ab^2c+3\,a^2c^2\right)\ln\left(|x|\right)}{a^6}-\frac{3\,a^5b^2-12\,a^6c-12\left(5\,ab^5c-27\,a^2b^3c^2+29\,a^3bc^3\right)x^5-6\left(10\,ab^6-59\,a^2b^4c+80\,a^3b^2c^2-12\,a^4c^3\right)x^4-2\left(15\,a^2b^5-86\,a\right.}{12\,(cx^2+bx+a)(b^2-4\,ac)a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)^2*x),x, algorithm="giac")
```

```
[Out] -(5*b^7 - 42*a*b^5*c + 105*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*
c*x + b)/sqrt(-b^2 + 4*a*c))/((a^6*b^2 - 4*a^7*c)*sqrt(-b^2 + 4*a
*c)) - 1/2*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*ln(c*x^2 + b*x + a)/a
^6 + (5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*ln(abs(x))/a^6 - 1/12*(3*a^
5*b^2 - 12*a^6*c - 12*(5*a*b^5*c - 27*a^2*b^3*c^2 + 29*a^3*b*c^3)
*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*b^2*c^2 - 12*a^4*c^3)*
x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*x^3 + (10*a^3
*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*c)*x
)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4)
```

3.29 $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=257

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}+\frac{bx\left(7b^2-12ac\right)\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3}-\frac{x\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}$$

```
[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((
105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(1920*c^4*x) - ((7*b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(240*c^2) + (x^2*(b + 8*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*c)
+ (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(9/2)*S
qrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.944702, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}+\frac{bx\left(7b^2-12ac\right)\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3}-\frac{x\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((
105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(1920*c^4*x) - ((7*b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(240*c^2) + (x^2*(b + 8*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*c)
+ (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTa
```

```
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(9/2)*S
qrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 91.2585, size = 241, normalized size = 0.94

$$\frac{b\left(-116ac+35b^2\right)\sqrt{ax^2+bx^3+cx^4}}{960c^3}+\frac{bx\left(-12ac+7b^2\right)\left(-4ac+b^2\right)\sqrt{a+bx+cx^2}\,\mathrm{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{\frac{9}{2}}\sqrt{ax^2+bx^3+cx^4}}+\frac{x^2\left(\frac{b}{2}+4cx\right)\sqrt{ax^2+bx^3+cx^4}}{20c}-\frac{x\left(-16ac+7b^2\right)\sqrt{ax^2+bx^3+cx^4}}{240c^2}-\frac{\sqrt{ax^2+bx^3+cx^4}\left(256a^2c^2-460ab^2c+105b^4\right)}{1920c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] b*(-116*a*c + 35*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(960*c**3)
+ b*x*(-12*a*c + 7*b**2)*(-4*a*c + b**2)*sqrt(a + b*x + c*x**2)*a
tanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(256*c**(9/2
)*sqrt(a*x**2 + b*x**3 + c*x**4)) + x**2*(b/2 + 4*c*x)*sqrt(a*x**
2 + b*x**3 + c*x**4)/(20*c) - x*(-16*a*c + 7*b**2)*sqrt(a*x**2 +
b*x**3 + c*x**4)/(240*c**2) - sqrt(a*x**2 + b*x**3 + c*x**4)*(256
*a**2*c**2 - 460*a*b**2*c + 105*b**4)/(1920*c**4*x)
```

Mathematica [A] time = 0.438264, size = 180, normalized size = 0.7

$$\frac{15x\left(48a^2bc^2-40ab^3c+7b^5\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)+2\sqrt{c}x(a+x(b+cx))\left(128c^2\left(-2a^2+acx\right.\right.}{3840c^{9/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(
115*a - 14*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2
 + a*c*x^2 + 3*c^2*x^4)) + 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)
*x*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)]])/(3840*c^(9/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.013, size = 310, normalized size = 1.2

$$\frac{1}{3840\,x}\sqrt{cx^4+bx^3+ax^2}\left(768\,x^2\,\left(cx^2+bx+a\right)^{3/2}c^{9/2}-672\,\left(cx^2+bx+a\right)^{3/2}c^{7/2}xb-512\,\left(cx^2+bx+a\right)^{3/2}c^{7/2}a+560\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/3840*(c*x^4+b*x^3+a*x^2)^(1/2)*(768*x^2*(c*x^2+b*x+a)^(3/2)*c^(
9/2)-672*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b-512*(c*x^2+b*x+a)^(3/2)*
c^(7/2)*a+560*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b^2+720*(c*x^2+b*x+a)^(
1/2)*c^(7/2)*x*a*b-420*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^3+360*(c*x
^2+b*x+a)^(1/2)*c^(5/2)*a*b^2-210*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^4
+720*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*
b*c^3-600*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))
*a*b^3*c^2+105*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(
1/2))*b^5*c)/x/(c*x^2+b*x+a)^(1/2)/c^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29767, size = 1, normalized size = 0.

$$\left[\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{c}x\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x+bc\right)+\left(8\,c^2x^3+8\,bcx^2+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right)+4\left(384\,c^5x^4+48\,bc^4x^3-105\right.}{7680\,c^5x}\right.$$

$$-\frac{15\left(7\,b^5-40\,ab^3c+48\,a^2bc^2\right)\sqrt{-c}x\arctan\left(\frac{\left(2\,cx^2+bx\right)\sqrt{-c}}{2\sqrt{cx^4+bx^3+ax^2}c}\right)-2\left(384\,c^5x^4+48\,bc^4x^3-105\,b^4c+460\,ab^2c^2-256\,a^2c^3-\right.}{3840\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(4
*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c) + (8*c^2*x^3 + 8*b*c
*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x) + 4*(384*c^5*x^4 + 48*b*c^4*x
^3 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*
a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 +
 a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)
*sqrt(-c)*x*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x
^3 + a*x^2)*c)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460
*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b
^3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x^2\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.301706, size = 382, normalized size = 1.49

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,x\mathrm{sign}\left(x\right)+\frac{b\mathrm{sign}\left(x\right)}{c}\right)x-\frac{7\,b^2c^2\mathrm{sign}\left(x\right)-16\,ac^3\mathrm{sign}\left(x\right)}{c^4}\right)x+\frac{35\,b^3c\mathrm{sign}\left(x\right)-116\,abc^2\mathrm{sign}}{c^4}\right.\right.$$
$$-\frac{\left(7\,b^5\mathrm{sign}\left(x\right)-40\,ab^3c\mathrm{sign}\left(x\right)+48\,a^2bc^2\mathrm{sign}\left(x\right)\right)\ln\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{9}{2}}}$$
$$+\frac{\left(105\,b^5\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-600\,ab^3c\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+720\,a^2bc^2\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+210\,\sqrt{a}b^4\sqrt{c}-920\,a^{\frac{3}{2}}b^2c^{\frac{3}{2}}+512}{3840\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sign(x) + b*sign(x)/c)
*x - (7*b^2*c^2*sign(x) - 16*a*c^3*sign(x))/c^4)*x + (35*b^3*c*si
gn(x) - 116*a*b*c^2*sign(x))/c^4)*x - (105*b^4*sign(x) - 460*a*b^
2*c*sign(x) + 256*a^2*c^2*sign(x))/c^4) - 1/256*(7*b^5*sign(x) -
```

```
40*a*b^3*c*sign(x) + 48*a^2*b*c^2*sign(x))*ln(abs(-2*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2) + 1/3840*(105*b^5*l
n(abs(-b + 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*ln(abs(-b + 2*sqrt(a
)*sqrt(c))) + 720*a^2*b*c^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 210
*sqrt(a)*b^4*sqrt(c) - 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5
/2))*sign(x)/c^(9/2)
```

3.30 $\int x\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=205

$$-\frac{x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}$$

```
[Out] -((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*c^2) + (b*(15
*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + (x*(b +
 6*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*c) - ((b^2 - 4*a*c)*(5*b
^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^
4])
```

Rubi [A] time = 0.58968, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{x\left(b^2-4ac\right)\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*c^2) + (b*(15
*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + (x*(b +
 6*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*c) - ((b^2 - 4*a*c)*(5*b
^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^
4])
```

Rubi in Sympy [A] time = 59.5143, size = 190, normalized size = 0.93

$$\frac{b\left(-52ac+15b^2\right)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}+\frac{x\left(\frac{b}{2}+3cx\right)\sqrt{ax^2+bx^3+cx^4}}{12c}-\frac{\left(-12ac+5b^2\right)\sqrt{ax^2+bx^3+cx^4}}{96c^2}-\frac{x\left(-4ac+b^2\right)\left(-4ac+5b^2\right)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{\frac{7}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] b*(-52*a*c + 15*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(192*c**3*x)
 + x*(b/2 + 3*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(12*c) - (-12*a
*c + 5*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(96*c**2) - x*(-4*a*c
 + b**2)*(-4*a*c + 5*b**2)*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*
x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(128*c**(7/2)*sqrt(a*x**2
+ b*x**3 + c*x**4))
```

Mathematica [A] time = 0.332886, size = 150, normalized size = 0.73

$$\frac{2\sqrt{c}x(a+x(b+cx))\left(b\left(8c^2x^2-52ac\right)+24c^2x\left(a+2cx^2\right)+15b^3-10b^2cx\right)-3x\left(16a^2c^2-24ab^2c+5b^4\right)\sqrt{a+x(b+cx)}\,\mathrm{lo}}{384c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a
 + 2*c*x^2) + b*(-52*a*c + 8*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2*c +
16*a^2*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqr
t[a + x*(b + c*x)]])/(384*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.011, size = 265, normalized size = 1.3

$$\frac{1}{384\,x}\sqrt{cx^4+bx^3+ax^2}\left(96\,x\left(cx^2+bx+a\right)^{3/2}c^{7/2}-80\,\left(cx^2+bx+a\right)^{3/2}c^{5/2}b-48\,\sqrt{cx^2+bx+a}c^{7/2}xa+60\,\sqrt{cx^2+bx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(96*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)
-80*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b-48*(c*x^2+b*x+a)^(1/2)*c^(7/2)*
x*a+60*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^2-24*(c*x^2+b*x+a)^(1/2)*c
^(5/2)*a*b+30*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^3-48*ln(1/2*(2*(c*x^2
+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*c^3+72*ln(1/2*(2*(c*x
^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*c^2-15*ln(1/2*(2*
(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^4*c)/x/(c*x^2+b*x
+a)^(1/2)/c^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292408, size = 1, normalized size = 0.

$$\left[\frac{3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\sqrt{c}x\log\left(\frac{4\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x+bc\right)-\left(8\,c^2x^3+8\,bcx^2+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right)+4\left(48\,c^4x^3+8\,bc^3x^2+15\,b^3c-5\right.}{768\,c^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log((4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c) - (8*c^2*x^3 + 8*b*c*x^2
+ (b^2 + 4*a*c)*x)*sqrt(c))/x) + 4*(48*c^4*x^3 + 8*b*c^3*x^2 + 15
*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*sqrt(c*x^4 + b*
x^3 + a*x^2))/(c^4*x), 1/384*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)
*sqrt(-c)*x*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x
^3 + a*x^2)*c)) + 2*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b
*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(
c^4*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.303799, size = 311, normalized size = 1.52

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,x\mathrm{sign}\left(x\right)+\frac{b\mathrm{sign}\left(x\right)}{c}\right)x-\frac{5\,b^2c\mathrm{sign}\left(x\right)-12\,ac^2\mathrm{sign}\left(x\right)}{c^3}\right)x+\frac{15\,b^3\mathrm{sign}\left(x\right)-52\,abc\mathrm{sign}\left(x\right)}{c^3}\right)$$
$$+\frac{\left(5\,b^4\mathrm{sign}\left(x\right)-24\,ab^2c\mathrm{sign}\left(x\right)+16\,a^2c^2\mathrm{sign}\left(x\right)\right)\ln\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{7}{2}}}$$
$$-\frac{\left(15\,b^4\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^3\sqrt{c}-104\,a^{\frac{3}{2}}bc^{\frac{3}{2}}\right)\mathrm{sign}\left(x\right)}{384\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x,x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*sign(x) + b*sign(x)/c)*x -
 (5*b^2*c*sign(x) - 12*a*c^2*sign(x))/c^3)*x + (15*b^3*sign(x) -
52*a*b*c*sign(x))/c^3) + 1/128*(5*b^4*sign(x) - 24*a*b^2*c*sign(x
) + 16*a^2*c^2*sign(x))*ln(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*sqrt(c) - b))/c^(7/2) - 1/384*(15*b^4*ln(abs(-b + 2*sqrt(a)*
sqrt(c))) - 72*a*b^2*c*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 48*a^2*c
^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104
*a^(3/2)*b*c^(3/2))*sign(x)/c^(7/2)
```

3.31 $\int \sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=163

$$\frac{b\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx}$$

```
[Out] -(b*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c^2*x) + ((a + b*
x + c*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c
)*Sqrt[a*x^2 + b*x^3 + c*x^4]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(16*c^(5/2)*x*Sqrt[a + b*x + c*x^2])
```

Rubi [A] time = 0.124519, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c^2*x) + ((a + b*
x + c*x^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c
)*Sqrt[a*x^2 + b*x^3 + c*x^4]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(16*c^(5/2)*x*Sqrt[a + b*x + c*x^2])
```

Rubi in Sympy [A] time = 24.5548, size = 146, normalized size = 0.9

$$-\frac{b\,(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{b\left(-4ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}\,\mathrm{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{\frac{5}{2}}x\sqrt{a+bx+cx^2}}+\frac{\left(a+bx+cx^2\right)\sqrt{ax^2+bx^3+cx^4}}{3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] -b*(b + 2*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(8*c**2*x) + b*(-4*
a*c + b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)*atanh((b + 2*c*x)/(2*s
qrt(c)*sqrt(a + b*x + c*x**2)))/(16*c**(5/2)*x*sqrt(a + b*x + c*x
**2)) + (a + b*x + c*x**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(3*c*x)
```

Mathematica [A] time = 0.332574, size = 117, normalized size = 0.72

$$\frac{3x\left(b^3-4abc\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)+2\sqrt{c}x(a+x(b+cx))\left(8c\left(a+cx^2\right)-3b^2+2bcx\right)}{48c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-3*b^2 + 2*b*c*x + 8*c*(a + c*x^2
)) + 3*(b^3 - 4*a*b*c)*x*Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*
Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(48*c^(5/2)*Sqrt[x^2*(a + x*(b +
c*x))])
```

Maple [A] time = 0.008, size = 167, normalized size = 1.

$$\frac{1}{48\,x}\sqrt{cx^4+bx^3+ax^2}\left(16\,\left(cx^2+bx+a\right)^{3/2}c^{5/2}-12\,\sqrt{cx^2+bx+a}c^{5/2}xb-6\,\sqrt{cx^2+bx+a}c^{3/2}b^2-12\,\ln\left(1/2\,\frac{2\,\sqrt{cx^2+}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(16*(c*x^2+b*x+a)^(3/2)*c^(5/2)-12
*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b-6*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^
2-12*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b*
c^2+3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^3
*c)/x/(c*x^2+b*x+a)^(1/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27975, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^{3}-4\,abc\right)\sqrt{c}x\log\left(\frac{4\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(2\,c^{2}x+bc\right)-\left(8\,c^{2}x^{3}+8\,bcx^{2}+\left(b^{2}+4\,ac\right)x\right)\sqrt{c}}{x}\right)-4\left(8\,c^{3}x^{2}+2\,bc^{2}x-3\,b^{2}c+8\,ac^{2}\right)\sqrt{cx^{4}+bx}}{96\,c^{3}x}\right.$$

$$\left.-\frac{3\left(b^{3}-4\,abc\right)\sqrt{-c}x\arctan\left(\frac{\left(2\,cx^{2}+bx\right)\sqrt{-c}}{2\sqrt{cx^{4}+bx^{3}+ax^{2}}c}\right)-2\left(8\,c^{3}x^{2}+2\,bc^{2}x-3\,b^{2}c+8\,ac^{2}\right)\sqrt{cx^{4}+bx^{3}+ax^{2}}}{48\,c^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(c)*x*log((4*sqrt(c*x^4 + b*x^3 + a
*x^2)*(2*c^2*x + b*c) - (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)
*sqrt(c))/x) - 4*(8*c^3*x^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt
(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -1/48*(3*(b^3 - 4*a*b*c)*sqrt(-
c)*x*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*
x^2)*c)) - 2*(8*c^3*x^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x
^4 + b*x^3 + a*x^2))/(c^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{ax^{2}+bx^{3}+cx^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.300165, size = 224, normalized size = 1.37

$$\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(4\,x\mathrm{sign}\left(x\right)+\frac{b\mathrm{sign}\left(x\right)}{c}\right)x-\frac{3\,b^2\mathrm{sign}\left(x\right)-8\,ac\mathrm{sign}\left(x\right)}{c^2}\right)-\frac{\left(b^3\mathrm{sign}\left(x\right)-4\,abc\mathrm{sign}\left(x\right)\right)\ln\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}+\frac{\left(3\,b^3\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-12\,abc\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sign}\left(x\right)}{48\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*sign(x) + b*sign(x)/c)*x - (3*
b^2*sign(x) - 8*a*c*sign(x))/c^2) - 1/16*(b^3*sign(x) - 4*a*b*c*s
ign(x))*ln(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b
))/c^(5/2) + 1/48*(3*b^3*ln(abs(-b + 2*sqrt(a)*sqrt(c))) - 12*a*b
*c*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a
^(3/2)*c^(3/2))*sign(x)/c^(5/2)
```

3.32 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx$

Optimal. Leaf size=119

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c
)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.131767, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c
)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(8*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 19.825, size = 107, normalized size = 0.9

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x\left(-4ac+b^2\right)\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{\frac{3}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] (b + 2*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(4*c*x) - x*(-4*a*c +
b**2)*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a
+ b*x + c*x**2)))/(8*c**(3/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.236237, size = 100, normalized size = 0.84

$$\frac{x\left(2\sqrt{c}(b+2cx)(a+x(b+cx))-\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\right)}{8c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] (x*(2*Sqrt[c]*(b + 2*c*x)*(a + x*(b + c*x)) - (b^2 - 4*a*c)*Sqrt[
a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
))/(8*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.007, size = 146, normalized size = 1.2

$$\frac{1}{8\,x}\sqrt{cx^4+bx^3+ax^2}\left(4\sqrt{cx^2+bx+a}c^{5/2}x+2\sqrt{cx^2+bx+a}c^{3/2}b+4\ln\left(1/2\,\frac{2\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx+b}{\sqrt{c}}\right)ac^2-\ln\left(\frac{1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x)
```

```
[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x+2*
(c*x^2+b*x+a)^(1/2)*c^(3/2)*b+4*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(
1/2)+2*c*x+b)/c^(1/2))*a*c^2-ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2
)+2*c*x+b)/c^(1/2))*b^2*c)/(c*x^2+b*x+a)^(1/2)/c^(5/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285538, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-4ac\right)\sqrt{c}x\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}\left(2c^2x+bc\right)+\left(8c^2x^3+8bcx^2+\left(b^2+4ac\right)x\right)\sqrt{c}}{x}\right)-4\sqrt{cx^4+bx^3+ax^2}\left(2c^2x+bc\right)}{16c^2x}, \frac{\left(b^2-4ac\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c^2*x + b*c) + (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sq
rt(c))/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x
), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*arctan(1/2*(2*c*x^2 + b*x)*sqrt(
-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) + 2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c^2*x + b*c))/(c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)
```

GIAC/XCAS [A] time = 0.294393, size = 169, normalized size = 1.42

$$\frac{1}{8}\left(2\sqrt{cx^2+bx+a}\left(2x+\frac{b}{c}\right)+\frac{\left(b^2-4ac\right)\ln\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}\right)\mathrm{sign}\left(x\right)-\frac{\left(b^2\ln\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-4ac\ln\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}b\sqrt{c}\right)\mathrm{sign}\left(x\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*ln(abs(-
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2))*sign
(x) - 1/8*(b^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*ln(abs(-b
+ 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*b*sqrt(c))*sign(x)/c^(3/2)
```

3.33 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} \, dx$

Optimal. Leaf size=173

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*
ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^
2 + b*x^3 + c*x^4] + (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*Sqrt[a*x^2 + b*
x^3 + c*x^4])
```

Rubi [A] time = 0.260683, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*
ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^
2 + b*x^3 + c*x^4] + (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*Sqrt[a*x^2 + b*
x^3 + c*x^4])
```

Rubi in Sympy [A] time = 37.0255, size = 158, normalized size = 0.91

$$-\frac{\sqrt{a}x\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] -sqrt(a)*x*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sq
rt(a + b*x + c*x**2)))/sqrt(a*x**2 + b*x**3 + c*x**4) + b*x*sqrt(
```

```
a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x
**2)))/(2*sqrt(c)*sqrt(a*x**2 + b*x**3 + c*x**4)) + sqrt(a*x**2 +
 b*x**3 + c*x**4)/x
```

Mathematica [A] time = 0.164966, size = 144, normalized size = 0.83

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}-2\sqrt{a}\sqrt{c}\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)+b\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)+\right.}{2\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] + 2*Sqr
t[a]*Sqrt[c]*Log[x] - 2*Sqrt[a]*Sqrt[c]*Log[2*a + b*x + 2*Sqrt[a]
*Sqrt[a + x*(b + c*x)]] + b*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*
(b + c*x)]]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.007, size = 126, normalized size = 0.7

$$-\frac{1}{2x}\sqrt{cx^4+bx^3+ax^2}\left(2\sqrt{a}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\sqrt{c}-2\sqrt{cx^2+bx+a}\sqrt{c}-b\ln\left(\frac{1}{2}\left(2\sqrt{cx^2+bx+a}\sqrt{c}+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x)
```

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(2*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x)*c^(1/2)-2*(c*x^2+b*x+a)^(1/2)*c^(1/2)-b*ln
(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2)))/x/(c*x^2+b
*x+a)^(1/2)/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.312332, size = 1, normalized size = 0.01

$$\left[\frac{b\sqrt{c}x\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}(2c^2x+bc)+(8c^2x^3+8bcx^2+(b^2+4ac)x)\sqrt{c}}{x}\right)+2\sqrt{a}cx\log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a}{x^3}\right)}{4cx}\right.$$

$$-\frac{b\sqrt{-c}x\arctan\left(\frac{(2cx^2+bx)\sqrt{-c}}{2\sqrt{cx^4+bx^3+ax^2}c}\right)-\sqrt{a}cx\log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)-2\sqrt{cx^4+bx^3+ax^2}c}{2cx},$$

$$-\frac{4\sqrt{-a}cx\arctan\left(\frac{bx^2+2ax}{2\sqrt{cx^4+bx^3+ax^2}\sqrt{-a}}\right)-b\sqrt{c}x\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}(2c^2x+bc)+(8c^2x^3+8bcx^2+(b^2+4ac)x)\sqrt{c}}{x}\right)-4\sqrt{cx^4+bx^3+a}}{4cx}$$

$$\left.-\frac{2\sqrt{-a}cx\arctan\left(\frac{bx^2+2ax}{2\sqrt{cx^4+bx^3+ax^2}\sqrt{-a}}\right)+b\sqrt{-c}x\arctan\left(\frac{(2cx^2+bx)\sqrt{-c}}{2\sqrt{cx^4+bx^3+ax^2}c}\right)-2\sqrt{cx^4+bx^3+ax^2}c}{2cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x +
b*c) + (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x) + 2*
sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqr
t(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*c)/(c*x), -1/2*(b*sqrt(-c)*x*arctan(1/2*(2*c*x^
2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) - sqrt(a)*c*x*
log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*
x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 2*sqrt(c*x^4 + b*x^3 + a
*x^2)*c)/(c*x), -1/4*(4*sqrt(-a)*c*x*arctan(1/2*(b*x^2 + 2*a*x)/(
sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-a))) - b*sqrt(c)*x*log(-(4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c) + (8*c^2*x^3 + 8*b*c*x^2
+ (b^2 + 4*a*c)*x)*sqrt(c))/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)
/(c*x), -1/2*(2*sqrt(-a)*c*x*arctan(1/2*(b*x^2 + 2*a*x)/(sqrt(c*x
^4 + b*x^3 + a*x^2)*sqrt(-a))) + b*sqrt(-c)*x*arctan(1/2*(2*c*x^2
 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) - 2*sqrt(c*x^4
+ b*x^3 + a*x^2)*c)/(c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.34 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3} \, dx$

Optimal. Leaf size=173

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*Sqrt[a + b*x + c*x^2]*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]
*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^
2 + b*x^3 + c*x^4]
```

Rubi [A] time = 0.26131, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*Sqrt[a + b*x + c*x^2]*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]
*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^
2 + b*x^3 + c*x^4]
```

Rubi in Sympy [A] time = 36.7956, size = 160, normalized size = 0.92

$$\frac{\sqrt{c}x\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] sqrt(c)*x*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqr
t(a + b*x + c*x**2)))/sqrt(a*x**2 + b*x**3 + c*x**4) - sqrt(a*x**
```

```
2 + b*x**3 + c*x**4)/x**2 - b*x*sqrt(a + b*x + c*x**2)*atanh((2*a
 + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(2*sqrt(a)*sqrt(a*x**
2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.223059, size = 133, normalized size = 0.77

$$\frac{\sqrt{a+x(b+cx)}\left(-bx\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)-2\sqrt{a}\left(\sqrt{a+x(b+cx)}-\sqrt{c}x\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\right)\right)}{2\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(b*x*Log[x] - b*x*Log[2*a + b*x + 2*Sqrt[a
]*Sqrt[a + x*(b + c*x)]] - 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)] - Sqr
t[c]*x*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])))/(2*Sqr
t[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.009, size = 173, normalized size = 1.

$$-\frac{1}{2\,ax^2}\sqrt{cx^4+bx^3+ax^2}\left(-2\,\sqrt{cx^2+bx+a}c^{5/2}x^2+\sqrt{a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)c^{\frac{3}{2}}xb+2\,\left(cx^2+bx+a\right)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(-2*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x^
2+a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*c^(3/2)*x
*b+2*(c*x^2+b*x+a)^(3/2)*c^(3/2)-2*(c*x^2+b*x+a)^(1/2)*c^(3/2)*x*
b-2*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*x*a*c
^2)/x^2/(c*x^2+b*x+a)^(1/2)/a/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.332382, size = 1, normalized size = 0.01

$$\left[\frac{2\,a\sqrt{c}x^2\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+\sqrt{a}bx^2\log\left(\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)-\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right.}{x^3}\right.}{4\,ax^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4
+ b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + sqrt
(a)*b*x^2*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2) - (8
*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) - 4*sqrt(c*
x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/4*(4*a*sqrt(-c)*x^2*arctan(1/2
*(2*c*x^2 + b*x)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-c))) + sqrt(a
)*b*x^2*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2) - (8*a
*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) - 4*sqrt(c*x^
4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*(b*
x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2)*a)) + a*sqrt(c
)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 2*sqrt(c*x^4 + b*x^3
 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*(b*x^2 + 2*a
*x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2)*a)) + 2*a*sqrt(-c)*x^2*
arctan(1/2*(2*c*x^2 + b*x)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-c))
) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.35 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4} dx$

Optimal. Leaf size=114

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(4*a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[
a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(3/2))
```

Rubi [A] time = 0.237801, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(4*a*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[
a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(3/2))
```

Rubi in Sympy [A] time = 41.9542, size = 128, normalized size = 1.12

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{x\left(-4ac+b^2\right)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{\frac{3}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] -sqrt(a*x**2 + b*x**3 + c*x**4)/(2*x**3) - b*sqrt(a*x**2 + b*x**3
 + c*x**4)/(4*a*x**2) + x*(-4*a*c + b**2)*sqrt(a + b*x + c*x**2)*
atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(8*a**(3/2)
*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.171924, size = 125, normalized size = 1.1

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(x^2\log(x)\left(-\left(b^2-4ac\right)\right)+x^2\left(b^2-4ac\right)\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)-2\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}\right)}{8a^{3/2}x^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(
b + c*x)] - (b^2 - 4*a*c)*x^2*Log[x] + (b^2 - 4*a*c)*x^2*Log[2*a
+ b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]]))/(8*a^(3/2)*x^3*Sqrt[a
+ x*(b + c*x)])
```

Maple [B] time = 0.009, size = 206, normalized size = 1.8

$$\frac{1}{8\,a^2x^3}\sqrt{cx^4+bx^3+ax^2}\left(-4\,a^{3/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)cx^2-2\,\sqrt{cx^2+bx+a}cx^3b+4\,\sqrt{cx^2+bx+a}cx^2a+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x)
```

```
[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(-4*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x)*c*x^2-2*(c*x^2+b*x+a)^(1/2)*c*x^3*b+4*(c*x
^2+b*x+a)^(1/2)*c*x^2*a+a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+
a)^(1/2))/x)*x^2*b^2+2*(c*x^2+b*x+a)^(3/2)*x*b-2*(c*x^2+b*x+a)^(1
/2)*x^2*b^2-4*(c*x^2+b*x+a)^(3/2)*a)/x^3/(c*x^2+b*x+a)^(1/2)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292596, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^{2}-4\,ac\right)\sqrt{a}x^{3}\log\left(\frac{4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(abx+2\,a^{2}\right)-\left(8\,abx^{2}+\left(b^{2}+4\,ac\right)x^{3}+8\,a^{2}x\right)\sqrt{a}}{x^{3}}\right)+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(abx+2\,a^{2}\right)}{16\,a^{2}x^{3}},\right.$$

$$\left.-\frac{\left(b^{2}-4\,ac\right)\sqrt{-a}x^{3}\arctan\left(\frac{\left(bx^{2}+2\,ax\right)\sqrt{-a}}{2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}a}\right)+2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(abx+2\,a^{2}\right)}{8\,a^{2}x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log((4*sqrt(c*x^4 + b*x^3 + a*x
^2)*(a*b*x + 2*a^2) - (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*s
qrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^
2*x^3), -1/8*((b^2 - 4*a*c)*sqrt(-a)*x^3*arctan(1/2*(b*x^2 + 2*a*
x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2)*a)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^{2}\left(a+bx+cx^{2}\right)}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4,x, algorithm="giac")
```

[Out] Timed out

3.36 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$

Optimal. Leaf size=155

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(12*a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(24*a^2*x^2) - (b*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a
]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(5/2))
```

Rubi [A] time = 0.412583, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}+\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(12*a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/
(24*a^2*x^2) - (b*(b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a
]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(5/2))
```

Rubi in Sympy [A] time = 64.5217, size = 167, normalized size = 1.08

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{\left(-8ac+3b^2\right)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{bx\left(-4ac+b^2\right)\sqrt{a+bx+cx^2}\,\mathrm{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{\frac{5}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] -sqrt(a*x**2 + b*x**3 + c*x**4)/(3*x**4) - b*sqrt(a*x**2 + b*x**3
 + c*x**4)/(12*a*x**3) + (-8*a*c + 3*b**2)*sqrt(a*x**2 + b*x**3 +
 c*x**4)/(24*a**2*x**2) - b*x*(-4*a*c + b**2)*sqrt(a + b*x + c*x*
*2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(16*a**
(5/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.38975, size = 145, normalized size = 0.94

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(-2\sqrt{a}\sqrt{a+x(b+cx)}\left(8a^2+2ax(b+4cx)-3b^2x^2\right)+3bx^3\log(x)\left(b^2-4ac\right)-3bx^3\left(b^2-4ac\right)\log\left(2\sqrt{\right.\right.}{48a^{5/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(8
*a^2 - 3*b^2*x^2 + 2*a*x*(b + 4*c*x)) + 3*b*(b^2 - 4*a*c)*x^3*Log
[x] - 3*b*(b^2 - 4*a*c)*x^3*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*
(b + c*x)]]))/(48*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.009, size = 234, normalized size = 1.5

$$-\frac{1}{48\,x^4a^3}\sqrt{cx^4+bx^3+ax^2}\left(-12\,a^{3/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)cx^3b-6\,\sqrt{cx^2+bx+a}cx^4b^2+12\,\sqrt{cx^2+bx+a}cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x)
```

```
[Out] -1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(-12*a^(3/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x)*c*x^3*b-6*(c*x^2+b*x+a)^(1/2)*c*x^4*b^2
+12*(c*x^2+b*x+a)^(1/2)*c*x^3*a*b+3*a^(1/2)*ln((2*a+b*x+2*a^(1/2)
*(c*x^2+b*x+a)^(1/2))/x)*x^3*b^3+6*(c*x^2+b*x+a)^(3/2)*x^2*b^2-6*
(c*x^2+b*x+a)^(1/2)*x^3*b^3-12*(c*x^2+b*x+a)^(3/2)*x*a*b+16*(c*x^
2+b*x+a)^(3/2)*a^2)/x^4/(c*x^2+b*x+a)^(1/2)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292678, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^3-4\,abc\right)\sqrt{a}x^4\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)+\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right)+4\sqrt{cx^4+bx^3+ax^2}\left(2\,a^2bx+8\,a^3-\left(3\,ab\right.\right.}{96\,a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^4*log(-(4*sqrt(c*x^4 + b*x^3
+ a*x^2)*(a*b*x + 2*a^2) + (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2
*x)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*
a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^4), 1/48*(3*(b^3 - 4*a*b*c
)*sqrt(-a)*x^4*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 +
b*x^3 + a*x^2)*a)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8
*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.37 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$

Optimal. Leaf size=205

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(4*x^5) - (b*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(24*a*x^4) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(96*a^2*x^3) - (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(192*a^3*x^2) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a +
 b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*a^(7/2))
```

Rubi [A] time = 0.619445, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(4*x^5) - (b*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(24*a*x^4) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(96*a^2*x^3) - (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(192*a^3*x^2) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a +
 b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*a^(7/2))
```

Rubi in Sympy [A] time = 84.7958, size = 214, normalized size = 1.04

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{(-12ac+5b^2)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b(-52ac+15b^2)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{x(-4ac+b^2)(-4ac+5b^2)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{\frac{7}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
```

```
[Out] -sqrt(a*x**2 + b*x**3 + c*x**4)/(4*x**5) - b*sqrt(a*x**2 + b*x**3
 + c*x**4)/(24*a*x**4) + (-12*a*c + 5*b**2)*sqrt(a*x**2 + b*x**3
+ c*x**4)/(96*a**2*x**3) - b*(-52*a*c + 15*b**2)*sqrt(a*x**2 + b*
x**3 + c*x**4)/(192*a**3*x**2) + x*(-4*a*c + b**2)*(-4*a*c + 5*b*
*2)*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a +
b*x + c*x**2)))/(128*a**(7/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.392298, size = 186, normalized size = 0.91

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(-3x^4\log(x)\left(16a^2c^2-24ab^2c+5b^4\right)+3x^4\left(16a^2c^2-24ab^2c+5b^4\right)\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)\right.}{384a^{7/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(4
8*a^3 + 15*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*
x)) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^4*Log[x] + 3*(5*b^4 -
 24*a*b^2*c + 16*a^2*c^2)*x^4*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a +
x*(b + c*x)]]))/(384*a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])
```

Maple [B] time = 0.011, size = 387, normalized size = 1.9

$$-\frac{1}{384\,x^5a^4}\sqrt{cx^4+bx^3+ax^2}\left(-48\,a^{5/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)c^2x^4-24\,\sqrt{cx^2+bx+a}c^2x^5ab+72\,a^{3/2}\ln\left(\frac{2\,a+\cdots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x)
```

```
[Out] -1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(-48*a^(5/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)*c^2*x^4-24*(c*x^2+b*x+a)^(1/2)*c^2*x^5
*a*b+72*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*c*x
^4*b^2+48*(c*x^2+b*x+a)^(1/2)*c^2*x^4*a^2+30*(c*x^2+b*x+a)^(1/2)*
c*x^5*b^3+24*(c*x^2+b*x+a)^(3/2)*c*x^3*a*b-84*(c*x^2+b*x+a)^(1/2)
*c*x^4*a*b^2-15*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2)
)/x)*x^4*b^4-48*(c*x^2+b*x+a)^(3/2)*c*x^2*a^2-30*(c*x^2+b*x+a)^(3
```

```
/2)*x^3*b^3+30*(c*x^2+b*x+a)^(1/2)*x^4*b^4+60*(c*x^2+b*x+a)^(3/2)
*x^2*a*b^2-80*(c*x^2+b*x+a)^(3/2)*x*a^2*b+96*(c*x^2+b*x+a)^(3/2)*
a^3)/x^5/(c*x^2+b*x+a)^(1/2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.30804, size = 1, normalized size = 0.

$$\left[\frac{3\left(5\,b^4 - 24\,ab^2c + 16\,a^2c^2\right)\sqrt{a}x^5\log\left(-\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)+\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right) - 4\left(8\,a^3bx + 48\,a^4 + \left(15\,ab^3 - \right.\right.}{768\,a^4x^5}\right.$$

$$-\frac{3\left(5\,b^4 - 24\,ab^2c + 16\,a^2c^2\right)\sqrt{-a}x^5\arctan\left(\frac{\left(bx^2+2\,ax\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^3+ax^2}a}\right) + 2\left(8\,a^3bx + 48\,a^4 + \left(15\,ab^3 - 52\,a^2bc\right)x^3 - 2\left(5\,a^2b^2 - 12\,a^3\right.\right.}{384\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(4*s
qrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2) + (8*a*b*x^2 + (b^2 +
4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) - 4*(8*a^3*b*x + 48*a^4 + (15
*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*x
^4 + b*x^3 + a*x^2))/(a^4*x^5), -1/384*(3*(5*b^4 - 24*a*b^2*c + 1
6*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt
(c*x^4 + b*x^3 + a*x^2)*a)) + 2*(8*a^3*b*x + 48*a^4 + (15*a*b^3 -
 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*x^4 + b*x
^3 + a*x^2))/(a^4*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.38 $\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=422

$$\frac{3x\left(b^2-4ac\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+bx^3+cx^4}}{71680c^4}+\frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{35840c^3}-\frac{b\left(-58816a^3c^3+81648a^2b^2c^2-30660ab^4c+3465b^6\right)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x}+\frac{\left(-6720a^3c^3+18896a^2b^2c^2-8988ab^4c+1155b^6\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5}-\frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^2}+\frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}$$

```
[Out] ((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*Sqr
t[a*x^2 + b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b
^4*c + 81648*a^2*b^2*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(573440*c^6*x) - (b*(231*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)
*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(71680*c^4) + ((99*b^4 - 568*a*b^
2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])/(35840*c^3) -
 (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[a*x^2
 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3
+ c*x^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c
 + 16*a^2*c^2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2)*Sqrt[a*x^2 + b*x^3
+ c*x^4])
```

Rubi [A] time = 1.87124, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$

$$\frac{3x\left(b^2-4ac\right)^2\left(16a^2c^2-72ab^2c+33b^4\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{bx\left(2416a^2c^2-1560ab^2c+231b^4\right)\sqrt{ax^2+bx^3+cx^4}}{71680c^4} + \frac{x^2\left(560a^2c^2-568ab^2c+99b^4\right)\sqrt{ax^2+bx^3+cx^4}}{35840c^3} - \frac{b\left(-58816a^3c^3+81648a^2b^2c^2-30660ab^4c+3465b^6\right)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x} + \frac{\left(-6720a^3c^3+18896a^2b^2c^2-8988ab^4c+1155b^6\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{x^3\left(10cx\left(11b^2-28ac\right)+b\left(68ac+11b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} + \frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*Sqr
t[a*x^2 + b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b
^4*c + 81648*a^2*b^2*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(573440*c^6*x) - (b*(231*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)
*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(71680*c^4) + ((99*b^4 - 568*a*b^
2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])/(35840*c^3) -
 (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[a*x^2
 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3
+ c*x^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c
 + 16*a^2*c^2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(32768*c^(13/2)*Sqrt[a*x^2 + b*x^3
+ c*x^4])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.647349, size = 304, normalized size = 0.72

$$\frac{x\sqrt{a+x(b+cx)}\left(105\left(16a^2c^2-72ab^2c+33b^4\right)\left(b^2-4ac\right)^2\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)+2\sqrt{c}\sqrt{a+x(b+cx)}\left(-16b\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*
b^7 + 2310*b^6*c*x + 84*b^5*c*(365*a - 22*c*x^2) + 24*b^4*c^2*x*(
-749*a + 66*c*x^2) + 32*b^2*c^3*x*(1181*a^2 - 284*a*c*x^2 + 40*c^
2*x^4) - 16*b^3*c^2*(5103*a^2 - 780*a*c*x^2 + 88*c^2*x^4) + 4480*
c^4*x*(-3*a^3 + 2*a^2*c*x^2 + 24*a*c^2*x^4 + 16*c^3*x^6) + 64*b*c
^3*(919*a^3 - 302*a^2*c*x^2 + 104*a*c^2*x^4 + 1360*c^3*x^6)) + 10
5*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*Log[b + 2*c*
x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(1146880*c^(13/2)*Sqrt[x^2
*(a + x*(b + c*x))])
```

Maple [A] time = 0.014, size = 649, normalized size = 1.5

$$\frac{1}{1146880\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(26880\,\ln\left(1/2\,\frac{2\,\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx+b}{\sqrt{c}}\right)a^4c^5+3465\,\ln\left(1/2\,\frac{2\,\sqrt{cx^2+bx+a}\sqrt{c}+2\,c}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/1146880*(c*x^4+b*x^3+a*x^2)^(3/2)*(26880*ln(1/2*(2*(c*x^2+b*x+a
)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^4*c^5+3465*ln(1/2*(2*(c*x^2+b
*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^8*c+143360*x^3*(c*x^2+b*x
+a)^(5/2)*c^(13/2)-59136*(c*x^2+b*x+a)^(5/2)*c^(7/2)*b^3+18480*(c
*x^2+b*x+a)^(3/2)*c^(5/2)*b^5+117600*ln(1/2*(2*(c*x^2+b*x+a)^(1/2
)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^4*c^3-35280*ln(1/2*(2*(c*x^2+b*
x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^6*c^2+17920*(c*x^2+b*x+a
)^(3/2)*c^(11/2)*x*a^2+95232*(c*x^2+b*x+a)^(5/2)*c^(9/2)*a*b+1344
0*(c*x^2+b*x+a)^(1/2)*c^(9/2)*a^3*b-63840*(c*x^2+b*x+a)^(1/2)*c^(
7/2)*a^2*b^3+42840*(c*x^2+b*x+a)^(1/2)*c^(5/2)*a*b^5-112640*(c*x^
2+b*x+a)^(5/2)*c^(11/2)*x^2*b-71680*(c*x^2+b*x+a)^(5/2)*c^(11/2)*
x*a+84480*(c*x^2+b*x+a)^(5/2)*c^(9/2)*x*b^2+26880*(c*x^2+b*x+a)^(
1/2)*c^(11/2)*x*a^3-13860*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^6-40320
*(c*x^2+b*x+a)^(3/2)*c^(7/2)*a*b^3+36960*(c*x^2+b*x+a)^(3/2)*c^(7
/2)*x*b^4+8960*(c*x^2+b*x+a)^(3/2)*c^(9/2)*a^2*b-134400*ln(1/2*(2
*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*b^2*c^4-127680
```

```
*(c*x^2+b*x+a)^(1/2)*c^(9/2)*x*a^2*b^2+85680*(c*x^2+b*x+a)^(1/2)*
c^(7/2)*x*a*b^4-80640*(c*x^2+b*x+a)^(3/2)*c^(9/2)*x*a*b^2-6930*(c
*x^2+b*x+a)^(1/2)*c^(3/2)*b^7)/x^3/(c*x^2+b*x+a)^(3/2)/c^(15/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.374168, size = 1, normalized size = 0.

$$\left[\frac{105\left(33\,b^8 - 336\,ab^6c + 1120\,a^2b^4c^2 - 1280\,a^3b^2c^3 + 256\,a^4c^4\right)\sqrt{c}x\log\left(-\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x+bc\right)+\left(8\,c^2x^3+8\,bcx^2+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right.}{}\right.$$

$$-\frac{105\left(33\,b^8 - 336\,ab^6c + 1120\,a^2b^4c^2 - 1280\,a^3b^2c^3 + 256\,a^4c^4\right)\sqrt{-c}x\arctan\left(\frac{\left(2\,cx^2+bx\right)\sqrt{-c}}{2\,\sqrt{cx^4+bx^3+ax^2}c}\right) - 2\left(71680\,c^8x^7 + 87040\,bc^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a
^3*b^2*c^3 + 256*a^4*c^4)*sqrt(c)*x*log(-(4*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c^2*x + b*c) + (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x
)*sqrt(c))/x) + 4*(71680*c^8*x^7 + 87040*b*c^7*x^6 - 3465*b^7*c +
 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280*(b^
2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(9
9*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1
560*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^
4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 +
 a*x^2))/(c^7*x), -1/1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^
2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*sqrt(-c)*x*arctan(1/2
*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) - 2*(7
1680*c^8*x^7 + 87040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 8
1648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^
5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(99*b^4*c^4 - 568*a*b^
```

```
2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416
*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^
2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(a + b*x + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.339766, size = 703, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*c*x*sign(x)
 + 17*b*sign(x))*x + (b^2*c^6*sign(x) + 84*a*c^7*sign(x))/c^7)*x
- (11*b^3*c^5*sign(x) - 52*a*b*c^6*sign(x))/c^7)*x + (99*b^4*c^4*
sign(x) - 568*a*b^2*c^5*sign(x) + 560*a^2*c^6*sign(x))/c^7)*x - (
231*b^5*c^3*sign(x) - 1560*a*b^3*c^4*sign(x) + 2416*a^2*b*c^5*sig
n(x))/c^7)*x + (1155*b^6*c^2*sign(x) - 8988*a*b^4*c^3*sign(x) + 1
8896*a^2*b^2*c^4*sign(x) - 6720*a^3*c^5*sign(x))/c^7)*x - (3465*b
^7*c*sign(x) - 30660*a*b^5*c^2*sign(x) + 81648*a^2*b^3*c^3*sign(x
) - 58816*a^3*b*c^4*sign(x))/c^7) - 3/32768*(33*b^8*sign(x) - 336
*a*b^6*c*sign(x) + 1120*a^2*b^4*c^2*sign(x) - 1280*a^3*b^2*c^3*si
gn(x) + 256*a^4*c^4*sign(x))*ln(abs(-2*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*sqrt(c) - b))/c^(13/2) + 1/1146880*(3465*b^8*ln(abs(-b
+ 2*sqrt(a)*sqrt(c))) - 35280*a*b^6*c*ln(abs(-b + 2*sqrt(a)*sqrt(
c))) + 117600*a^2*b^4*c^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) - 13440
0*a^3*b^2*c^3*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 26880*a^4*c^4*ln(
abs(-b + 2*sqrt(a)*sqrt(c))) + 6930*sqrt(a)*b^7*sqrt(c) - 61320*a
^(3/2)*b^5*c^(3/2) + 163296*a^(5/2)*b^3*c^(5/2) - 117632*a^(7/2)*
b*c^(7/2))*sign(x)/c^(13/2)
```

3.39 $\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=364

$$-\frac{b\left(1168a^2c^2-728ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{\left(-2048a^3c^3+5488a^2b^2c^2-2520ab^4c+315b^6\right)\sqrt{ax^2+bx^3+cx^4}}{35840c^5x}-\frac{3bx\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^3}-\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{2240c^2}+\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

```
[Out] -(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c
*x^4])/(17920*c^4) + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2
- 2048*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*
b^2 - 32*a*c)*(3*b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(448
0*c^3) - (b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])/(22
40*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x
^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 - (3*b*(b^2 -
4*a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2)*Sqrt[a*x^2
 + b*x^3 + c*x^4])
```

Rubi [A] time = 1.59755, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{b\left(1168a^2c^2-728ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{\left(-2048a^3c^3+5488a^2b^2c^2-2520ab^4c+315b^6\right)\sqrt{ax^2+bx^3+cx^4}}{35840c^5x}-\frac{3bx\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{x\left(7b^2-32ac\right)\left(3b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^3}-\frac{bx^2\left(9b^2-44ac\right)\sqrt{ax^2+bx^3+cx^4}}{2240c^2}+\frac{x^3\left(24ac+b^2+10bcx\right)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] -(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c
*x^4])/(17920*c^4) + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2
- 2048*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*
b^2 - 32*a*c)*(3*b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(448
0*c^3) - (b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])/(22
40*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x
^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 - (3*b*(b^2 -
4*a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2)*Sqrt[a*x^2
 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 171.091, size = 348, normalized size = 0.96

$$-\frac{bx^2\left(-44ac+9b^2\right)\sqrt{ax^2+bx^3+cx^4}}{2240c^2}-\frac{b\sqrt{ax^2+bx^3+cx^4}\left(1168a^2c^2-728ab^2c+105b^4\right)}{17920c^4}-\frac{3bx\left(-4ac+b^2\right)^2\left(-4ac+3b^2\right)\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{\frac{11}{2}}\sqrt{ax^2+bx^3+cx^4}}+\frac{x\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{7}+\frac{x^3\left(12ac+\frac{b^2}{2}+5bcx\right)\sqrt{ax^2+bx^3+cx^4}}{140c}+\frac{x\left(-32ac+7b^2\right)\left(-4ac+3b^2\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^3}+\frac{\sqrt{ax^2+bx^3+cx^4}\left(-2048a^3c^3+5488a^2b^2c^2-2520ab^4c+315b^6\right)}{35840c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] -b*x**2*(-44*a*c + 9*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(2240*c
**2) - b*sqrt(a*x**2 + b*x**3 + c*x**4)*(1168*a**2*c**2 - 728*a*b
**2*c + 105*b**4)/(17920*c**4) - 3*b*x*(-4*a*c + b**2)**2*(-4*a*c
 + 3*b**2)*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sq
rt(a + b*x + c*x**2)))/(2048*c**(11/2)*sqrt(a*x**2 + b*x**3 + c*x
**4)) + x*(a*x**2 + b*x**3 + c*x**4)**(3/2)/7 + x**3*(12*a*c + b*
*2/2 + 5*b*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(140*c) + x*(-32*a
*c + 7*b**2)*(-4*a*c + 3*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(44
80*c**3) + sqrt(a*x**2 + b*x**3 + c*x**4)*(-2048*a**3*c**3 + 5488
*a**2*b**2*c**2 - 2520*a*b**4*c + 315*b**6)/(35840*c**5*x)
```

Mathematica [A] time = 0.488518, size = 239, normalized size = 0.66

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(16b^2c^2\left(343a^2-62acx^2+8c^2x^4\right)+32bc^3x\left(-73a^2+22acx^2+200c^2x^4\right)+168b^4c\left(cx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^
6 - 210*b^5*c*x + 16*b^3*c^2*x*(91*a - 9*c*x^2) + 168*b^4*c*(-15*
a + c*x^2) + 1024*c^3*(a + c*x^2)^2*(-2*a + 5*c*x^2) + 16*b^2*c^2
*(343*a^2 - 62*a*c*x^2 + 8*c^2*x^4) + 32*b*c^3*x*(-73*a^2 + 22*a*
c*x^2 + 200*c^2*x^4)) - 105*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*Log
[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(71680*c^(11/2)*S
qrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.012, size = 479, normalized size = 1.3

$$\frac{1}{71680\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(10240\,x^2\left(cx^2+bx+a\right)^{5/2}c^{11/2}-7680\left(cx^2+bx+a\right)^{5/2}c^{9/2}xb-4096\left(cx^2+bx+a\right)^{5/2}c^{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/71680*(c*x^4+b*x^3+a*x^2)^(3/2)*(10240*x^2*(c*x^2+b*x+a)^(5/2)*
c^(11/2)-7680*(c*x^2+b*x+a)^(5/2)*c^(9/2)*x*b-4096*(c*x^2+b*x+a)^
(5/2)*c^(9/2)*a+5376*(c*x^2+b*x+a)^(5/2)*c^(7/2)*b^2+4480*(c*x^2+
b*x+a)^(3/2)*c^(9/2)*x*a*b-3360*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b^3
+2240*(c*x^2+b*x+a)^(3/2)*c^(7/2)*a*b^2-1680*(c*x^2+b*x+a)^(3/2)*
c^(5/2)*b^4+6720*(c*x^2+b*x+a)^(1/2)*c^(9/2)*x*a^2*b-6720*(c*x^2+
b*x+a)^(1/2)*c^(7/2)*x*a*b^3+1260*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b
^5+3360*(c*x^2+b*x+a)^(1/2)*c^(7/2)*a^2*b^2-3360*(c*x^2+b*x+a)^(1
/2)*c^(5/2)*a*b^4+630*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^6+6720*ln(1/2
*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*b*c^4-8400*
ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^3*c
^3+2940*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a
*b^5*c^2-315*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/
2))*b^7*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(13/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.337796, size = 1, normalized size = 0.

$$\left[-\frac{105 \left(3 b^7 - 28 ab^5 c + 80 a^2 b^3 c^2 - 64 a^3 bc^3\right) \sqrt{c} x \log\left(-\frac{4 \sqrt{cx^4+bx^3+ax^2}\left(2 c^2 x+bc\right)+\left(8 c^2 x^3+8 bcx^2+\left(b^2+4 ac\right) x\right) \sqrt{c}}{x}\right) - 4 \left(5120 c^7 x^6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c
^3)*sqrt(c)*x*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c)
 + (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x) - 4*(512
0*c^7*x^6 + 6400*b*c^6*x^5 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^
2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b
^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^
2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x)*
sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*
b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-c)*x*arctan(1/2*(2*c
*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) + 2*(5120*c
^7*x^6 + 6400*b*c^6*x^5 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b
^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*
c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c
^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x)*sqr
t(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ax^2 + bx^3 + cx^4\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.323435, size = 579, normalized size = 1.59

$$\frac{1}{35840}\sqrt{cx^2+bx+a}\Bigg(2\Bigg(4\Bigg(2\Bigg(8\Bigg(10\,(4\,cx\mathrm{sign}\,(x)+5\,b\mathrm{sign}\,(x))x+\frac{b^2c^5\mathrm{sign}\,(x)+64\,ac^6\mathrm{sign}\,(x)}{c^6}\Bigg)x-\frac{9\,b^3c^4\mathrm{sign}\,(x)-44\,ab}{c^6}$$

$$+\frac{3\left(3\,b^7\mathrm{sign}\,(x)-28\,ab^5c\mathrm{sign}\,(x)+80\,a^2b^3c^2\mathrm{sign}\,(x)-64\,a^3bc^3\mathrm{sign}\,(x)\right)\ln\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{2048\,c^{\frac{11}{2}}}$$

$$-\frac{\left(315\,b^7\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-2940\,ab^5c\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+8400\,a^2b^3c^2\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-6720\,a^3bc^3\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right.\right.\right.}{71680\,c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*sign(x) + 5*
b*sign(x))*x + (b^2*c^5*sign(x) + 64*a*c^6*sign(x))/c^6)*x - (9*b
^3*c^4*sign(x) - 44*a*b*c^5*sign(x))/c^6)*x + (21*b^4*c^3*sign(x)
 - 124*a*b^2*c^4*sign(x) + 128*a^2*c^5*sign(x))/c^6)*x - (105*b^5
*c^2*sign(x) - 728*a*b^3*c^3*sign(x) + 1168*a^2*b*c^4*sign(x))/c^
6)*x + (315*b^6*c*sign(x) - 2520*a*b^4*c^2*sign(x) + 5488*a^2*b^2
*c^3*sign(x) - 2048*a^3*c^4*sign(x))/c^6) + 3/2048*(3*b^7*sign(x)
 - 28*a*b^5*c*sign(x) + 80*a^2*b^3*c^2*sign(x) - 64*a^3*b*c^3*sig
n(x))*ln(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))
/c^(11/2) - 1/71680*(315*b^7*ln(abs(-b + 2*sqrt(a)*sqrt(c))) - 29
40*a*b^5*c*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 8400*a^2*b^3*c^2*ln(
abs(-b + 2*sqrt(a)*sqrt(c))) - 6720*a^3*b*c^3*ln(abs(-b + 2*sqrt(
a)*sqrt(c))) + 630*sqrt(a)*b^6*sqrt(c) - 5040*a^(3/2)*b^4*c^(3/2)
 + 10976*a^(5/2)*b^2*c^(5/2) - 4096*a^(7/2)*c^(7/2))*sign(x)/c^(1
1/2)
```

3.40 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=288

$$-\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x}+\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3}+\frac{x\left(7b^2-4ac\right)\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{960c^2}+\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}$$

```
[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(3840*c^3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x
^2 + b*x^3 + c*x^4])/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*
(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^2) + ((3*
b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(60*c*x) + ((b^2 - 4*a
*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^2 + b
*x^3 + c*x^4])
```

Rubi [A] time = 0.828588, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{b\left(1296a^2c^2-760ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x}+\frac{\left(240a^2c^2-216ab^2c+35b^4\right)\sqrt{ax^2+bx^3+cx^4}}{3840c^3}+\frac{x\left(7b^2-4ac\right)\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{x\left(6cx\left(7b^2-20ac\right)+b\left(12ac+7b^2\right)\right)\sqrt{ax^2+bx^3+cx^4}}{960c^2}+\frac{(3b+10cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{60cx}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(3840*c^3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x
^2 + b*x^3 + c*x^4])/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*
(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^2) + ((3*
```

```
b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(60*c*x) + ((b^2 - 4*a
*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^2 + b
*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 107.947, size = 274, normalized size = 0.95

$$-\frac{b\sqrt{ax^2+bx^3+cx^4}\left(1296a^2c^2-760ab^2c+105b^4\right)}{7680c^4x}+\frac{\left(\frac{3b}{2}+5cx\right)\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{30cx}-\frac{x\left(\frac{b\left(12ac+7b^2\right)}{4}+\frac{3cx\left(-20ac+7b^2\right)}{2}\right)\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{\sqrt{ax^2+bx^3+cx^4}\left(240a^2c^2-216ab^2c+35b^4\right)}{3840c^3}+\frac{x\left(-4ac+b^2\right)^2\left(-4ac+7b^2\right)\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{\frac{9}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] -b*sqrt(a*x**2 + b*x**3 + c*x**4)*(1296*a**2*c**2 - 760*a*b**2*c
+ 105*b**4)/(7680*c**4*x) + (3*b/2 + 5*c*x)*(a*x**2 + b*x**3 + c*
x**4)**(3/2)/(30*c*x) - x*(b*(12*a*c + 7*b**2)/4 + 3*c*x*(-20*a*c
 + 7*b**2)/2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(240*c**2) + sqrt(a*
x**2 + b*x**3 + c*x**4)*(240*a**2*c**2 - 216*a*b**2*c + 35*b**4)/
(3840*c**3) + x*(-4*a*c + b**2)**2*(-4*a*c + 7*b**2)*sqrt(a + b*x
 + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/
(1024*c**(9/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.386622, size = 211, normalized size = 0.73

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(16bc^2\left(-81a^2+18acx^2+104c^2x^4\right)+160c^3x\left(3a^2+14acx^2+8c^2x^4\right)+8b^3c\left(95a-7c\right.\right.\right.}{15360c^{9/2}\sqrt{x^2(a+x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b
^5 + 70*b^4*c*x + 8*b^3*c*(95*a - 7*c*x^2) + 48*b^2*c^2*x*(-9*a +
```

```
 c*x^2) + 160*c^3*x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4) + 16*b*c^2*(
-81*a^2 + 18*a*c*x^2 + 104*c^2*x^4)) + 15*(b^2 - 4*a*c)^2*(7*b^2
- 4*a*c)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(1536
0*c^(9/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.012, size = 431, normalized size = 1.5

$$\frac{1}{15360\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(2560\,x\left(cx^2+bx+a\right)^{5/2}c^{9/2}-1792\left(cx^2+bx+a\right)^{5/2}c^{7/2}b-640\left(cx^2+bx+a\right)^{3/2}c^{9/2}xa\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x)
```

```
[Out] 1/15360*(c*x^4+b*x^3+a*x^2)^(3/2)*(2560*x*(c*x^2+b*x+a)^(5/2)*c^(
9/2)-1792*(c*x^2+b*x+a)^(5/2)*c^(7/2)*b-640*(c*x^2+b*x+a)^(3/2)*c
^(9/2)*x*a+1120*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b^2-320*(c*x^2+b*x+
a)^(3/2)*c^(7/2)*a*b+560*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b^3-960*(c*x
^2+b*x+a)^(1/2)*c^(9/2)*x*a^2+1920*(c*x^2+b*x+a)^(1/2)*c^(7/2)*x*
a*b^2-420*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^4-480*(c*x^2+b*x+a)^(1/
2)*c^(7/2)*a^2*b+960*(c*x^2+b*x+a)^(1/2)*c^(5/2)*a*b^3-210*(c*x^2
+b*x+a)^(1/2)*c^(3/2)*b^5-960*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/
2)+2*c*x+b)/c^(1/2))*a^3*c^4+2160*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c
^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^2*c^3-900*ln(1/2*(2*(c*x^2+b*x+a)^
(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^4*c^2+105*ln(1/2*(2*(c*x^2+b*
x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^6*c)/x^3/(c*x^2+b*x+a)^(3/
2)/c^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.310667, size = 1, normalized size = 0.

$$\left[-\frac{15\left(7\,b^{6}-60\,ab^{4}c+144\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}\right)\sqrt{c}x\log\left(\frac{4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(2\,c^{2}x+bc\right)-\left(8\,c^{2}x^{3}+8\,bcx^{2}+\left(b^{2}+4\,ac\right)x\right)\sqrt{c}}{x}\right)-4\left(1280\,c^{6}x^{5}+1\right.}{}\right.$$

$$-\frac{15\left(7\,b^{6}-60\,ab^{4}c+144\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}\right)\sqrt{-c}x\arctan\left(\frac{\left(2\,cx^{2}+bx\right)\sqrt{-c}}{2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}c}\right)-2\left(1280\,c^{6}x^{5}+1664\,bc^{5}x^{4}-105\,b^{5}c+760\,ab^{3}\right.}{1536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)
*sqrt(c)*x*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c) - (
8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x) - 4*(1280*c^
6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c
^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*
x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*sqrt(c*x^4
+ b*x^3 + a*x^2))/(c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c + 144
*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*x*arctan(1/2*(2*c*x^2 + b*x)*
sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) - 2*(1280*c^6*x^5 + 166
4*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*
b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35
*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a
*x^2))/(c^5*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)
```

GIAC/XCAS [A] time = 0.319411, size = 493, normalized size = 1.71

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\Bigg(2\Bigg(4\Bigg(2\Bigg(8\,(10\,cx\mathrm{sign}\,(x)+13\,b\mathrm{sign}\,(x))x+\frac{3\,b^2c^4\mathrm{sign}\,(x)+140\,ac^5\mathrm{sign}\,(x)}{c^5}\Bigg)x-\frac{7\,b^3c^3\mathrm{sign}\,(x)-36\,ab}{c^5}$$

$$-\frac{\left(7\,b^6\mathrm{sign}\,(x)-60\,ab^4c\mathrm{sign}\,(x)+144\,a^2b^2c^2\mathrm{sign}\,(x)-64\,a^3c^3\mathrm{sign}\,(x)\right)\ln\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{9}{2}}}$$

$$+\frac{\left(105\,b^6\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-900\,ab^4c\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+2160\,a^2b^2c^2\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-960\,a^3c^3\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+\right.}{15360\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*sign(x) + 13*b*s
ign(x))*x + (3*b^2*c^4*sign(x) + 140*a*c^5*sign(x))/c^5)*x - (7*b
^3*c^3*sign(x) - 36*a*b*c^4*sign(x))/c^5)*x + (35*b^4*c^2*sign(x)
 - 216*a*b^2*c^3*sign(x) + 240*a^2*c^4*sign(x))/c^5)*x - (105*b^5
*c*sign(x) - 760*a*b^3*c^2*sign(x) + 1296*a^2*b*c^3*sign(x))/c^5)
 - 1/1024*(7*b^6*sign(x) - 60*a*b^4*c*sign(x) + 144*a^2*b^2*c^2*s
ign(x) - 64*a^3*c^3*sign(x))*ln(abs(-2*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*sqrt(c) - b))/c^(9/2) + 1/15360*(105*b^6*ln(abs(-b + 2*
sqrt(a)*sqrt(c))) - 900*a*b^4*c*ln(abs(-b + 2*sqrt(a)*sqrt(c))) +
 2160*a^2*b^2*c^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) - 960*a^3*c^3*l
n(abs(-b + 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^5*sqrt(c) - 1520*a
^(3/2)*b^3*c^(3/2) + 2592*a^(5/2)*b*c^(5/2))*sign(x)/c^(9/2)
```

3.41 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=198

$$-\frac{3bx\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3}+\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}$$

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*
c^3*x) - (b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^
3) + (a*x^2 + b*x^3 + c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)
^2*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.297191, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$-\frac{3bx\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{16c^2x^3}+\frac{\left(ax^2+bx^3+cx^4\right)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*
c^3*x) - (b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^
3) + (a*x^2 + b*x^3 + c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)
^2*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 40.0156, size = 187, normalized size = 0.94

$$-\frac{b\left(b+2cx\right)\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{16c^2x^3}+\frac{3b\left(b+2cx\right)\left(-4ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{3bx\left(-4ac+b^2\right)^2\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{\frac{7}{2}}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(ax^2+bx^3+cx^4\right)^{\frac{5}{2}}}{5cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)
```

```
[Out] -b*(b + 2*c*x)*(a*x**2 + b*x**3 + c*x**4)**(3/2)/(16*c**2*x**3) +
 3*b*(b + 2*c*x)*(-4*a*c + b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(
128*c**3*x) - 3*b*x*(-4*a*c + b**2)**2*sqrt(a + b*x + c*x**2)*ata
nh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(256*c**(7/2)*
sqrt(a*x**2 + b*x**3 + c*x**4)) + (a*x**2 + b*x**3 + c*x**4)**(5/
2)/(5*c*x**5)
```

Mathematica [A] time = 0.285907, size = 161, normalized size = 0.81

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(4b^2c\left(2cx^2-25a\right)+8bc^2x\left(7a+22cx^2\right)+128c^2\left(a+cx^2\right)^2+15b^4-10b^3cx\right)-15b\left(\right.}{1280c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4
 - 10*b^3*c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2)
 + 8*b*c^2*x*(7*a + 22*c*x^2)) - 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c
*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(1280*c^(7/2)*Sqrt[x^2*(a
 + x*(b + c*x))])
```

Maple [A] time = 0.008, size = 289, normalized size = 1.5

$$\frac{1}{1280\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(256\left(cx^2+bx+a\right)^{5/2}c^{7/2}-160\left(cx^2+bx+a\right)^{3/2}c^{7/2}xb-80\left(cx^2+bx+a\right)^{3/2}c^{5/2}b^2-240\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x)
```

```
[Out] 1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(256*(c*x^2+b*x+a)^(5/2)*c^(7/2)
-160*(c*x^2+b*x+a)^(3/2)*c^(7/2)*x*b-80*(c*x^2+b*x+a)^(3/2)*c^(5/
2)*b^2-240*(c*x^2+b*x+a)^(1/2)*c^(7/2)*x*a*b+60*(c*x^2+b*x+a)^(1/
2)*c^(5/2)*x*b^3-120*(c*x^2+b*x+a)^(1/2)*c^(5/2)*a*b^2+30*(c*x^2+
b*x+a)^(1/2)*c^(3/2)*b^4-240*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2
)+2*c*x+b)/c^(1/2))*a^2*b*c^3+120*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c
^(1/2)+2*c*x+b)/c^(1/2))*a*b^3*c^2-15*ln(1/2*(2*(c*x^2+b*x+a)^(1/
```

```
2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^5*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(9/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.337554, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\sqrt{c}x\log\left(\frac{4\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x+bc\right)-\left(8\,c^2x^3+8\,bcx^2+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right)+4\left(128\,c^5x^4+176\,bc^4x^3+15\,b^4c\right.}{2560\,c^4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log((4*sqr
t(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c) - (8*c^2*x^3 + 8*b*c*x^2
 + (b^2 + 4*a*c)*x)*sqrt(c))/x) + 4*(128*c^5*x^4 + 176*b*c^4*x^3
+ 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)
*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))
/(c^4*x), 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*x*
arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*
c)) + 2*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 +
 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b
*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)
```

GIAC/XCAS [A] time = 0.315092, size = 383, normalized size = 1.93

$$\frac{1}{640}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\,cx\text{sign}(x)+11\,b\text{sign}(x)\right)x+\frac{b^2c^3\text{sign}(x)+32\,ac^4\text{sign}(x)}{c^4}\right)x-\frac{5\,b^3c^2\text{sign}(x)-28\,abc^3\text{sign}(x}{c^4}\right.\right.$$
$$+\frac{3\left(b^5\text{sign}(x)-8\,ab^3c\text{sign}(x)+16\,a^2bc^2\text{sign}(x)\right)\ln\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{7}{2}}}$$
$$-\frac{\left(15\,b^5\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-120\,ab^3c\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+240\,a^2bc^2\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+30\,\sqrt{a}b^4\sqrt{c}-200\,a^{\frac{3}{2}}b^2c^{\frac{3}{2}}+256\,a}{1280\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x*sign(x) + 11*b*sign(x
))*x + (b^2*c^3*sign(x) + 32*a*c^4*sign(x))/c^4)*x - (5*b^3*c^2*s
ign(x) - 28*a*b*c^3*sign(x))/c^4)*x + (15*b^4*c*sign(x) - 100*a*b
^2*c^2*sign(x) + 128*a^2*c^3*sign(x))/c^4) + 3/256*(b^5*sign(x) -
 8*a*b^3*c*sign(x) + 16*a^2*b*c^2*sign(x))*ln(abs(-2*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2) - 1/1280*(15*b^5*ln
(abs(-b + 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c*ln(abs(-b + 2*sqrt(a)
*sqrt(c))) + 240*a^2*b*c^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*s
qrt(a)*b^4*sqrt(c) - 200*a^(3/2)*b^2*c^(3/2) + 256*a^(5/2)*c^(5/2
))*sign(x)/c^(7/2)
```

3.42 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=165

$$\frac{3x\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^
2*x) + ((b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) + (3
*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])
```

Rubi [A] time = 0.199709, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{3x\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3\left(b^2-4ac\right)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{8cx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^
2*x) + ((b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) + (3
*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(5/2)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])
```

Rubi in Sympy [A] time = 30.5433, size = 155, normalized size = 0.94

$$\frac{(b+2cx)\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{8cx^3}-\frac{3(b+2cx)\left(-4ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{3x\left(-4ac+b^2\right)^2\sqrt{a+bx+cx^2}\,\mathrm{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{\frac{5}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] (b + 2*c*x)*(a*x**2 + b*x**3 + c*x**4)**(3/2)/(8*c*x**3) - 3*(b +
 2*c*x)*(-4*a*c + b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(64*c**2*x
) + 3*x*(-4*a*c + b**2)**2*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*
x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(128*c**(5/2)*sqrt(a*x**2
+ b*x**3 + c*x**4))
```

Mathematica [A] time = 0.211458, size = 130, normalized size = 0.79

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\left(4c\left(5a+2cx^2\right)-3b^2+8bcx\right)+3\left(b^2-4ac\right)^2\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2\right.\right.}{128c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)]*(-3*b^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) + 3*(b^2 - 4*a*c)^2
*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(128*c^(5/2)*
Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.007, size = 265, normalized size = 1.6

$$\frac{1}{128\,x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(32\,x\left(cx^2+bx+a\right)^{3/2}c^{7/2}+16\left(cx^2+bx+a\right)^{3/2}c^{5/2}b+48\sqrt{cx^2+bx+a}c^{7/2}xa-12\sqrt{cx^2+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)
+16*(c*x^2+b*x+a)^(3/2)*c^(5/2)*b+48*(c*x^2+b*x+a)^(1/2)*c^(7/2)*
x*a-12*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b^2+24*(c*x^2+b*x+a)^(1/2)*c
^(5/2)*a*b-6*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^3+48*ln(1/2*(2*(c*x^2+
b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*c^3-24*ln(1/2*(2*(c*x^
2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*c^2+3*ln(1/2*(2*(c
*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^4*c)/x^3/(c*x^2+b*x
+a)^(3/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298431, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{c}x\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}\left(2c^2x+bc\right)+\left(8c^2x^3+8bcx^2+\left(b^2+4ac\right)x\right)\sqrt{c}}{x}\right)+4\left(16c^4x^3+24bc^3x^2-3b^3c+20\right.}{256c^3x}\right.$$

$$-\frac{3\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{-c}x\arctan\left(\frac{\left(2cx^2+bx\right)\sqrt{-c}}{2\sqrt{cx^4+bx^3+ax^2}c}\right)-2\left(16c^4x^3+24bc^3x^2-3b^3c+20abc^2+2\left(b^2c^2+20ac^3\right)x\right)\sqrt{c}}{128c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c) + (8*c^2*x^3 + 8*b*c*x^2 +
(b^2 + 4*a*c)*x)*sqrt(c))/x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b
^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3
+ a*x^2))/(c^3*x), -1/128*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(
-c)*x*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a
*x^2)*c)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 +
 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**3, x)
```

GIAC/XCAS [A] time = 0.318725, size = 313, normalized size = 1.9

$$\frac{1}{64}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\,cx\mathrm{sign}\left(x\right)+3\,b\mathrm{sign}\left(x\right)\right)x+\frac{b^2c^2\mathrm{sign}\left(x\right)+20\,ac^3\mathrm{sign}\left(x\right)}{c^3}\right)x-\frac{3\,b^3c\mathrm{sign}\left(x\right)-20\,abc^2\mathrm{sign}\left(x\right)}{c^3}\right)-\frac{3\left(b^4\mathrm{sign}\left(x\right)-8\,ab^2c\mathrm{sign}\left(x\right)+16\,a^2c^2\mathrm{sign}\left(x\right)\right)\ln\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{5}{2}}}+\frac{\left(3\,b^4\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-24\,ab^2c\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+6\,\sqrt{a}b^3\sqrt{c}-40\,a^{\frac{3}{2}}bc^{\frac{3}{2}}\right)\mathrm{sign}\left(x\right)}{128\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x*sign(x) + 3*b*sign(x))*x
+ (b^2*c^2*sign(x) + 20*a*c^3*sign(x))/c^3)*x - (3*b^3*c*sign(x)
- 20*a*b*c^2*sign(x))/c^3) - 3/128*(b^4*sign(x) - 8*a*b^2*c*sign(
x) + 16*a^2*c^2*sign(x))*ln(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))*sqrt(c) - b))/c^(5/2) + 1/128*(3*b^4*ln(abs(-b + 2*sqrt(a)*
sqrt(c))) - 24*a*b^2*c*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 48*a^2*c
^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^3*sqrt(c) - 40*a
^(3/2)*b*c^(3/2))*sign(x)/c^(5/2)
```

3.43 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=227

$$-\frac{a^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}$$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (
a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x +
c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sq
rt[a*x^2 + b*x^3 + c*x^4] - (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*
x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.46656, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292

$$-\frac{a^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (
a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x +
c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sq
rt[a*x^2 + b*x^3 + c*x^4] - (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*
x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 59.5492, size = 209, normalized size = 0.92

$$-\frac{a^{\frac{3}{2}} x \sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} - \frac{bx\left(-12ac+b^2\right)\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{\frac{3}{2}}\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{3x^3} + \frac{\left(4ac+\frac{b^2}{2}+bcx\right)\sqrt{ax^2+bx^3+cx^4}}{4cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] -a**(3/2)*x*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*s
qrt(a + b*x + c*x**2)))/sqrt(a*x**2 + b*x**3 + c*x**4) - b*x*(-12
*a*c + b**2)*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*
sqrt(a + b*x + c*x**2)))/(16*c**(3/2)*sqrt(a*x**2 + b*x**3 + c*x*
*4)) + (a*x**2 + b*x**3 + c*x**4)**(3/2)/(3*x**3) + (4*a*c + b**2
/2 + b*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(4*c*x)
```

Mathematica [A] time = 0.326267, size = 248, normalized size = 1.09

$$x\sqrt{a+x(b+cx)}\left(-48a^{3/2}c^{3/2}\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)+48a^{3/2}c^{3/2}\log(x)-3b^3\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2c\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(6*b^2*Sqrt[c]*Sqrt[a + x*(b + c*x)] + 6
4*a*c^(3/2)*Sqrt[a + x*(b + c*x)] + 28*b*c^(3/2)*x*Sqrt[a + x*(b
+ c*x)] + 16*c^(5/2)*x^2*Sqrt[a + x*(b + c*x)] + 48*a^(3/2)*c^(3/
2)*Log[x] - 48*a^(3/2)*c^(3/2)*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a +
 x*(b + c*x)]] - 3*b^3*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b +
c*x)]] + 36*a*b*c*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]
]))/(48*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.009, size = 222, normalized size = 1.

$$\frac{1}{48x^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(16\left(cx^2+bx+a\right)^{3/2}c^{5/2}+12\sqrt{cx^2+bx+a}c^{5/2}xb-48a^{3/2}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(16*(c*x^2+b*x+a)^(3/2)*c^(5/2)+12
*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x*b-48*a^(3/2)*ln((2*a+b*x+2*a^(1/2)
*(c*x^2+b*x+a)^(1/2))/x)*c^(5/2)+48*(c*x^2+b*x+a)^(1/2)*c^(5/2)*a
+6*(c*x^2+b*x+a)^(1/2)*c^(3/2)*b^2+36*ln(1/2*(2*(c*x^2+b*x+a)^(1/
2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b*c^2-3*ln(1/2*(2*(c*x^2+b*x+a)^(1
/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^3*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(5
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.408867, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a
^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) -
3*(b^3 - 12*a*b*c)*sqrt(c)*x*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(2*c^2*x + b*c) + (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(
c))/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x
^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*x*log(-(8*a*b*
x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan
(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) +
2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^
3 + a*x^2))/(c^2*x), -1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*(b*x^2
 + 2*a*x)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-a))) + 3*(b^3 - 12*a
*b*c)*sqrt(c)*x*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*
c) + (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x) - 4*(8
*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 +
a*x^2))/(c^2*x), -1/48*(48*sqrt(-a)*a*c^2*x*arctan(1/2*(b*x^2 + 2
```

```
*a*x)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-a))) - 3*(b^3 - 12*a*b*c
)*sqrt(-c)*x*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*
x^3 + a*x^2)*c)) - 2*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2
)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.44 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=219

$$\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*
x^3 + c*x^4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^
2 + b*x^3 + c*x^4]) + (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.460006, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^4}+\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*
x^3 + c*x^4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^
2 + b*x^3 + c*x^4]) + (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 57.1001, size = 206, normalized size = 0.94

$$-\frac{3\sqrt{a}bx\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}+\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{\frac{3}{2}}}{x^4}+\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] -3*sqrt(a)*b*x*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a
)*sqrt(a + b*x + c*x**2)))/(2*sqrt(a*x**2 + b*x**3 + c*x**4)) + 3
*(3*b + 2*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(4*x) - (a*x**2 + b
*x**3 + c*x**4)**(3/2)/x**4 + 3*x*(4*a*c + b**2)*sqrt(a + b*x + c
*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(8*s
qrt(c)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.258069, size = 229, normalized size = 1.05

$$\frac{\sqrt{a+x(b+cx)}\left(3b^2x\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)+4c^{3/2}x^2\sqrt{a+x(b+cx)}+10b\sqrt{c}x\sqrt{a+x(b+cx)}-8a\sqrt{c}\sqrt{a+x(b}\right.}{8\sqrt{c}\sqrt{x^2(a+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-8*a*Sqrt[c]*Sqrt[a + x*(b + c*x)] + 10*b
*Sqrt[c]*x*Sqrt[a + x*(b + c*x)] + 4*c^(3/2)*x^2*Sqrt[a + x*(b +
c*x)] + 12*Sqrt[a]*b*Sqrt[c]*x*Log[x] - 12*Sqrt[a]*b*Sqrt[c]*x*Lo
g[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]] + 3*b^2*x*Log[b +
2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]] + 12*a*c*x*Log[b + 2*c*x
 + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(8*Sqrt[c]*Sqrt[x^2*(a + x*
(b + c*x))])
```

Maple [A] time = 0.009, size = 254, normalized size = 1.2

$$-\frac{1}{8\,ax^4}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(-8\left(cx^2+bx+a\right)^{3/2}c^{5/2}x^2+8\left(cx^2+bx+a\right)^{5/2}c^{3/2}-8\left(cx^2+bx+a\right)^{3/2}c^{3/2}xb-12\sqrt{cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x)
```

```
[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(-8*(c*x^2+b*x+a)^(3/2)*c^(5/2)*x^
2+8*(c*x^2+b*x+a)^(5/2)*c^(3/2)-8*(c*x^2+b*x+a)^(3/2)*c^(3/2)*x*b
-12*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x^2*a+12*a^(3/2)*ln((2*a+b*x+2*a^
(1/2)*(c*x^2+b*x+a)^(1/2))/x)*c^(3/2)*x*b-18*(c*x^2+b*x+a)^(1/2)*
c^(3/2)*x*a*b-3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^
(1/2))*c*x*a*b^2-12*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b
)/c^(1/2))*x*a^2*c^2)/x^4/(c*x^2+b*x+a)^(3/2)/a/c^(3/2)
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

<u>Fricas</u> [A] time = 0.358942, size = 1, normalized size = 0.

$$\left[\frac{12\,\sqrt{a}bcx^2\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}(bx+2\,a)\sqrt{a}}{x^3}\right)+3\,(b^2+4\,ac)\sqrt{c}x^2\log\left(-\frac{4\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x+bc)+(8\,c^2x^3+\cdots}{x}\right.\cdots}{16\,cx^2}\right.$$

$$-\frac{24\,\sqrt{-a}bcx^2\arctan\left(\frac{bx^2+2\,ax}{2\,\sqrt{cx^4+bx^3+ax^2}\sqrt{-a}}\right)-3\,(b^2+4\,ac)\sqrt{c}x^2\log\left(-\frac{4\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x+bc)+(8\,c^2x^3+8\,bcx^2+(b^2+4\,ac)x)\sqrt{c}}{x}\right)-4\cdots}{16\,cx^2}$$

$$-\frac{12\,\sqrt{-a}bcx^2\arctan\left(\frac{bx^2+2\,ax}{2\,\sqrt{cx^4+bx^3+ax^2}\sqrt{-a}}\right)+3\,(b^2+4\,ac)\sqrt{-c}x^2\arctan\left(\frac{(2\,cx^2+bx)\sqrt{-c}}{2\,\sqrt{cx^4+bx^3+ax^2}c}\right)-2\,\sqrt{cx^4+bx^3+ax^2}(2\,c^2x^2+5\,b\cdots}{8\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8
*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)
```

```
+ 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(2*c^2*x + b*c) + (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt
(c))/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*
a*c))/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*
a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*(2*c*x^2 +
b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) + 2*sqrt(c*x^4 + b
*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), -1/16*(24*s
qrt(-a)*b*c*x^2*arctan(1/2*(b*x^2 + 2*a*x)/(sqrt(c*x^4 + b*x^3 +
a*x^2)*sqrt(-a))) - 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(4*sqrt(c*x^
4 + b*x^3 + a*x^2)*(2*c^2*x + b*c) + (8*c^2*x^3 + 8*b*c*x^2 + (b^
2 + 4*a*c)*x)*sqrt(c))/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*
x^2 + 5*b*c*x - 4*a*c))/(c*x^2), -1/8*(12*sqrt(-a)*b*c*x^2*arctan
(1/2*(b*x^2 + 2*a*x)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-a))) + 3*
(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(s
qrt(c*x^4 + b*x^3 + a*x^2)*c)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2 \left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.45 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=219

$$-\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}-\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}+\frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

```
[Out] (-3*(b - 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b
*x^3 + c*x^4)^(3/2)/(2*x^5) - (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c
*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*
Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*b*Sqrt[c]*x*Sqrt[a + b*
x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(2*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.453306, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$-\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}-\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}+\frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b
*x^3 + c*x^4)^(3/2)/(2*x^5) - (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c
*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*
Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*b*Sqrt[c]*x*Sqrt[a + b*
x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(2*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 60.8086, size = 207, normalized size = 0.95

$$\frac{3b\sqrt{c}x\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{3\left(b-2cx\right)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{2x^5}-\frac{3x\left(4ac+b^2\right)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] 3*b*sqrt(c)*x*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)
*sqrt(a + b*x + c*x**2)))/(2*sqrt(a*x**2 + b*x**3 + c*x**4)) - 3*
(b - 2*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(4*x**2) - (a*x**2 + b
*x**3 + c*x**4)**(3/2)/(2*x**5) - 3*x*(4*a*c + b**2)*sqrt(a + b*x
 + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/
(8*sqrt(a)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.356234, size = 173, normalized size = 0.79

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(3x^2\log(x)\left(4ac+b^2\right)-3x^2\left(4ac+b^2\right)\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)+2\sqrt{a}\left(6b\sqrt{c}x^2\log\left(2\sqrt{c}\sqrt{a\cdots}\right.\right.\right.}{8\sqrt{a}x^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*Log[x] - 3*(b^2
 + 4*a*c)*x^2*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]] +
2*Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(-2*a + x*(-5*b + 4*c*x)) + 6*b*
Sqrt[c]*x^2*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])))/(
8*Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.01, size = 338, normalized size = 1.5

$$-\frac{1}{8a^2x^5}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(12a^{5/2}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)c^{5/2}x^2-2\left(cx^2+bx+a\right)^{3/2}c^{5/2}x^3b-4\left(cx^2+bx\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(12*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*
(c*x^2+b*x+a)^(1/2))/x)*c^(5/2)*x^2-2*(c*x^2+b*x+a)^(3/2)*c^(5/2)
*x^3*b-4*(c*x^2+b*x+a)^(3/2)*c^(5/2)*x^2*a-6*(c*x^2+b*x+a)^(1/2)*
c^(5/2)*x^3*a*b+3*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)*c^(3/2)*x^2*b^2+2*(c*x^2+b*x+a)^(5/2)*c^(3/2)*x*b-2*(c*x^2
+b*x+a)^(3/2)*c^(3/2)*x^2*b^2-12*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x^2*
a^2+4*(c*x^2+b*x+a)^(5/2)*a*c^(3/2)-6*(c*x^2+b*x+a)^(1/2)*c^(3/2)
*x^2*a*b^2-12*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1
/2))*c^2*x^2*a^2*b)/x^5/(c*x^2+b*x+a)^(3/2)/a^2/c^(3/2)
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

<u>Fricas</u> [A] time = 0.344875, size = 1, normalized size = 0.

$$\left[\frac{12\,ab\sqrt{c}x^3\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}(2\,cx+b)\sqrt{c}+\left(b^2+4\,ac\right)x}{x}\right)+3\,\left(b^2+4\,ac\right)\sqrt{a}x^3\log\left(\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)-\left(8\,abx^2+(\cdots\right.}{x^3}\right.}{16\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*
x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) +
3*(b^2 + 4*a*c)*sqrt(a)*x^3*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a
*b*x + 2*a^2) - (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a)
)/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a
^2))/(a*x^3), 1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/2*(2*c*x^2 + b*x
)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-c))) + 3*(b^2 + 4*a*c)*sqrt(
a)*x^3*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2) - (8*a*
b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) + 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*
a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
```

```
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4
*a*c)*sqrt(-a)*x^3*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^
4 + b*x^3 + a*x^2)*a)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2
 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2
*(2*c*x^2 + b*x)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-c))) + 3*(b^2
 + 4*a*c)*sqrt(-a)*x^3*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(
c*x^4 + b*x^3 + a*x^2)*a)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c
*x^2 - 5*a*b*x - 2*a^2))/(a*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.46 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=257

$$\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(-8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}+\frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5}$$

```
[Out] ((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) -
 (a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^
4)^(3/2))/(4*a*x^5) + (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(16*a^(3/2
)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (c^(3/2)*x*Sqrt[a + b*x + c*x^2]
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x
^2 + b*x^3 + c*x^4]
```

Rubi [A] time = 0.644078, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{bx\left(b^2-12ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(-8ac+b^2+2bcx\right)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}+\frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```


```
[Out] ((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) -
 (a*x^2 + b*x^3 + c*x^4)^(3/2)/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^
4)^(3/2))/(4*a*x^5) + (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(16*a^(3/2
)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (c^(3/2)*x*Sqrt[a + b*x + c*x^2]
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x
^2 + b*x^3 + c*x^4]
```

Rubi in Sympy [A] time = 79.3191, size = 238, normalized size = 0.93

$$\frac{c^{\frac{3}{2}}x\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{3x^6}-\frac{b\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{4ax^5}+\frac{\left(-4ac+\frac{b^2}{2}+bcx\right)\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{bx\left(-12ac+b^2\right)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{\frac{3}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] c**(3/2)*x*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sq
rt(a + b*x + c*x**2)))/sqrt(a*x**2 + b*x**3 + c*x**4) - (a*x**2 +
 b*x**3 + c*x**4)**(3/2)/(3*x**6) - b*(a*x**2 + b*x**3 + c*x**4)*
*(3/2)/(4*a*x**5) + (-4*a*c + b**2/2 + b*c*x)*sqrt(a*x**2 + b*x**
3 + c*x**4)/(4*a*x**2) + b*x*(-12*a*c + b**2)*sqrt(a + b*x + c*x*
*2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(16*a**
(3/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.752317, size = 187, normalized size = 0.73

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(-2\sqrt{a}\left(\sqrt{a+x(b+cx)}\left(8a^2+2ax(7b+16cx)+3b^2x^2\right)-24ac^{3/2}x^3\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\right.\right.}{48a^{3/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-3*b*(b^2 - 12*a*c)*x^3*Log[x] + 3*
b*(b^2 - 12*a*c)*x^3*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*
x)]] - 2*Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*a^2 + 3*b^2*x^2 + 2*a*
x*(7*b + 16*c*x)) - 24*a*c^(3/2)*x^3*Log[b + 2*c*x + 2*Sqrt[c]*Sq
rt[a + x*(b + c*x)]])))/(48*a^(3/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Maple [A] time = 0.013, size = 435, normalized size = 1.7

$$\frac{1}{48\,x^6a^3}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(-36\,a^{5/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)c^{5/2}x^3b+32\,\left(cx^2+bx+a\right)^{3/2}c^{7/2}x^4a-2\,\left(cx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(-36*a^(5/2)*ln((2*a+b*x+2*a^(1/2)
*(c*x^2+b*x+a)^(1/2))/x)*c^(5/2)*x^3*b+32*(c*x^2+b*x+a)^(3/2)*c^(
7/2)*x^4*a-2*(c*x^2+b*x+a)^(3/2)*c^(5/2)*x^4*b^2+48*(c*x^2+b*x+a)
^(1/2)*c^(7/2)*x^4*a^2+3*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x
+a)^(1/2))/x)*c^(3/2)*x^3*b^3-32*(c*x^2+b*x+a)^(5/2)*c^(5/2)*x^2*
a+28*(c*x^2+b*x+a)^(3/2)*c^(5/2)*x^3*a*b-6*(c*x^2+b*x+a)^(1/2)*c^
(5/2)*x^4*a*b^2+2*(c*x^2+b*x+a)^(5/2)*c^(3/2)*x^2*b^2-2*(c*x^2+b*
x+a)^(3/2)*c^(3/2)*x^3*b^3+60*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x^3*a^2
*b+4*(c*x^2+b*x+a)^(5/2)*c^(3/2)*x*a*b-6*(c*x^2+b*x+a)^(1/2)*c^(3
/2)*x^3*a*b^3+48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c
^(1/2))*x^3*a^3*c^3-16*(c*x^2+b*x+a)^(5/2)*a^2*c^(3/2))/x^6/(c*x^
2+b*x+a)^(3/2)/a^3/c^(3/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.367622, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*
x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) -
3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(a*b*x + 2*a^2) - (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt
(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (
3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), 1/96*(96*a^2*sqrt(-c)*c*x^4*
arctan(1/2*(2*c*x^2 + b*x)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-c))
) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log((4*sqrt(c*x^4 + b*x^3 + a*
x^2)*(a*b*x + 2*a^2) - (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*
sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3
 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4
```

```
*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-
a)*x^4*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 +
a*x^2)*a)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 +
(3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(48*a^2*sqrt(-c)*c*x^4
*arctan(1/2*(2*c*x^2 + b*x)/(sqrt(c*x^4 + b*x^3 + a*x^2)*sqrt(-c)
)) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*(b*x^2 + 2*a*x)*s
qrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2)*a)) - 2*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.47 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=197

$$-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}$$

```
[Out] -((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a*x^3) + (b*(3*
b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^2*x^2) - ((b + 6
*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*x^4) - (a*x^2 + b*x^3 + c*x
^4)^(3/2)/(4*x^7) - (3*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2
*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*a^(5/2))
```

Rubi [A] time = 0.590535, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}+\frac{b\left(3b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{\left(b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a*x^3) + (b*(3*
b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^2*x^2) - ((b + 6
*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*x^4) - (a*x^2 + b*x^3 + c*x
^4)^(3/2)/(4*x^7) - (3*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2
*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(128*a^(5/2))
```

Rubi in Sympy [A] time = 83.2664, size = 207, normalized size = 1.05

$$-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{\left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}}{4x^7}-\frac{\left(-12ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b\left(-20ac+3b^2\right)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{3x\left(-4ac+b^2\right)^2\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{\frac{5}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] -(b + 6*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(8*x**4) - (a*x**2 +
b*x**3 + c*x**4)**(3/2)/(4*x**7) - (-12*a*c + b**2)*sqrt(a*x**2 +
 b*x**3 + c*x**4)/(32*a*x**3) + b*(-20*a*c + 3*b**2)*sqrt(a*x**2
+ b*x**3 + c*x**4)/(64*a**2*x**2) - 3*x*(-4*a*c + b**2)**2*sqrt(a
 + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x*
*2)))/(128*a**(5/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.315644, size = 156, normalized size = 0.79

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(-2\sqrt{a}(2a+bx)\sqrt{a+x(b+cx)}\left(8a^2+4ax(2b+5cx)-3b^2x^2\right)+3x^4\log(x)\left(b^2-4ac\right)^2-3x^4\left(b^2-4ac\right)\right.}{128a^{5/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(
b + c*x)]*(8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x)) + 3*(b^2 - 4*
a*c)^2*x^4*Log[x] - 3*(b^2 - 4*a*c)^2*x^4*Log[2*a + b*x + 2*Sqrt[
a]*Sqrt[a + x*(b + c*x)]]))/(128*a^(5/2)*x^5*Sqrt[a + x*(b + c*x)
])
```

Maple [B] time = 0.012, size = 501, normalized size = 2.5

$$-\frac{1}{128\,x^7a^4}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(48\,a^{7/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)c^2x^4-24\,a^{5/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*a^(7/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x)*c^2*x^4-24*a^(5/2)*ln((2*a+b*x+2*a^(1/2
)*(c*x^2+b*x+a)^(1/2))/x)*c*x^4*b^2+24*(c*x^2+b*x+a)^(3/2)*c^2*x^
5*a*b-16*(c*x^2+b*x+a)^(3/2)*c^2*x^4*a^2-2*(c*x^2+b*x+a)^(3/2)*c*
x^5*b^3+24*(c*x^2+b*x+a)^(1/2)*c^2*x^5*a^2*b+3*a^(3/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^4*b^4-24*(c*x^2+b*x+a)^(5/2
)*c*x^3*a*b+20*(c*x^2+b*x+a)^(3/2)*c*x^4*a*b^2-48*(c*x^2+b*x+a)^(
```

```
1/2)*c^2*x^4*a^3-6*(c*x^2+b*x+a)^(1/2)*c*x^5*a*b^3+16*(c*x^2+b*x+
a)^(5/2)*c*x^2*a^2+2*(c*x^2+b*x+a)^(5/2)*x^3*b^3-2*(c*x^2+b*x+a)^
(3/2)*x^4*b^4+36*(c*x^2+b*x+a)^(1/2)*c*x^4*a^2*b^2+4*(c*x^2+b*x+a
)^(5/2)*x^2*a*b^2-6*(c*x^2+b*x+a)^(1/2)*x^4*a*b^4-16*(c*x^2+b*x+a
)^(5/2)*x*a^2*b+32*(c*x^2+b*x+a)^(5/2)*a^3)/x^7/(c*x^2+b*x+a)^(3/
2)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.327811, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{a}x^5\log\left(\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)-\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right)-4\left(24\,a^3bx+16\,a^4-\left(3\,ab^3-20\right.\right.}{256\,a^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log((4*sqrt(
c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2) - (8*a*b*x^2 + (b^2 + 4*a*
c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b
^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b
*x^3 + a*x^2))/(a^3*x^5), 1/128*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)
*sqrt(-a)*x^5*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b
*x^3 + a*x^2)*a)) - 2*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*
c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))
/(a^3*x^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] Timed out
```

3.48 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=249

$$\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}}+\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}-\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}-\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}$$

```
[Out] -((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(80*a*x^4) + (b*(5*b
^2 - 28*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(320*a^2*x^3) - ((15*b^
4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(640*
a^3*x^2) - (3*(b + 4*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*x^5) -
 (a*x^2 + b*x^3 + c*x^4)^(3/2)/(5*x^8) + (3*b*(b^2 - 4*a*c)^2*Arc
Tanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2
56*a^(7/2))
```

Rubi [A] time = 0.807728, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}}+\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}-\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}-\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(80*a*x^4) + (b*(5*b
^2 - 28*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(320*a^2*x^3) - ((15*b^
4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(640*
a^3*x^2) - (3*(b + 4*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(40*x^5) -
 (a*x^2 + b*x^3 + c*x^4)^(3/2)/(5*x^8) + (3*b*(b^2 - 4*a*c)^2*Arc
Tanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2
56*a^(7/2))
```

Rubi in Sympy [A] time = 118.298, size = 262, normalized size = 1.05

$$-\frac{3(2b+8cx)\sqrt{ax^2+bx^3+cx^4}}{80x^5}-\frac{(ax^2+bx^3+cx^4)^{\frac{3}{2}}}{5x^8}-\frac{(-8ac+b^2)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b(-28ac+5b^2)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{\sqrt{ax^2+bx^3+cx^4}\,(128a^2c^2-100ab^2c+15b^4)}{640a^3x^2}+\frac{3bx(-4ac+b^2)^2\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{256a^{\frac{7}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] -3*(2*b + 8*c*x)*sqrt(a*x**2 + b*x**3 + c*x**4)/(80*x**5) - (a*x*
*2 + b*x**3 + c*x**4)**(3/2)/(5*x**8) - (-8*a*c + b**2)*sqrt(a*x*
*2 + b*x**3 + c*x**4)/(80*a*x**4) + b*(-28*a*c + 5*b**2)*sqrt(a*x
**2 + b*x**3 + c*x**4)/(320*a**2*x**3) - sqrt(a*x**2 + b*x**3 + c
*x**4)*(128*a**2*c**2 - 100*a*b**2*c + 15*b**4)/(640*a**3*x**2) +
 3*b*x*(-4*a*c + b**2)**2*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x
)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(256*a**(7/2)*sqrt(a*x**2 +
 b*x**3 + c*x**4))
```

Mathematica [A] time = 0.409058, size = 193, normalized size = 0.78

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(-2\sqrt{a}\sqrt{a+x(b+cx)}\left(128a^4+16a^3x(11b+16cx)+8a^2x^2\left(b^2+7bcx+16c^2x^2\right)-10ab^2x^3(b+10cx)+\right.\right.}{1280a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-2*Sqrt[a]*Sqrt[a + x*(b + c*x)]*(1
28*a^4 + 15*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b
+ 16*c*x) + 8*a^2*x^2*(b^2 + 7*b*c*x + 16*c^2*x^2)) - 15*b*(b^2 -
 4*a*c)^2*x^5*Log[x] + 15*b*(b^2 - 4*a*c)^2*x^5*Log[2*a + b*x + 2
*Sqrt[a]*Sqrt[a + x*(b + c*x)]]))/(1280*a^(7/2)*x^6*Sqrt[a + x*(b
 + c*x)])
```

Maple [B] time = 0.014, size = 534, normalized size = 2.1

$$\frac{1}{1280\,x^8a^5}\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(240\,a^{7/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)c^2x^5b-120\,a^{5/2}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x)
```

```
[Out] 1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(240*a^(7/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)*c^2*x^5*b-120*a^(5/2)*ln((2*a+b*x+2*a^
(1/2)*(c*x^2+b*x+a)^(1/2))/x)*c*x^5*b^3+120*(c*x^2+b*x+a)^(3/2)*c
^2*x^6*a*b^2-80*(c*x^2+b*x+a)^(3/2)*c^2*x^5*a^2*b-10*(c*x^2+b*x+a
)^(3/2)*c*x^6*b^4+120*(c*x^2+b*x+a)^(1/2)*c^2*x^6*a^2*b^2+15*a^(3
/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x^5*b^5-120*(c*
x^2+b*x+a)^(5/2)*c*x^4*a*b^2+100*(c*x^2+b*x+a)^(3/2)*c*x^5*a*b^3-
240*(c*x^2+b*x+a)^(1/2)*c^2*x^5*a^3*b-30*(c*x^2+b*x+a)^(1/2)*c*x^
6*a*b^4+80*(c*x^2+b*x+a)^(5/2)*c*x^3*a^2*b+10*(c*x^2+b*x+a)^(5/2)
*x^4*b^4-10*(c*x^2+b*x+a)^(3/2)*x^5*b^5+180*(c*x^2+b*x+a)^(1/2)*c
*x^5*a^2*b^3+20*(c*x^2+b*x+a)^(5/2)*x^3*a*b^3-30*(c*x^2+b*x+a)^(1
/2)*x^5*a*b^5-80*(c*x^2+b*x+a)^(5/2)*x^2*a^2*b^2+160*(c*x^2+b*x+a
)^(5/2)*x*a^3*b-256*(c*x^2+b*x+a)^(5/2)*a^4)/x^8/(c*x^2+b*x+a)^(3
/2)/a^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.316086, size = 1, normalized size = 0.

$$\left[\frac{15\left(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2}\right)\sqrt{a}x^{6}\log\left(-\frac{4\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(abx+2\,a^{2}\right)+\left(8\,abx^{2}+\left(b^{2}+4\,ac\right)x^{3}+8\,a^{2}x\right)\sqrt{a}}{x^{3}}\right)-4\left(176\,a^{4}bx+128\,a^{5}+\left(15\,ab\right.}{2560\,a^{4}x^{6}}\right.$$

$$-\frac{15\left(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2}\right)\sqrt{-a}x^{6}\arctan\left(\frac{\left(bx^{2}+2\,ax\right)\sqrt{-a}}{2\sqrt{cx^{4}+bx^{3}+ax^{2}}a}\right)+2\left(176\,a^{4}bx+128\,a^{5}+\left(15\,ab^{4}-100\,a^{2}b^{2}c+128\,a^{3}c^{2}\right)x^{4}-\right.}{1280\,a^{4}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(4*
sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2) + (8*a*b*x^2 + (b^2 +
 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 +
 (15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28
*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 +
a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*s
qrt(-a)*x^6*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b*x
^3 + a*x^2)*a)) + 2*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*
b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^
3*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**9, x)
```

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] Timed out
```

3.49 $\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}} \, dx$

Optimal. Leaf size=143

$$\frac{x\left(3b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x
^4])/(4*c^2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[
a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.291617, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{x\left(3b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x
^4])/(4*c^2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[
a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 31.519, size = 129, normalized size = 0.9

$$-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}+\frac{x\left(-ac+\frac{3b^2}{4}\right)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{\frac{5}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] -3*b*sqrt(a*x**2 + b*x**3 + c*x**4)/(4*c**2*x) + sqrt(a*x**2 + b*
x**3 + c*x**4)/(2*c) + x*(-a*c + 3*b**2/4)*sqrt(a + b*x + c*x**2)
*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/(2*c**(5/2
```

```
)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.169847, size = 103, normalized size = 0.72

$$\frac{x\left(\left(3b^2-4ac\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)+2\sqrt{c}(2cx-3b)(a+x(b+cx))\right)}{8c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (3*b^2 - 4*a*c)*
Sqrt[a + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c
*x)]]))/(8*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.011, size = 144, normalized size = 1.

$$\frac{x}{8}\sqrt{cx^2+bx+a}\left(4\sqrt{cx^2+bx+a}c^{5/2}x-6\sqrt{cx^2+bx+a}c^{3/2}b-4\ln\left(1/2\,\frac{2\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx+b}{\sqrt{c}}\right)ac^2+3\ln\left(1/2\,\frac{2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b*x+a)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*c^(5/2)*x-6*(c*x
^2+b*x+a)^(1/2)*c^(3/2)*b-4*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)
+2*c*x+b)/c^(1/2))*a*c^2+3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+
2*c*x+b)/c^(1/2))*b^2*c)/(c*x^4+b*x^3+a*x^2)^(1/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299294, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,b^{2}-4\,ac\right)\sqrt{c}x\log\left(\frac{4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(2\,c^{2}x+bc\right)-\left(8\,c^{2}x^{3}+8\,bcx^{2}+\left(b^{2}+4\,ac\right)x\right)\sqrt{c}}{x}\right)-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(2\,c^{2}x-3\,bc\right)}{16\,c^{3}x},\ -\frac{\left(3\,b^{2}-4\,ac\right)\sqrt{-c}x\arctan\left(\frac{\left(2\,cx^{2}+bx\right)\sqrt{-c}}{2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}c}\right)-2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(2\,c^{2}x-3\,bc\right)}{8\,c^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(c)*x*log((4*sqrt(c*x^4 + b*x^3 + a*x
^2)*(2*c^2*x + b*c) - (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*s
qrt(c))/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^
3*x), -1/8*((3*b^2 - 4*a*c)*sqrt(-c)*x*arctan(1/2*(2*c*x^2 + b*x)
*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) - 2*sqrt(c*x^4 + b*x^3
 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{x^{2}\left(a+bx+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\sqrt{cx^{4}+bx^{3}+ax^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")

[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

3.50 $\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.128508, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 17.9324, size = 92, normalized size = 0.89

$$-\frac{bx\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{\frac{3}{2}}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] -b*x*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a +
 b*x + c*x**2)))/(2*c**(3/2)*sqrt(a*x**2 + b*x**3 + c*x**4)) + sq
rt(a*x**2 + b*x**3 + c*x**4)/(c*x)
```

Mathematica [A] time = 0.0915276, size = 87, normalized size = 0.84

$$\frac{x\left(2\sqrt{c}(a+x(b+cx))-b\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\right)}{2c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(a + x*(b + c*x)) - b*Sqrt[a + x*(b + c*x)]*Log[b +
 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(2*c^(3/2)*Sqrt[x^2*(
a + x*(b + c*x))])
```

Maple [A] time = 0.008, size = 88, normalized size = 0.9

$$\frac{x}{2}\sqrt{cx^2+bx+a}\left(2\sqrt{cx^2+bx+a}c^{3/2}-b\ln\left(\frac{1}{2}\left(2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b\right)\frac{1}{\sqrt{c}}\right)c\right)\frac{1}{\sqrt{cx^4+bx^3+ax^2}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*c^(3/2)-b*ln(1/2
*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*c)/(c*x^4+b*x^3
+a*x^2)^(1/2)/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293534, size = 1, normalized size = 0.01

$$\left[\frac{b\sqrt{c}x\log\left(\frac{4\sqrt{cx^4+bx^3+ax^2}(2c^2x+bc)-(8c^2x^3+8bcx^2+(b^2+4ac)x)\sqrt{c}}{x}\right)+4\sqrt{cx^4+bx^3+ax^2}c}{4c^2x}, \frac{b\sqrt{-c}x\arctan\left(\frac{(2cx^2+bx)\sqrt{-c}}{2\sqrt{cx^4+bx^3+ax^2}c}\right)+2}{2c^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b
*c) - (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x) + 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1
/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) + 2*
sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c^2*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.28169, size = 146, normalized size = 1.42

$$\frac{b\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{b\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-2\sqrt{a}c}{\left(\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)^2-c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] b*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*
c) + (b*(sqrt(c + b/x + a/x^2) - sqrt(a)/x) - 2*sqrt(a)*c)/(((sqr
```

```
t(c + b/x + a/x^2) - sqrt(a)/x)^2 - c)*c)
```

3.51 $\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=71

$$\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.0629483, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 10.3357, size = 66, normalized size = 0.93

$$\frac{x\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] x*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*
x + c*x**2)))/(sqrt(c)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.0398958, size = 66, normalized size = 0.93

$$\frac{x\sqrt{a+bx+cx^2}\log\left(2\sqrt{c}\sqrt{a+bx+cx^2}+b+2cx\right)}{\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2]])/(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.007, size = 65, normalized size = 0.9

$$x\sqrt{cx^2+bx+a}\ln\left(\frac{1}{2}\left(2\sqrt{cx^2+bx+a}\sqrt{c}+2cx+b\right)\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{cx^4+bx^3+ax^2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^
2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285901, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}(2c^2x+bc)+\left(8c^2x^3+8bcx^2+\left(b^2+4ac\right)x\right)\sqrt{c}}{x}\right)}{2\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\left(2cx^2+bx\right)\sqrt{-c}}{2\sqrt{cx^4+bx^3+ax^2}c}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c) + (8*c^2
*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x)/sqrt(c), -sqrt(-c
)*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2
)*c))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.281321, size = 50, normalized size = 0.7

$$-\frac{2\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] -2*arctan((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/sqrt(-c)
```

3.52 $\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
]/Sqrt[a])
```

Rubi [A] time = 0.0325963, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
]/Sqrt[a])
```

Rubi in Sympy [A] time = 19.1527, size = 68, normalized size = 1.51

$$-\frac{x\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] -x*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b
*x + c*x**2)))/(sqrt(a)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.0701684, size = 70, normalized size = 1.56

$$\frac{x\sqrt{a+x(b+cx)}\left(\log(x)-\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(Log[x] - Log[2*a + b*x + 2*Sqrt[a]*Sqrt
[a + x*(b + c*x)]]))/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.008, size = 66, normalized size = 1.5

$$-x\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{cx^4+bx^3+ax^2}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277752, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)-\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\left(bx^2+2\,ax\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^3+ax^2}\,a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2) - (8*a*b*
x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3)/sqrt(a), sqrt(-a
)*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2
)*a))/a]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.276477, size = 80, normalized size = 1.78

$$-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\,\mathrm{sign}\left(x\right)}{\sqrt{-a}} + \frac{2\,\arctan\left(-\frac{\sqrt{c}x - \sqrt{cx^2 + bx + a}}{\sqrt{-a}}\right)}{\sqrt{-a}\mathrm{sign}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sign(x)/sqrt(-a) + 2*arctan(-(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sign(x))
```

3.53 $\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=77

$$\frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x
))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

Rubi [A] time = 0.093948, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x
))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

Rubi in Sympy [A] time = 28.7087, size = 94, normalized size = 1.22

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}+\frac{bx\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{\frac{3}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] -sqrt(a*x**2 + b*x**3 + c*x**4)/(a*x**2) + b*x*sqrt(a + b*x + c*x
**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(2*a**
(3/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.130446, size = 106, normalized size = 1.38

$$\frac{-2\sqrt{a}(a+x(b+cx))-bx\log(x)\sqrt{a+x(b+cx)}+bx\sqrt{a+x(b+cx)}\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)}{2a^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[a]*(a + x*(b + c*x)) - b*x*Sqrt[a + x*(b + c*x)]*Log[x]
+ b*x*Sqrt[a + x*(b + c*x)]*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*
(b + c*x)]])/(2*a^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.009, size = 88, normalized size = 1.1

$$-\frac{1}{2}\sqrt{cx^2+bx+a}\left(2\sqrt{cx^2+bx+a}a^{3/2}-b\ln\left(\frac{1}{x}\left(2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)xa\right)\frac{1}{\sqrt{cx^4+bx^3+ax^2}}a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/2*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*a^(3/2)-b*ln((2*a
+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*x*a)/(c*x^4+b*x^3+a*x^2)^(
1/2)/a^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29666, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{a}bx^2\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}(abx+2a^2)+(8abx^2+(b^2+4ac)x^3+8a^2x)\sqrt{a}}{x^3}\right)-4\sqrt{cx^4+bx^3+ax^2}a}{4a^2x^2},\right.$$
$$\left.-\frac{\sqrt{-a}bx^2\arctan\left(\frac{(bx^2+2ax)\sqrt{-a}}{2\sqrt{cx^4+bx^3+ax^2}a}\right)+2\sqrt{cx^4+bx^3+ax^2}a}{2a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x +
2*a^2) + (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3)
- 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^
2*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2
)*a)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.54 $\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=119

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3
+ c*x^4])/(4*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/
(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(5/2))
```

Rubi [A] time = 0.250008, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3
+ c*x^4])/(4*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/
(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(5/2))
```

Rubi in Sympy [A] time = 42.7591, size = 134, normalized size = 1.13

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{x\left(-ac+\frac{3b^2}{4}\right)\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{\frac{5}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] -sqrt(a*x**2 + b*x**3 + c*x**4)/(2*a*x**3) + 3*b*sqrt(a*x**2 + b*
x**3 + c*x**4)/(4*a**2*x**2) - x*(-a*c + 3*b**2/4)*sqrt(a + b*x +
 c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(2
*a**(5/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.230935, size = 138, normalized size = 1.16

$$\frac{x^2\log(x)\left(3b^2-4ac\right)\sqrt{a+x(b+cx)}+x^2\left(4ac-3b^2\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)-2\sqrt{a}(2a-3bx)(}{8a^{5/2}x\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x)) + (3*b^2 - 4*a*c)*x^2
*Sqrt[a + x*(b + c*x)]*Log[x] + (-3*b^2 + 4*a*c)*x^2*Sqrt[a + x*(
b + c*x)]*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])/(8*a^
(5/2)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.01, size = 152, normalized size = 1.3

$$-\frac{1}{8x}\sqrt{cx^2+bx+a}\left(4\sqrt{cx^2+bx+a}\,a^{5/2}-6\sqrt{cx^2+bx+a}\,a^{3/2}xb-4c\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)x^2a^2+3\ln\left(\frac{2a}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+b*x+a)^(1/2)*(4*(c*x^2+b*x+a)^(1/2)*a^(5/2)-6*(c*x^2+
b*x+a)^(1/2)*a^(3/2)*x*b-4*c*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^
(1/2))/x)*x^2*a^2+3*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
*x^2*a*b^2)/x/(c*x^4+b*x^3+a*x^2)^(1/2)/a^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289729, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,b^2-4\,ac\right)\sqrt{a}x^3\log\left(-\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)+\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\left(3\,abx-2\,a^2\right)}{16\,a^3x^3},\ \frac{\left(3\,b^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(4*sqrt(c*x^4 + b*x^3 +
a*x^2)*(a*b*x + 2*a^2) + (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2)
)/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)*sqrt(-a)*x^3*arctan(1/2*(b*x^2
+ 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2)*a)) + 2*sqrt(c*x^4
 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.55 $\int \frac{x^7}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{3x\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c^3x\left(b^2-4ac\right)}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)}+\frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)}$$

```
[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) +
 ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c^2*(b^2 - 4*a
*c)) - (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(
b^2 - 4*a*c)*x) - (2*b*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4
*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2)*Sqrt[a*x^2
+ b*x^3 + c*x^4])
```

Rubi [A] time = 0.785475, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{3x\left(5b^2-4ac\right)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4c^3x\left(b^2-4ac\right)}+\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2\left(b^2-4ac\right)}+\frac{2x^4(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) +
 ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c^2*(b^2 - 4*a
*c)) - (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(
b^2 - 4*a*c)*x) - (2*b*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4
*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2)*Sqrt[a*x^2
+ b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 78.8596, size = 241, normalized size = 0.92

$$-\frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(-4ac+b^2)}-\frac{b(-52ac+15b^2)\sqrt{ax^2+bx^3+cx^4}}{4c^3x(-4ac+b^2)}+\frac{2x^4(2a+bx)}{(-4ac+b^2)\sqrt{ax^2+bx^3+cx^4}}+\frac{(-24ac+10b^2)\sqrt{ax^2+bx^3+cx^4}}{4c^2(-4ac+b^2)}+\frac{3x(-8ac+10b^2)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{\frac{7}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] -2*b*x*sqrt(a*x**2 + b*x**3 + c*x**4)/(c*(-4*a*c + b**2)) - b*(-5
2*a*c + 15*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(4*c**3*x*(-4*a*c
 + b**2)) + 2*x**4*(2*a + b*x)/((-4*a*c + b**2)*sqrt(a*x**2 + b*x
**3 + c*x**4)) + (-24*a*c + 10*b**2)*sqrt(a*x**2 + b*x**3 + c*x**
4)/(4*c**2*(-4*a*c + b**2)) + 3*x*(-8*a*c + 10*b**2)*sqrt(a + b*x
 + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x**2)))/
(16*c**(7/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.326922, size = 181, normalized size = 0.69

$$\frac{x\left(2\sqrt{c}\left(4a^2c(6cx-13b)+a\left(15b^3-62b^2cx-20bc^2x^2+8c^3x^3\right)+b^2x\left(15b^2+5bcx-2c^2x^2\right)\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\right.}{8c^{7/2}(4ac-b^2)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x
- 2*c^2*x^2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)
) - 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*Sqrt[a + x*(b + c*x)]*Log
[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(8*c^(7/2)*(-b^2
+ 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.012, size = 283, normalized size = 1.1

$$-\frac{x^3(cx^2+bx+a)}{32ac-8b^2}\left(-16c^{9/2}x^3a+4c^{7/2}x^3b^2+40c^{7/2}x^2ab-10c^{5/2}x^2b^3-48c^{7/2}xa^2+124c^{5/2}xab^2-30c^{3/2}xb^4+48\sqrt{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/8*x^3*(c*x^2+b*x+a)/c^(9/2)*(-16*c^(9/2)*x^3*a+4*c^(7/2)*x^3*b
^2+40*c^(7/2)*x^2*a*b-10*c^(5/2)*x^2*b^3-48*c^(7/2)*x*a^2+124*c^(
5/2)*x*a*b^2-30*c^(3/2)*x*b^4+48*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c
*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*c^3-72*(c*x^2+b*x
+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))
*a*b^2*c^2+15*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c
^(1/2)+2*c*x+b)/c^(1/2))*b^4*c+104*c^(5/2)*a^2*b-30*c^(3/2)*a*b^3
)/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.348499, size = 1, normalized size = 0.

$$\left[-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^3+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^2+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x\right)\sqrt{c}\log\left(\frac{4\sqrt{cx^4+bx^3+ax^2}}{}\right.}{16\left(\left(b^2c^5-4\right.\right.}\right.$$

$$-\frac{3\left(\left(5\,b^4c-24\,ab^2c^2+16\,a^2c^3\right)x^3+\left(5\,b^5-24\,ab^3c+16\,a^2bc^2\right)x^2+\left(5\,ab^4-24\,a^2b^2c+16\,a^3c^2\right)x\right)\sqrt{-c}\arctan\left(\frac{(2\,cx^2+bx)}{2\sqrt{cx^4+bx^3}}\right.}{8\left(\left(b^2c^5-4\,ac^6\right)x^3+\left(b^3c^4-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 2
4*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*
c^2)*x)*sqrt(c)*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c
) - (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x) + 4*(15
*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2
- 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt
(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*
```

```
a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24
*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2
)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(-c)*arctan(
1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqrt(c*x^4 + b*x^3 + a*x^2)*c)) + 2
*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*
c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*
sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4
- 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

3.56 $\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=201

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2x\left(b^2-4ac\right)}+\frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)}-\frac{3bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 -
8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*
x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.481831, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\left(3b^2-8ac\right)\sqrt{ax^2+bx^3+cx^4}}{c^2x\left(b^2-4ac\right)}+\frac{2x^3(2a+bx)}{\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(b^2-4ac\right)}-\frac{3bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 -
8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*
x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 53.7688, size = 187, normalized size = 0.93

$$-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\left(-4ac+b^2\right)}-\frac{3bx\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{\frac{5}{2}}\sqrt{ax^2+bx^3+cx^4}}+\frac{2x^3\left(2a+bx\right)}{\left(-4ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}}+\frac{\left(-16ac+6b^2\right)\sqrt{ax^2+bx^3+cx^4}}{2c^2x\left(-4ac+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] -2*b*sqrt(a*x**2 + b*x**3 + c*x**4)/(c*(-4*a*c + b**2)) - 3*b*x*s
qrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x +
 c*x**2)))/(2*c**(5/2)*sqrt(a*x**2 + b*x**3 + c*x**4)) + 2*x**3*(
2*a + b*x)/((-4*a*c + b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)) + (-1
6*a*c + 6*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(2*c**2*x*(-4*a*c
+ b**2))
```

Mathematica [A] time = 0.201403, size = 139, normalized size = 0.69

$$\frac{x\left(2\sqrt{c}\left(8a^2c+a\left(-3b^2+10bcx+4c^2x^2\right)-b^2x(3b+cx)\right)+3b\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right.\right.}{2c^{5/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x
 + 4*c^2*x^2)) + 3*b*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Log[b +
2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(2*c^(5/2)*(-b^2 + 4*a
*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.01, size = 199, normalized size = 1.

$$-\frac{x^3\left(cx^2+bx+a\right)}{8\,ac-2\,b^2}\left(-8\,c^{7/2}x^2a+2\,c^{5/2}x^2b^2-20\,c^{5/2}xab+6\,c^{3/2}xb^3+12\,\sqrt{cx^2+bx+a}\ln\left(1/2\,\frac{2\,\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx}{\sqrt{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/2*x^3*(c*x^2+b*x+a)/c^(7/2)*(-8*c^(7/2)*x^2*a+2*c^(5/2)*x^2*b^
2-20*c^(5/2)*x*a*b+6*c^(3/2)*x*b^3+12*(c*x^2+b*x+a)^(1/2)*ln(1/2*
(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b*c^2-3*(c*x^2
+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1
/2))*b^3*c-16*c^(5/2)*a^2+6*c^(3/2)*a*b^2)/(c*x^4+b*x^3+a*x^2)^(3
/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.329475, size = 1, normalized size = 0.

$$\left[\frac{3\left(\left(b^{3}c-4\,abc^{2}\right)x^{3}+\left(b^{4}-4\,ab^{2}c\right)x^{2}+\left(ab^{3}-4\,a^{2}bc\right)x\right)\sqrt{c}\log\left(\frac{4\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(2\,c^{2}x+bc\right)-\left(8\,c^{2}x^{3}+8\,bcx^{2}+\left(b^{2}+4\,ac\right)x\right)\sqrt{c}}{x}\right)+4\,\surd}{4\left(\left(b^{2}c^{4}-4\,ac^{5}\right)x^{3}+\left(b^{3}c^{3}-4\,abc^{4}\right)x^{2}+\left(ab^{2}c^{3}-4\,a^{2}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3
 - 4*a^2*b*c)*x)*sqrt(c)*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^
2*x + b*c) - (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x
) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b^2*c - 8*a^2*c^2 + (b^2*c
^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x))/((b^2*c^4 - 4*a*c^
5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^4)*x),
1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3
- 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqr
t(c*x^4 + b*x^3 + a*x^2)*c)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a
*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*
c^2)*x))/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (
a*b^2*c^3 - 4*a^2*c^4)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{6}}{\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.287723, size = 263, normalized size = 1.31

$$\frac{2\left(\frac{b^3c^2-3\,abc^3}{b^2c^4-4\,ac^5}+\frac{ab^2c^2-2\,a^2c^3}{(b^2c^4-4\,ac^5)x}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}+\frac{3\,b\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}+\frac{b\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-2\sqrt{a}c}{\left(\left(\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)^2-c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*((b^3*c^2 - 3*a*b*c^3)/(b^2*c^4 - 4*a*c^5) + (a*b^2*c^2 - 2*a^2
*c^3)/((b^2*c^4 - 4*a*c^5)*x))/sqrt(c + b/x + a/x^2) + 3*b*arctan
((sqrt(c + b/x + a/x^2) - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c^2) + (
b*(sqrt(c + b/x + a/x^2) - sqrt(a)/x) - 2*sqrt(a)*c)/(((sqrt(c +
b/x + a/x^2) - sqrt(a)/x)^2 - c)*c^2)
```

3.57 $\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx\,(b^2-4ac)}+\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[
a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.274087, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx\,(b^2-4ac)}+\frac{x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 (2*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[
a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 33.3719, size = 139, normalized size = 0.91

$$-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx\,(-4ac+b^2)}+\frac{2x^2\,(2a+bx)}{(-4ac+b^2)\sqrt{ax^2+bx^3+cx^4}}+\frac{x\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{\frac{3}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] -2*b*sqrt(a*x**2 + b*x**3 + c*x**4)/(c*x*(-4*a*c + b**2)) + 2*x**
2*(2*a + b*x)/((-4*a*c + b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)) +
```

```
x*sqrt(a + b*x + c*x**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*
x + c*x**2)))/(c**(3/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.180332, size = 110, normalized size = 0.72

$$-\frac{x\left(2\sqrt{c}\left(-ab+2acx+b^2(-x)\right)+\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\right)}{c^{3/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(2*Sqrt[c]*(-(a*b) - b^2*x + 2*a*c*x) + (b^2 - 4*a*c)*Sqrt[a
 + x*(b + c*x)]*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])
)/(c^(3/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))]))
```

Maple [A] time = 0.011, size = 166, normalized size = 1.1

$$\frac{x^3\left(cx^2+bx+a\right)}{4\,ac-b^2}\left(-4\,c^{5/2}xa+2\,c^{3/2}xb^2+4\,\sqrt{cx^2+bx+a}\ln\left(1/2\,\frac{2\,\sqrt{cx^2+bx+a}\sqrt{c}+2\,cx+b}{\sqrt{c}}\right)ac^2-\sqrt{cx^2+bx+a}\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b*x+a)/c^(5/2)*(-4*c^(5/2)*x*a+2*c^(3/2)*x*b^2+4*(c*x^
2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(
1/2))*a*c^2-(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(
1/2)+2*c*x+b)/c^(1/2))*b^2*c+2*c^(3/2)*a*b)/(c*x^4+b*x^3+a*x^2)^(
3/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.32409, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x\right)\sqrt{c}\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}\left(2\,c^2x+bc\right)+\left(8\,c^2x^3+8\,bcx^2+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right)-4\sqrt{cx^4}}{2\left(\left(b^2c^3-4\,ac^4\right)x^3+\left(b^3c^2-4\,abc^3\right)x^2+\left(ab^2c^2-4\,a^2c^3\right)x\right)}\right.$$

$$-\frac{\left(\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x\right)\sqrt{-c}\arctan\left(\frac{\left(2\,cx^2+bx\right)\sqrt{-c}}{2\sqrt{cx^4+bx^3+ax^2}c}\right)+2\sqrt{cx^4+bx^3+ax^2}\left(abc+\left(b^2c-2\,ac\right.\right.}{\left(b^2c^3-4\,ac^4\right)x^3+\left(b^3c^2-4\,abc^3\right)x^2+\left(ab^2c^2-4\,a^2c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a
^2*c)*x)*sqrt(c)*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b
*c) + (8*c^2*x^3 + 8*b*c*x^2 + (b^2 + 4*a*c)*x)*sqrt(c))/x) - 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c
^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^
2*c^3)*x), -(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^
2 - 4*a^2*c)*x)*sqrt(-c)*arctan(1/2*(2*c*x^2 + b*x)*sqrt(-c)/(sqr
t(c*x^4 + b*x^3 + a*x^2)*c)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b
*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 -
4*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.292568, size = 149, normalized size = 0.97

$$-\frac{2\left(\frac{abc}{(b^2c^2-4\,ac^3)x}+\frac{b^2c-2\,ac^2}{b^2c^2-4\,ac^3}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}-\frac{2\,\arctan\left(\frac{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}{\sqrt{-c}}\right)}{\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(a*b*c/((b^2*c^2 - 4*a*c^3)*x) + (b^2*c - 2*a*c^2)/(b^2*c^2 -
4*a*c^3))/sqrt(c + b/x + a/x^2) - 2*arctan((sqrt(c + b/x + a/x^2)
 - sqrt(a)/x)/sqrt(-c))/(sqrt(-c)*c)
```

3.58 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.138836, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 9.37527, size = 36, normalized size = 0.9

$$\frac{x(4a+2bx)}{(-4ac+b^2)\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] x*(4*a + 2*b*x)/((-4*a*c + b**2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.0410305, size = 37, normalized size = 0.92

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.005, size = 53, normalized size = 1.3

$$-2\,{\frac { \left(c{x}^{2}+bx+a \right) \left(bx+2\,a \right) {x}^{3}}{ \left(4\,ac-{b}^{2} \right) \left(c{x}^{4}+b{x}^{3}+a{x}^{2} \right) ^{3/2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2*(c*x^2+b*x+a)*(b*x+2*a)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290717, size = 99, normalized size = 2.48

$$\frac{2 \, \sqrt{c x^{4} + b x^{3} + a x^{2}}{\left(b x + 2 \, a\right)}}{{\left(b^{2} c - 4 \, a c^{2}\right)} x^{3} + {\left(b^{3} - 4 \, a b c\right)} x^{2} + {\left(a b^{2} - 4 \, a^{2} c\right)} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3
+ (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.285969, size = 61, normalized size = 1.52

$$\frac{2\left(\frac{b}{b^2-4\,ac}+\frac{2\,a}{(b^2-4\,ac)x}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b/(b^2 - 4*a*c) + 2*a/((b^2 - 4*a*c)*x))/sqrt(c + b/x + a/x^2)
```

3.59 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=39

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi [A] time = 0.0558066, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rubi in Sympy [A] time = 9.38745, size = 37, normalized size = 0.95

$$-\frac{2x\,(b+2cx)}{(-4ac+b^2)\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] -2*x*(b + 2*c*x)/((-4*a*c + b**2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.0298336, size = 36, normalized size = 0.92

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]

[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.005, size = 52, normalized size = 1.3

$$2\,{\frac { \left(c{x}^{2}+bx+a \right) \left(2\,cx+b \right) {x}^{3}}{ \left(4\,ac-{b}^{2} \right) \left(c{x}^{4}+b{x}^{3}+a{x}^{2} \right) ^{3/2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x)

[Out] 2*(c*x^2+b*x+a)*(2*c*x+b)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284836, size = 97, normalized size = 2.49

$$-\frac{2 \, \sqrt{c x^{4} + b x^{3} + a x^{2}}{\left(2 \, c x + b\right)}}{{\left(b^{2} c - 4 \, a c^{2}\right)} x^{3} + {\left(b^{3} - 4 \, a b c\right)} x^{2} + {\left(a b^{2} - 4 \, a^{2} c\right)} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="fricas")

[Out] -2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)/((b^2*c - 4*a*c^2)*x^3
 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.284596, size = 61, normalized size = 1.56

$$-\frac{2\left(\frac{2c}{b^2-4ac}+\frac{b}{(b^2-4ac)x}\right)}{\sqrt{c+\frac{b}{x}+\frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*c/(b^2 - 4*a*c) + b/((b^2 - 4*a*c)*x))/sqrt(c + b/x + a/x^2
)
```

3.60 $\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

```
[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 +
 c*x^4]) - ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
+ c*x^4])]/a^(3/2)
```

Rubi [A] time = 0.110074, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 +
 c*x^4]) - ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
+ c*x^4])]/a^(3/2)
```

Rubi in Sympy [A] time = 29.4479, size = 112, normalized size = 1.19

$$\frac{2x\left(-2ac+b^2+bcx\right)}{a\left(-4ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{x\sqrt{a+bx+cx^2}\,\text{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{\frac{3}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] 2*x*(-2*a*c + b**2 + b*c*x)/(a*(-4*a*c + b**2)*sqrt(a*x**2 + b*x*
*3 + c*x**4)) - x*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqr
t(a)*sqrt(a + b*x + c*x**2)))/(a**(3/2)*sqrt(a*x**2 + b*x**3 + c*
x**4))
```

Mathematica [A] time = 0.261168, size = 132, normalized size = 1.4

$$-\frac{x\left(2\sqrt{a}\left(-2ac+b^2+bcx\right)+\log(x)\left(b^2-4ac\right)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+\right.\right.}{a^{3/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(2*Sqrt[a]*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*
(b + c*x)]*Log[x] - (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Log[2*a +
 b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]]))/(a^(3/2)*(-b^2 + 4*a*c)
*Sqrt[x^2*(a + x*(b + c*x))]))
```

Maple [A] time = 0.009, size = 166, normalized size = 1.8

$$-\frac{x^3\left(cx^2+bx+a\right)}{4\,ac-b^2}\left(2\,a^{3/2}xbc+4\,\sqrt{cx^2+bx+a}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)a^2c-\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -x^3*(c*x^2+b*x+a)*(2*a^(3/2)*x*b*c+4*(c*x^2+b*x+a)^(1/2)*ln((2*a
+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*a^2*c-(c*x^2+b*x+a)^(1/2)*
ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*a*b^2-4*a^(5/2)*c+2
*a^(3/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(5/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298579, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)x^3+\left(b^3-4\,abc\right)x^2+\left(ab^2-4\,a^2c\right)x\right)\sqrt{a}\log\left(\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)-\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4}}{2\left(\left(a^2b^2c-4\,a^3c^2\right)x^3+\left(a^2b^3-4\,a^3bc\right)x^2+\left(a^3b^2-4\,a^4c\right)x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a
^2*c)*x)*sqrt(a)*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^
2) - (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) + 4*
sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^
2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a
^4*c)*x), (((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2
- 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(
c*x^4 + b*x^3 + a*x^2)*a)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c
*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 -
4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.61 $\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c
*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2
- 4*a*c)*x^2) + (3*b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^
2 + b*x^3 + c*x^4])])/(2*a^(5/2))
```

Rubi [A] time = 0.26817, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c
*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2
- 4*a*c)*x^2) + (3*b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^
2 + b*x^3 + c*x^4])])/(2*a^(5/2))
```

Rubi in Sympy [A] time = 57.4486, size = 162, normalized size = 1.12

$$\frac{2(-2ac+b^2+bcx)}{a(-4ac+b^2)\sqrt{ax^2+bx^3+cx^4}}-\frac{(-8ac+3b^2)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2(-4ac+b^2)}+\frac{3bx\sqrt{a+bx+cx^2}\,\mathrm{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{\frac{5}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x)/(a*(-4*a*c + b**2)*sqrt(a*x**2 + b*x**3
 + c*x**4)) - (-8*a*c + 3*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)/(a
**2*x**2*(-4*a*c + b**2)) + 3*b*x*sqrt(a + b*x + c*x**2)*atanh((2
```

```
*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/(2*a**(5/2)*sqrt(a*
x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.280128, size = 163, normalized size = 1.13

$$\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx-8c^2x^2\right)+3b^2x(b+cx)\right)+3bx\log(x)\left(b^2-4ac\right)\sqrt{a+x(b+cx)}-3bx\left(b^2-4ac\right)\sqrt{a+x(b+}{2a^{5/2}\left(4ac-b^2\right)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*
c^2*x^2)) + 3*b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*Log[x] - 3*
b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*Log[2*a + b*x + 2*Sqrt[a]
*Sqrt[a + x*(b + c*x)]])/(2*a^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a +
x*(b + c*x))])
```

Maple [A] time = 0.01, size = 201, normalized size = 1.4

$$\frac{x^2\left(cx^2+bx+a\right)}{8\,ac-2\,b^2}\left(-16\,a^{5/2}x^2c^2+6\,a^{3/2}x^2b^2c+12\,\sqrt{cx^2+bx+a}\ln\left(\frac{2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)xa^2bc-3\,\sqrt{cx^2+bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 1/2*x^2*(c*x^2+b*x+a)*(-16*a^(5/2)*x^2*c^2+6*a^(3/2)*x^2*b^2*c+12
*(c*x^2+b*x+a)^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
)*x*a^2*b*c-3*(c*x^2+b*x+a)^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)*x*a*b^3-20*a^(5/2)*x*b*c+6*a^(3/2)*x*b^3-8*a^(7/2)
*c+2*a^(5/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(7/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A] time = 0.308961, size = 1, normalized size = 0.01

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^4+\left(b^4-4\,ab^2c\right)x^3+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{a}\log\left(-\frac{4\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)+\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right)-}{4\left((a^3b^2c-4\,a^4c^2)x^4+(a^3b^3-4\,a^4bc)x^3+(a^4b^2-4\right.}\right.$$

$$-\frac{3\left(\left(b^3c-4\,abc^2\right)x^4+\left(b^4-4\,ab^2c\right)x^3+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{-a}\arctan\left(\frac{\left(bx^2+2\,ax\right)\sqrt{-a}}{2\sqrt{cx^4+bx^3+ax^2}\,a}\right)+2\sqrt{cx^4+bx^3+ax^2}\left(a^2b^2-4\,a\right.}{2\left((a^3b^2c-4\,a^4c^2)x^4+(a^3b^3-4\,a^4bc)x^3+(a^4b^2-4\,a^5c)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3
 - 4*a^2*b*c)*x^2)*sqrt(a)*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a
*b*x + 2*a^2) + (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a)
)/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a^2*b^2 - 4*a^3*c + (3*a*
b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b*c)*x))/((a^3*b^2*c -
 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 - 4*a^5*c)
*x^2), -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*(b*x^2 + 2*a*x)*sqr
t(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2)*a)) + 2*sqrt(c*x^4 + b*x^3 + a
*x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3
 - 10*a^2*b*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^
4*b*c)*x^3 + (a^4*b^2 - 4*a^5*c)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.62 $\int \frac{1}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 +
 c*x^4]) - ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*
(b^2 - 4*a*c)*x^3) + (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(4*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(
2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(7/2))
```

Rubi [A] time = 0.484992, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 +
 c*x^4]) - ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*
(b^2 - 4*a*c)*x^3) + (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*
x^4])/(4*a^3*(b^2 - 4*a*c)*x^2) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(
2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(8*a^(7/2))
```

Rubi in Sympy [A] time = 79.8979, size = 221, normalized size = 1.06

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax\left(-4ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(-12ac+5b^2\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3\left(-4ac+b^2\right)}+\frac{b\left(-52ac+15b^2\right)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2\left(-4ac+b^2\right)}-\frac{3x\left(-4ac+5b^2\right)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{\frac{7}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x)/(a*x*(-4*a*c + b**2)*sqrt(a*x**2 + b*x*
*3 + c*x**4)) - (-12*a*c + 5*b**2)*sqrt(a*x**2 + b*x**3 + c*x**4)
/(2*a**2*x**3*(-4*a*c + b**2)) + b*(-52*a*c + 15*b**2)*sqrt(a*x**
2 + b*x**3 + c*x**4)/(4*a**3*x**2*(-4*a*c + b**2)) - 3*x*(-4*a*c
+ 5*b**2)*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqr
t(a + b*x + c*x**2)))/(8*a**(7/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.569977, size = 220, normalized size = 1.05

$$-\frac{-3x^2\log(x)\left(16a^2c^2-24ab^2c+5b^4\right)\sqrt{a+x(b+cx)}+3x^2\left(16a^2c^2-24ab^2c+5b^4\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{a}\sqrt{a+x(b+cx}\right.}{8a^{7/2}x\left(b^2-4ac\right)\sqrt{x^2(a+}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] -(-2*Sqrt[a]*(8*a^3*c + 15*b^3*x^2*(b + c*x) + a*b*x*(5*b^2 - 62*
b*c*x - 52*c^2*x^2) - 2*a^2*(b^2 + 10*b*c*x - 12*c^2*x^2)) - 3*(5
*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^2*Sqrt[a + x*(b + c*x)]*Log[x]
+ 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^2*Sqrt[a + x*(b + c*x)]*L
og[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])/(8*a^(7/2)*(b^2
- 4*a*c)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.013, size = 292, normalized size = 1.4

$$-\frac{x\left(cx^2+bx+a\right)}{32\,ac-8\,b^2}\left(48\,a^{7/2}x^2c^2-104\,a^{5/2}x^3bc^2+16\,a^{9/2}c-40\,a^{7/2}xbc-124\,a^{5/2}x^2b^2c+30\,a^{3/2}x^3b^3c-4\,a^{7/2}b^2+10\,a^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/8*x*(c*x^2+b*x+a)*(48*a^(7/2)*x^2*c^2-104*a^(5/2)*x^3*b*c^2+16
*a^(9/2)*c-40*a^(7/2)*x*b*c-124*a^(5/2)*x^2*b^2*c+30*a^(3/2)*x^3*
b^3*c-4*a^(7/2)*b^2+10*a^(5/2)*x*b^3+30*a^(3/2)*x^2*b^4-48*ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a
^3*c^2+72*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*
x+a)^(1/2)*x^2*a^2*b^2*c-15*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(
```

```
1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^2*a*b^4)/(c*x^4+b*x^3+a*x^2)^(3/2)
/a^(9/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)
```

Fricas [A] time = 0.353024, size = 1, normalized size = 0.

$$\left[-\frac{3\left(\left(5\,b^4c - 24\,ab^2c^2 + 16\,a^2c^3\right)x^5 + \left(5\,b^5 - 24\,ab^3c + 16\,a^2bc^2\right)x^4 + \left(5\,ab^4 - 24\,a^2b^2c + 16\,a^3c^2\right)x^3\right)\sqrt{a}\log\left(-\frac{4\sqrt{cx^4+bx^3+\cdots}}{\cdots}\right.}{16\left(\left(a^4b^2c - \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 2
4*a*b^3*c + 16*a^2*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*
c^2)*x^3)*sqrt(a)*log(-(4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*
a^2) + (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) +
4*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*
b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*b*c)*x)
*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 + (a^4
*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3), 1/8*(3*((5*b^4*
c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2
*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(-a)
*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2)
*a)) - 2*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 -
 (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*
b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5
 + (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ax^2 + bx^3 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.614045, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2),x, algorithm="giac")

[Out] sage0*x
```

3.63 $\int \frac{1}{x(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{5b\left(7b^2-12ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2x^4\left(b^2-4ac\right)}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4x^2\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3
 + c*x^4]) - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^
2*(b^2 - 4*a*c)*x^4) + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])/(12*a^3*(b^2 - 4*a*c)*x^3) - ((105*b^4 - 460*a*b^2*c + 2
56*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^4*(b^2 - 4*a*c)*x^
2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqr
t[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))
```

Rubi [A] time = 0.739871, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{5b\left(7b^2-12ac\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}}+\frac{b\left(35b^2-116ac\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(b^2-4ac\right)}-\frac{\left(7b^2-16ac\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2x^4\left(b^2-4ac\right)}-\frac{\left(256a^2c^2-460ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{24a^4x^2\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3
 + c*x^4]) - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^
2*(b^2 - 4*a*c)*x^4) + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])/(12*a^3*(b^2 - 4*a*c)*x^3) - ((105*b^4 - 460*a*b^2*c + 2
56*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^4*(b^2 - 4*a*c)*x^
2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqr
t[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))
```

Rubi in Sympy [A] time = 119.222, size = 282, normalized size = 1.04

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^2\left(-4ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(-16ac+7b^2\right)\sqrt{ax^2+bx^3+cx^4}}{3a^2x^4\left(-4ac+b^2\right)}+\frac{b\left(-116ac+35b^2\right)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(-4ac+b^2\right)}-\frac{\sqrt{ax^2+bx^3+cx^4}\left(256a^2c^2-460ab^2c+105b^4\right)}{24a^4x^2\left(-4ac+b^2\right)}+\frac{5bx\left(-12ac+7b^2\right)\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{\frac{9}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x)/(a*x**2*(-4*a*c + b**2)*sqrt(a*x**2 + b
*x**3 + c*x**4)) - (-16*a*c + 7*b**2)*sqrt(a*x**2 + b*x**3 + c*x*
*4)/(3*a**2*x**4*(-4*a*c + b**2)) + b*(-116*a*c + 35*b**2)*sqrt(a
*x**2 + b*x**3 + c*x**4)/(12*a**3*x**3*(-4*a*c + b**2)) - sqrt(a*
x**2 + b*x**3 + c*x**4)*(256*a**2*c**2 - 460*a*b**2*c + 105*b**4)
/(24*a**4*x**2*(-4*a*c + b**2)) + 5*b*x*(-12*a*c + 7*b**2)*sqrt(a
 + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x*
*2)))/(16*a**(9/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.581639, size = 265, normalized size = 0.98

$$\frac{15bx^3\log(x)\left(48a^2c^2-40ab^2c+7b^4\right)\sqrt{a+x(b+cx)}-15bx^3\left(48a^2c^2-40ab^2c+7b^4\right)\sqrt{a+x(b+cx)}\log\left(2\sqrt{a}\sqrt{a+x(b+\right.}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*Sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^
2 - 106*b*c*x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2)
+ 2*a^2*x*(-7*b^3 - 86*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) +
15*b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*x^3*Sqrt[a + x*(b + c*x)]*
Log[x] - 15*b*(7*b^4 - 40*a*b^2*c + 48*a^2*c^2)*x^3*Sqrt[a + x*(b
 + c*x)]*Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])/(48*a^
(9/2)*(-b^2 + 4*a*c)*x^2*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.015, size = 340, normalized size = 1.3

$$-\frac{cx^2+bx+a}{192\,ac-48\,b^2}\left(-512\,a^{7/2}x^4c^3-256\,a^{9/2}x^2c^2-976\,a^{7/2}x^3bc^2+920\,a^{5/2}x^4b^2c^2+64\,a^{11/2}c-112\,a^{9/2}xbc+344\,a^{7/2}x^2b^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/48*(c*x^2+b*x+a)*(-512*a^(7/2)*x^4*c^3-256*a^(9/2)*x^2*c^2-976
*a^(7/2)*x^3*b*c^2+920*a^(5/2)*x^4*b^2*c^2+64*a^(11/2)*c-112*a^(9
/2)*x*b*c+344*a^(7/2)*x^2*b^2*c+1060*a^(5/2)*x^3*b^3*c-210*a^(3/2
)*x^4*b^4*c-16*a^(9/2)*b^2+28*a^(7/2)*x*b^3-70*a^(5/2)*x^2*b^4-21
0*a^(3/2)*x^3*b^5+720*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)*(c*x^2+b*x+a)^(1/2)*x^3*a^3*b*c^2-600*ln((2*a+b*x+2*a^(1/2)*(c
*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^3*a^2*b^3*c+105*ln((2
*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^3*
a*b^5)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(11/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

Fricas [A] time = 0.376666, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6
- 40*a*b^4*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48
*a^3*b*c^2)*x^4)*sqrt(a)*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*
```

```
x + 2*a^2) - (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x
^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 +
256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^
3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3
- 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*
c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4),
-1/48*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 -
 40*a*b^4*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*
a^3*b*c^2)*x^4)*sqrt(-a)*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqr
t(c*x^4 + b*x^3 + a*x^2)*a)) + 2*(8*a^4*b^2 - 32*a^5*c + (105*a*b
^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*
b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^
4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x
^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (
a^6*b^2 - 4*a^7*c)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)

[Out] Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x),x, algorithm="giac")

[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

3.64 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=343

$$\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{8a^3x^4\left(b^2-4ac\right)}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2x^5\left(b^2-4ac\right)}-\frac{15\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}}+\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{64a^5x^2\left(b^2-4ac\right)}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{32a^4x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3
 + c*x^4]) - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^
2*(b^2 - 4*a*c)*x^5) + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 +
c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^4) - ((105*b^4 - 448*a*b^2*c + 240
*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a^4*(b^2 - 4*a*c)*x^3)
 + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt[a*x^2 + b*x^3
+ c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^2*c +
 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(128*a^(11/2))
```

Rubi [A] time = 0.974915, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{b\left(21b^2-68ac\right)\sqrt{ax^2+bx^3+cx^4}}{8a^3x^4\left(b^2-4ac\right)}-\frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2x^5\left(b^2-4ac\right)}-\frac{15\left(16a^2c^2-56ab^2c+21b^4\right)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}}+\frac{b\left(1808a^2c^2-1680ab^2c+315b^4\right)\sqrt{ax^2+bx^3+cx^4}}{64a^5x^2\left(b^2-4ac\right)}-\frac{\left(240a^2c^2-448ab^2c+105b^4\right)\sqrt{ax^2+bx^3+cx^4}}{32a^4x^3\left(b^2-4ac\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3
 + c*x^4]) - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^
```

```
2*(b^2 - 4*a*c)*x^5) + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 +
c*x^4])/(8*a^3*(b^2 - 4*a*c)*x^4) - ((105*b^4 - 448*a*b^2*c + 240
*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(32*a^4*(b^2 - 4*a*c)*x^3)
 + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt[a*x^2 + b*x^3
+ c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^2*c +
 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(128*a^(11/2))
```

Rubi in Sympy [A] time = 145.7, size = 352, normalized size = 1.03

$$\frac{2\left(-2ac+b^2+bcx\right)}{ax^3\left(-4ac+b^2\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(-20ac+9b^2\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2x^5\left(-4ac+b^2\right)}+\frac{b\left(-68ac+21b^2\right)\sqrt{ax^2+bx^3+cx^4}}{8a^3x^4\left(-4ac+b^2\right)}-\frac{\sqrt{ax^2+bx^3+cx^4}\left(240a^2c^2-448ab^2c+105b^4\right)}{32a^4x^3\left(-4ac+b^2\right)}+\frac{b\sqrt{ax^2+bx^3+cx^4}\left(1808a^2c^2-1680ab^2c+315b^4\right)}{64a^5x^2\left(-4ac+b^2\right)}-\frac{15x\sqrt{a+bx+cx^2}\left(16a^2c^2-56ab^2c+21b^4\right)\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{\frac{11}{2}}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] 2*(-2*a*c + b**2 + b*c*x)/(a*x**3*(-4*a*c + b**2)*sqrt(a*x**2 + b
*x**3 + c*x**4)) - (-20*a*c + 9*b**2)*sqrt(a*x**2 + b*x**3 + c*x*
*4)/(4*a**2*x**5*(-4*a*c + b**2)) + b*(-68*a*c + 21*b**2)*sqrt(a*
x**2 + b*x**3 + c*x**4)/(8*a**3*x**4*(-4*a*c + b**2)) - sqrt(a*x*
*2 + b*x**3 + c*x**4)*(240*a**2*c**2 - 448*a*b**2*c + 105*b**4)/(
32*a**4*x**3*(-4*a*c + b**2)) + b*sqrt(a*x**2 + b*x**3 + c*x**4)*
(1808*a**2*c**2 - 1680*a*b**2*c + 315*b**4)/(64*a**5*x**2*(-4*a*c
 + b**2)) - 15*x*sqrt(a + b*x + c*x**2)*(16*a**2*c**2 - 56*a*b**2
*c + 21*b**4)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)
))/(128*a**(11/2)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.925578, size = 322, normalized size = 0.94

$$\frac{-15x^4\log(x)\left(-64a^3c^3+240a^2b^2c^2-140ab^4c+21b^6\right)\sqrt{a+x(b+cx)}+15x^4\left(-64a^3c^3+240a^2b^2c^2-140ab^4c+21b^6\right)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[a]*(64*a^5*c + 315*b^5*x^4*(b + c*x) + 105*a*b^3*x^3*(b^
2 - 18*b*c*x - 16*c^2*x^2) - 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2)
+ 8*a^3*x*(3*b^3 + 26*b^2*c*x + 98*b*c^2*x^2 - 60*c^3*x^3) + 2*a^
2*b*x^2*(-21*b^3 - 308*b^2*c*x + 1352*b*c^2*x^2 + 904*c^3*x^3)) -
 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*x^4*Sqr
t[a + x*(b + c*x)]*Log[x] + 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^
2*c^2 - 64*a^3*c^3)*x^4*Sqrt[a + x*(b + c*x)]*Log[2*a + b*x + 2*S
qrt[a]*Sqrt[a + x*(b + c*x)]])/(128*a^(11/2)*(-b^2 + 4*a*c)*x^3*S
qrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.017, size = 446, normalized size = 1.3

$$-\frac{cx^2+bx+a}{128\,x\,(4\,ac-b^2)}\Big(3616\,a^{7/2}x^5bc^3-3360\,a^{5/2}x^5b^3c^2+630\,a^{3/2}x^5b^5c-960\,a^{9/2}x^4c^3+5408\,a^{7/2}x^4b^2c^2-3780\,a^{5/2}x^4b^4c+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/128/x*(c*x^2+b*x+a)*(3616*a^(7/2)*x^5*b*c^3-3360*a^(5/2)*x^5*b
^3*c^2+630*a^(3/2)*x^5*b^5*c-960*a^(9/2)*x^4*c^3+5408*a^(7/2)*x^4
*b^2*c^2-3780*a^(5/2)*x^4*b^4*c+630*a^(3/2)*x^4*b^6+960*ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a^4*
c^3-3600*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x
+a)^(1/2)*x^4*a^3*b^2*c^2+2100*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a
)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a^2*b^4*c-315*ln((2*a+b*x+2*a
^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*x^4*a*b^6+1568
*a^(9/2)*x^3*b*c^2-1232*a^(7/2)*x^3*b^3*c+210*a^(5/2)*x^3*b^5-320
*a^(11/2)*x^2*c^2+416*a^(9/2)*x^2*b^2*c-84*a^(7/2)*x^2*b^4-192*a^
(11/2)*x*b*c+48*a^(9/2)*x*b^3+128*a^(13/2)*c-32*a^(11/2)*b^2)/(c*
x^4+b*x^3+a*x^2)^(3/2)/a^(13/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Fricas [A] time = 0.450788, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] [1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*
c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3
)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)
*x^5)*sqrt(a)*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2)
- (8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3) - 4*(16
*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*
b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480
*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3
 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3
- 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*
c^2)*x^7 + (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5),
1/128*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c
^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)
*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*
x^5)*sqrt(-a)*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b
*x^3 + a*x^2)*a)) - 2*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 168
0*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c
 + 2704*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b
^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^
5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x
^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*c)*x^6 + (
a^7*b^2 - 4*a^8*c)*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(x^2 \left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^3 + ax^2)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

3.65 $\int x^m \left(ax+bx^3+cx^5\right)\,dx$

Optimal. Leaf size=37

$$\frac{ax^{m+2}}{m+2}+\frac{bx^{m+4}}{m+4}+\frac{cx^{m+6}}{m+6}$$

```
[Out] (a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6
+ m)
```

Rubi [A] time = 0.0298752, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{ax^{m+2}}{m+2}+\frac{bx^{m+4}}{m+4}+\frac{cx^{m+6}}{m+6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6
+ m)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int xx^m\left(a+bx^2+cx^4\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(c*x**5+b*x**3+a*x),x)
```

```
[Out] Integral(x*x**m*(a + b*x**2 + c*x**4), x)
```

Mathematica [A] time = 0.0368534, size = 35, normalized size = 0.95

$$x^m\left(\frac{ax^2}{m+2}+\frac{bx^4}{m+4}+\frac{cx^6}{m+6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x^m*((a*x^2)/(2 + m) + (b*x^4)/(4 + m) + (c*x^6)/(6 + m))
```

Maple [B] time = 0.004, size = 77, normalized size = 2.1

$$\frac{x^{2+m}\left(cm^2x^4+6\,cmx^4+bm^2x^2+8\,cx^4+8\,bmx^2+am^2+12\,bx^2+10\,am+24\,a\right)}{(6+m)\,(4+m)\,(2+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(c*x^5+b*x^3+a*x),x)
```

```
[Out] x^(2+m)*(c*m^2*x^4+6*c*m*x^4+b*m^2*x^2+8*c*x^4+8*b*m*x^2+a*m^2+12
*b*x^2+10*a*m+24*a)/(6+m)/(4+m)/(2+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292797, size = 96, normalized size = 2.59

$$\frac{\left(\left(cm^2+6\,cm+8\,c\right)x^6+\left(bm^2+8\,bm+12\,b\right)x^4+\left(am^2+10\,am+24\,a\right)x^2\right)x^m}{m^3+12\,m^2+44\,m+48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)*x^m,x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 6*c*m + 8*c)*x^6 + (b*m^2 + 8*b*m + 12*b)*x^4 + (a*m^2
+ 10*a*m + 24*a)*x^2)*x^m/(m^3 + 12*m^2 + 44*m + 48)
```

Sympy [A] time = 3.35989, size = 280, normalized size = 7.57

$$\begin{cases} -\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x) \\ -\frac{a}{2x^2} + b\log(x) + \frac{cx^2}{2} \\ a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4} \\ \frac{am^2x^2x^m}{m^3+12m^2+44m+48} + \frac{10amx^2x^m}{m^3+12m^2+44m+48} + \frac{24ax^2x^m}{m^3+12m^2+44m+48} + \frac{bm^2x^4x^m}{m^3+12m^2+44m+48} + \frac{8bmx^4x^m}{m^3+12m^2+44m+48} + \frac{12bx^4x^m}{m^3+12m^2+44m+48} + \frac{cm^2x^6x^m}{m^3+12m^2+44m+4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(c*x**5+b*x**3+a*x),x)
```

```
[Out] Piecewise((-a/(4*x**4) - b/(2*x**2) + c*log(x), Eq(m, -6)), (-a/(
2*x**2) + b*log(x) + c*x**2/2, Eq(m, -4)), (a*log(x) + b*x**2/2 +
 c*x**4/4, Eq(m, -2)), (a*m**2*x**2*x**m/(m**3 + 12*m**2 + 44*m +
 48) + 10*a*m*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 24*a*x**2*
x**m/(m**3 + 12*m**2 + 44*m + 48) + b*m**2*x**4*x**m/(m**3 + 12*m
**2 + 44*m + 48) + 8*b*m*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) +
 12*b*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + c*m**2*x**6*x**m/(
m**3 + 12*m**2 + 44*m + 48) + 6*c*m*x**6*x**m/(m**3 + 12*m**2 + 4
4*m + 48) + 8*c*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48), True))
```

GIAC/XCAS [A] time = 0.268786, size = 169, normalized size = 4.57

$$\frac{cm^2x^6e^{(m\ln(x))} + 6\,cmx^6e^{(m\ln(x))} + bm^2x^4e^{(m\ln(x))} + 8\,cx^6e^{(m\ln(x))} + 8\,bmx^4e^{(m\ln(x))} + am^2x^2e^{(m\ln(x))} + 12\,bx^4e^{(m\ln(x))} + 10\,am}{m^3 + 12\,m^2 + 44\,m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)*x^m,x, algorithm="giac")
```

```
[Out] (c*m^2*x^6*e^(m*ln(x)) + 6*c*m*x^6*e^(m*ln(x)) + b*m^2*x^4*e^(m*l
n(x)) + 8*c*x^6*e^(m*ln(x)) + 8*b*m*x^4*e^(m*ln(x)) + a*m^2*x^2*e
^(m*ln(x)) + 12*b*x^4*e^(m*ln(x)) + 10*a*m*x^2*e^(m*ln(x)) + 24*a
*x^2*e^(m*ln(x)))/(m^3 + 12*m^2 + 44*m + 48)
```

3.66 $\int x^2 \left(ax + bx^3 + cx^5\right) \, dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Rubi [A] time = 0.0179731, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5),x]

[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a \int^{x^2} x \, dx}{2} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(c*x**5+b*x**3+a*x),x)

[Out] a*Integral(x, (x, x**2))/2 + b*x**6/6 + c*x**8/8
```

Mathematica [A] time = 0.00279025, size = 25, normalized size = 1.

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

Maxima [A] time = 0.77696, size = 26, normalized size = 1.04

$$\frac{1}{8}\,cx^8 + \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)*x^2,x, algorithm="maxima")
```

```
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

Fricas [A] time = 0.237954, size = 1, normalized size = 0.04

$$\frac{1}{8}x^8c + \frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)*x^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c + 1/6*x^6*b + 1/4*x^4*a
```

Sympy [A] time = 0.068629, size = 19, normalized size = 0.76

$$\frac{ax^4}{4} + \frac{bx^6}{6} + \frac{cx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**5+b*x**3+a*x),x)
```

```
[Out] a*x**4/4 + b*x**6/6 + c*x**8/8
```

GIAC/XCAS [A] time = 0.24866, size = 26, normalized size = 1.04

$$\frac{1}{8}\,cx^8 + \frac{1}{6}\,bx^6 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)*x^2,x, algorithm="giac")
```

```
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

3.67 $\int x \left(ax + bx^3 + cx^5\right) \, dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rubi [A] time = 0.0164836, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a*x + b*x^3 + c*x^5),x]

[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rubi in Sympy [A] time = 6.74848, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**5+b*x**3+a*x),x)

[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

Mathematica [A] time = 0.00269298, size = 25, normalized size = 1.

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Maxima [A] time = 0.770818, size = 26, normalized size = 1.04

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)*x,x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A] time = 0.234849, size = 1, normalized size = 0.04

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)*x,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a
```

Sympy [A] time = 0.067234, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**5+b*x**3+a*x),x)
```

```
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

GIAC/XCAS [A] time = 0.249766, size = 26, normalized size = 1.04

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)*x,x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

3.68 $\int \left(ax + bx^3 + cx^5\right)\, dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi [A] time = 0.0119757, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[a*x + b*x^3 + c*x^5,x]

[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\int x\,dx + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(c*x**5+b*x**3+a*x,x)

[Out] a*Integral(x, x) + b*x**4/4 + c*x**6/6
```

Mathematica [A] time = 0.0000601568, size = 25, normalized size = 1.

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a*x + b*x^3 + c*x^5,x]
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^5+b*x^3+a*x,x)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Maxima [A] time = 0.762292, size = 26, normalized size = 1.04

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5 + b*x^3 + a*x,x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Fricas [A] time = 0.235292, size = 1, normalized size = 0.04

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5 + b*x^3 + a*x,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a
```

Sympy [A] time = 0.069569, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x**5+b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

GIAC/XCAS [A] time = 0.252827, size = 26, normalized size = 1.04

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^5 + b*x^3 + a*x,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

3.69 $\int \frac{ax+bx^3+cx^5}{x}\,dx$

Optimal. Leaf size=20

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] time = 0.0140265, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^3}{3} + \frac{cx^5}{5} + \int a\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**5+b*x**3+a*x)/x,x)

[Out] b*x**3/3 + c*x**5/5 + Integral(a, x)
```

Mathematica [A] time = 0.00075196, size = 20, normalized size = 1.

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)/x,x]

[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)/x,x)

[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Maxima [A] time = 0.767012, size = 22, normalized size = 1.1

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x,x, algorithm="maxima")

[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Fricas [A] time = 0.251218, size = 22, normalized size = 1.1

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x,x, algorithm="fricas")

[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Sympy [A] time = 0.066881, size = 15, normalized size = 0.75

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)/x,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

GIAC/XCAS [A] time = 0.251414, size = 22, normalized size = 1.1

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

3.70 $\int \frac{ax+bx^3+cx^5}{x^2}\,dx$

Optimal. Leaf size=21

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rubi [A] time = 0.0161841, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)/x^2,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\log\left(x^2\right)}{2} + \frac{c\int^{x^2} x\,dx}{2} + \frac{\int^{x^2} b\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)/x**2,x)
```

```
[Out] a*log(x**2)/2 + c*Integral(x, (x, x**2))/2 + Integral(b, (x, x**2
))/2
```

Mathematica [A] time = 0.0026517, size = 21, normalized size = 1.

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)/x^2,x]

[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$\frac{bx^2}{2} + \frac{cx^4}{4} + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)/x^2,x)

[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Maxima [A] time = 0.765729, size = 23, normalized size = 1.1

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x^2,x, algorithm="maxima")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Fricas [A] time = 0.254858, size = 23, normalized size = 1.1

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x^2,x, algorithm="fricas")

[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Sympy [A] time = 0.169227, size = 17, normalized size = 0.81

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)/x**2,x)
```

```
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

GIAC/XCAS [A] time = 0.278529, size = 27, normalized size = 1.29

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + \frac{1}{2}a\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*ln(x^2)
```

3.71 $\int \frac{ax+bx^3+cx^5}{x^3} \, dx$

Optimal. Leaf size=18

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rubi [A] time = 0.0149666, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)/x^3,x]

[Out] -(a/x) + b*x + (c*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a}{x}+\frac{cx^3}{3}+\int b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)/x**3,x)

[Out] -a/x + c*x**3/3 + Integral(b, x)
```

Mathematica [A] time = 0.00314479, size = 18, normalized size = 1.

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)/x^3,x]

[Out] -(a/x) + b*x + (c*x^3)/3
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)/x^3,x)

[Out] -a/x+b*x+1/3*c*x^3
```

Maxima [A] time = 0.772295, size = 22, normalized size = 1.22

$$\frac{1}{3}cx^3+bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x^3,x, algorithm="maxima")

[Out] 1/3*c*x^3 + b*x - a/x
```

Fricas [A] time = 0.250282, size = 27, normalized size = 1.5

$$\frac{cx^4+3\,bx^2-3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x^3,x, algorithm="fricas")

[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

Sympy [A] time = 0.981781, size = 12, normalized size = 0.67

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)/x**3,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

GIAC/XCAS [A] time = 0.252864, size = 22, normalized size = 1.22

$$\frac{1}{3}cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

3.72 $\int x^m \left(ax + bx^3 + cx^5\right)^2 \, dx$

Optimal. Leaf size=76

$$\frac{a^2 x^{m+3}}{m+3} + \frac{x^{m+7}\left(2ac + b^2\right)}{m+7} + \frac{2abx^{m+5}}{m+5} + \frac{2bcx^{m+9}}{m+9} + \frac{c^2 x^{m+11}}{m+11}$$

```
[Out] (a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a
*c)*x^(7 + m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 +
 m))/(11 + m)
```

Rubi [A] time = 0.0901415, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{a^2 x^{m+3}}{m+3} + \frac{x^{m+7}\left(2ac + b^2\right)}{m+7} + \frac{2abx^{m+5}}{m+5} + \frac{2bcx^{m+9}}{m+9} + \frac{c^2 x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a
*c)*x^(7 + m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 +
 m))/(11 + m)
```

Rubi in Sympy [A] time = 26.186, size = 66, normalized size = 0.87

$$\frac{a^2 x^{m+3}}{m+3} + \frac{2abx^{m+5}}{m+5} + \frac{2bcx^{m+9}}{m+9} + \frac{c^2 x^{m+11}}{m+11} + \frac{x^{m+7}\left(2ac + b^2\right)}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**(m + 3)/(m + 3) + 2*a*b*x**(m + 5)/(m + 5) + 2*b*c*x**(m
+ 9)/(m + 9) + c**2*x**(m + 11)/(m + 11) + x**(m + 7)*(2*a*c + b*
*2)/(m + 7)
```

Mathematica [A] time = 0.0686476, size = 70, normalized size = 0.92

$$x^m \left(\frac{a^2 x^3}{m+3} + \frac{x^7\left(2ac + b^2\right)}{m+7} + \frac{2abx^5}{m+5} + \frac{2bcx^9}{m+9} + \frac{c^2 x^{11}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] x^m*((a^2*x^3)/(3 + m) + (2*a*b*x^5)/(5 + m) + ((b^2 + 2*a*c)*x^7
)/(7 + m) + (2*b*c*x^9)/(9 + m) + (c^2*x^11)/(11 + m))
```

Maple [B] time = 0.01, size = 300, normalized size = 4.

$$x^{3+m}\left(c^{2}m^{4}x^{8}+24\,c^{2}m^{3}x^{8}+2\,bcm^{4}x^{6}+206\,c^{2}m^{2}x^{8}+52\,bcm^{3}x^{6}+744\,c^{2}mx^{8}+2\,acm^{4}x^{4}+b^{2}m^{4}x^{4}+472\,bcm^{2}x^{6}+945\,c^{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^m*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] x^(3+m)*(c^2*m^4*x^8+24*c^2*m^3*x^8+2*b*c*m^4*x^6+206*c^2*m^2*x^8
+52*b*c*m^3*x^6+744*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+472*b*c*m
^2*x^6+945*c^2*x^8+56*a*c*m^3*x^4+28*b^2*m^3*x^4+1772*b*c*m*x^6+2
*a*b*m^4*x^2+548*a*c*m^2*x^4+274*b^2*m^2*x^4+2310*b*c*x^6+60*a*b*
m^3*x^2+2184*a*c*m*x^4+1092*b^2*m*x^4+a^2*m^4+640*a*b*m^2*x^2+297
0*a*c*x^4+1485*b^2*x^4+32*a^2*m^3+2820*a*b*m*x^2+374*a^2*m^2+4158
*a*b*x^2+1888*a^2*m+3465*a^2)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2*x^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280001, size = 325, normalized size = 4.28

$$\left(\left(c^{2}m^{4}+24\,c^{2}m^{3}+206\,c^{2}m^{2}+744\,c^{2}m+945\,c^{2}\right)x^{11}+2\left(bcm^{4}+26\,bcm^{3}+236\,bcm^{2}+886\,bcm+1155\,bc\right)x^{9}+\left(\left(b^{2}+2\,ac\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2*x^m,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 24*c^2*m^3 + 206*c^2*m^2 + 744*c^2*m + 945*c^2)*x^11
+ 2*(b*c*m^4 + 26*b*c*m^3 + 236*b*c*m^2 + 886*b*c*m + 1155*b*c)*x
^9 + ((b^2 + 2*a*c)*m^4 + 28*(b^2 + 2*a*c)*m^3 + 274*(b^2 + 2*a*c
)*m^2 + 1485*b^2 + 2970*a*c + 1092*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*
m^4 + 30*a*b*m^3 + 320*a*b*m^2 + 1410*a*b*m + 2079*a*b)*x^5 + (a^
2*m^4 + 32*a^2*m^3 + 374*a^2*m^2 + 1888*a^2*m + 3465*a^2)*x^3)*x^
m/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

Sympy [A] time = 12.4578, size = 1377, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Piecewise((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4
*x**4) - b*c/x**2 + c**2*log(x), Eq(m, -11)), (-a**2/(6*x**6) - a
*b/(2*x**4) - a*c/x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2
/2, Eq(m, -9)), (-a**2/(4*x**4) - a*b/x**2 + 2*a*c*log(x) + b**2*
log(x) + b*c*x**2 + c**2*x**4/4, Eq(m, -7)), (-a**2/(2*x**2) + 2*
a*b*log(x) + a*c*x**2 + b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6, E
q(m, -5)), (a**2*log(x) + a*b*x**2 + a*c*x**4/2 + b**2*x**4/4 + b
*c*x**6/3 + c**2*x**8/8, Eq(m, -3)), (a**2*m**4*x**3*x**m/(m**5 +
 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 32*a**2*m**3*
x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395
) + 374*a**2*m**2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**
2 + 9129*m + 10395) + 1888*a**2*m*x**3*x**m/(m**5 + 35*m**4 + 470
*m**3 + 3010*m**2 + 9129*m + 10395) + 3465*a**2*x**3*x**m/(m**5 +
 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a*b*m**4*x*
*5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
+ 60*a*b*m**3*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 +
9129*m + 10395) + 640*a*b*m**2*x**5*x**m/(m**5 + 35*m**4 + 470*m*
*3 + 3010*m**2 + 9129*m + 10395) + 2820*a*b*m*x**5*x**m/(m**5 + 3
5*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 4158*a*b*x**5*x
**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*
a*c*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*
m + 10395) + 56*a*c*m**3*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3
010*m**2 + 9129*m + 10395) + 548*a*c*m**2*x**7*x**m/(m**5 + 35*m*
*4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2184*a*c*m*x**7*x**
m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2970
*a*c*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m +
10395) + b**2*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m*
*2 + 9129*m + 10395) + 28*b**2*m**3*x**7*x**m/(m**5 + 35*m**4 + 4
70*m**3 + 3010*m**2 + 9129*m + 10395) + 274*b**2*m**2*x**7*x**m/(
m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1092*b*
*2*m*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m +
```

```
10395) + 1485*b**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m*
*2 + 9129*m + 10395) + 2*b*c*m**4*x**9*x**m/(m**5 + 35*m**4 + 470
*m**3 + 3010*m**2 + 9129*m + 10395) + 52*b*c*m**3*x**9*x**m/(m**5
 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 472*b*c*m**
2*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 103
95) + 1772*b*c*m*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
 + 9129*m + 10395) + 2310*b*c*x**9*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + c**2*m**4*x**11*x**m/(m**5 + 35
*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 24*c**2*m**3*x**
11*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
+ 206*c**2*m**2*x**11*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
 + 9129*m + 10395) + 744*c**2*m*x**11*x**m/(m**5 + 35*m**4 + 470*
m**3 + 3010*m**2 + 9129*m + 10395) + 945*c**2*x**11*x**m/(m**5 +
35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395), True))
```

GIAC/XCAS [A] time = 0.286241, size = 620, normalized size = 8.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2*x^m,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x^11*e^(m*ln(x)) + 24*c^2*m^3*x^11*e^(m*ln(x)) + 2*b*c*m
^4*x^9*e^(m*ln(x)) + 206*c^2*m^2*x^11*e^(m*ln(x)) + 52*b*c*m^3*x^
9*e^(m*ln(x)) + 744*c^2*m*x^11*e^(m*ln(x)) + b^2*m^4*x^7*e^(m*ln(
x)) + 2*a*c*m^4*x^7*e^(m*ln(x)) + 472*b*c*m^2*x^9*e^(m*ln(x)) + 9
45*c^2*x^11*e^(m*ln(x)) + 28*b^2*m^3*x^7*e^(m*ln(x)) + 56*a*c*m^3
*x^7*e^(m*ln(x)) + 1772*b*c*m*x^9*e^(m*ln(x)) + 2*a*b*m^4*x^5*e^(
m*ln(x)) + 274*b^2*m^2*x^7*e^(m*ln(x)) + 548*a*c*m^2*x^7*e^(m*ln(
x)) + 2310*b*c*x^9*e^(m*ln(x)) + 60*a*b*m^3*x^5*e^(m*ln(x)) + 109
2*b^2*m*x^7*e^(m*ln(x)) + 2184*a*c*m*x^7*e^(m*ln(x)) + a^2*m^4*x^
3*e^(m*ln(x)) + 640*a*b*m^2*x^5*e^(m*ln(x)) + 1485*b^2*x^7*e^(m*l
n(x)) + 2970*a*c*x^7*e^(m*ln(x)) + 32*a^2*m^3*x^3*e^(m*ln(x)) + 2
820*a*b*m*x^5*e^(m*ln(x)) + 374*a^2*m^2*x^3*e^(m*ln(x)) + 4158*a*
b*x^5*e^(m*ln(x)) + 1888*a^2*m*x^3*e^(m*ln(x)) + 3465*a^2*x^3*e^(
m*ln(x)))/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

3.73 $\int x^2 \left(ax + bx^3 + cx^5\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9\left(2ac + b^2\right) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11
)/11 + (c^2*x^13)/13
```

Rubi [A] time = 0.0760657, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9\left(2ac + b^2\right) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11
)/11 + (c^2*x^13)/13
```

Rubi in Sympy [A] time = 14.0211, size = 51, normalized size = 0.94

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{2bcx^{11}}{11} + \frac{c^2x^{13}}{13} + x^9\left(\frac{2ac}{9} + \frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**
9*(2*a*c/9 + b**2/9)
```

Mathematica [A] time = 0.0120419, size = 54, normalized size = 1.

$$\frac{a^2x^5}{5} + \frac{1}{9}x^9\left(2ac + b^2\right) + \frac{2}{7}abx^7 + \frac{2}{11}bcx^{11} + \frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11
)/11 + (c^2*x^13)/13
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^5}{5}+\frac{2\,abx^7}{7}+\frac{\left(2\,ac+b^2\right)x^9}{9}+\frac{2\,bcx^{11}}{11}+\frac{c^2x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^
2*x^13
```

Maxima [A] time = 0.771598, size = 59, normalized size = 1.09

$$\frac{1}{13}\,c^2x^{13}+\frac{2}{11}\,bcx^{11}+\frac{1}{9}\left(b^2+2\,ac\right)x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x
^7 + 1/5*a^2*x^5
```

Fricas [A] time = 0.246728, size = 1, normalized size = 0.02

$$\frac{1}{13}x^{13}c^2+\frac{2}{11}x^{11}cb+\frac{1}{9}x^9b^2+\frac{2}{9}x^9ca+\frac{2}{7}x^7ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^2 + 2/11*x^11*c*b + 1/9*x^9*b^2 + 2/9*x^9*c*a + 2/7*x
^7*b*a + 1/5*x^5*a^2
```

Sympy [A] time = 0.112262, size = 51, normalized size = 0.94

$$\frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{2bcx^{11}}{11} + \frac{c^2x^{13}}{13} + x^9\left(\frac{2ac}{9} + \frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**
9*(2*a*c/9 + b**2/9)
```

GIAC/XCAS [A] time = 0.248123, size = 62, normalized size = 1.15

$$\frac{1}{13}\,c^2x^{13} + \frac{2}{11}\,bcx^{11} + \frac{1}{9}\,b^2x^9 + \frac{2}{9}\,acx^9 + \frac{2}{7}\,abx^7 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 2/7*a
*b*x^7 + 1/5*a^2*x^5
```

3.74 $\int x\left(ax+bx^3+cx^5\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^10)/5
+ (c^2*x^12)/12
```

Rubi [A] time = 0.130488, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{a^2x^4}{4}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^10)/5
+ (c^2*x^12)/12
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\int^{x^2}x\,dx}{2}+\frac{abx^6}{3}+\frac{bcx^{10}}{5}+\frac{c^2x^{12}}{12}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*Integral(x, (x, x**2))/2 + a*b*x**6/3 + b*c*x**10/5 + c**2*x
**12/12 + x**8*(a*c/4 + b**2/8)
```

Mathematica [A] time = 0.0125977, size = 48, normalized size = 0.89

$$\frac{1}{120}x^4\left(30a^2+15x^4\left(2ac+b^2\right)+40abx^2+24bcx^6+10c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^4*(30*a^2 + 40*a*b*x^2 + 15*(b^2 + 2*a*c)*x^4 + 24*b*c*x^6 + 1
0*c^2*x^8))/120
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{\left(2\,ac+b^2\right)x^8}{8}+\frac{bcx^{10}}{5}+\frac{c^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2
*x^12
```

Maxima [A] time = 0.773449, size = 59, normalized size = 1.09

$$\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\left(b^2+2\,ac\right)x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2*x,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^
6 + 1/4*a^2*x^4
```

Fricas [A] time = 0.247719, size = 1, normalized size = 0.02

$$\frac{1}{12}x^{12}c^2+\frac{1}{5}x^{10}cb+\frac{1}{8}x^8b^2+\frac{1}{4}x^8ca+\frac{1}{3}x^6ba+\frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2*x,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*c^2 + 1/5*x^10*c*b + 1/8*x^8*b^2 + 1/4*x^8*c*a + 1/3*x^
6*b*a + 1/4*x^4*a^2
```

Sympy [A] time = 0.1115, size = 46, normalized size = 0.85

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{bcx^{10}}{5} + \frac{c^2x^{12}}{12} + x^8\left(\frac{ac}{4} + \frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b*c*x**10/5 + c**2*x**12/12 + x**8*(a*
c/4 + b**2/8)
```

GIAC/XCAS [A] time = 0.249277, size = 62, normalized size = 1.15

$$\frac{1}{12}\,c^2x^{12} + \frac{1}{5}\,bcx^{10} + \frac{1}{8}\,b^2x^8 + \frac{1}{4}\,acx^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2*x,x, algorithm="giac")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 1/3*a*
b*x^6 + 1/4*a^2*x^4
```

3.75 $\int (ax + bx^3 + cx^5)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7\left(2ac + b^2\right) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)
/9 + (c^2*x^11)/11
```

Rubi [A] time = 0.0615135, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7\left(2ac + b^2\right) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)
/9 + (c^2*x^11)/11
```

Rubi in Sympy [A] time = 13.772, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*
(2*a*c/7 + b**2/7)
```

Mathematica [A] time = 0.00967181, size = 54, normalized size = 1.

$$\frac{a^2x^3}{3} + \frac{1}{7}x^7\left(2ac + b^2\right) + \frac{2}{5}abx^5 + \frac{2}{9}bcx^9 + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)
/9 + (c^2*x^11)/11
```

Maple [A] time = 0.002, size = 45, normalized size = 0.8

$$\frac{a^2x^3}{3} + \frac{2\,abx^5}{5} + \frac{\left(2\,ac + b^2\right)x^7}{7} + \frac{2\,bcx^9}{9} + \frac{c^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*
x^11
```

Maxima [A] time = 0.769016, size = 65, normalized size = 1.2

$$\frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\,b^2x^7 + \frac{1}{3}\,a^2x^3 + \frac{2}{35}\left(5\,cx^7 + 7\,bx^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 1/3*a^2*x^3 + 2/35*(5
*c*x^7 + 7*b*x^5)*a
```

Fricas [A] time = 0.247831, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}c^2 + \frac{2}{9}x^9cb + \frac{1}{7}x^7b^2 + \frac{2}{7}x^7ca + \frac{2}{5}x^5ba + \frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 2/9*x^9*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 2/5*x^5
*b*a + 1/3*x^3*a^2
```

Sympy [A] time = 0.103182, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*
(2*a*c/7 + b**2/7)
```

GIAC/XCAS [A] time = 0.276552, size = 62, normalized size = 1.15

$$\frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\,b^2x^7 + \frac{2}{7}\,acx^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b
*x^5 + 1/3*a^2*x^3
```

3.76 $\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 +
 (c^2*x^10)/10
```

Rubi [A] time = 0.0810699, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 +
 (c^2*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ab\int^{x^2} x\,dx+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)+\frac{\int^{x^2} a^2\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)**2/x,x)
```

```
[Out] a*b*Integral(x, (x, x**2)) + b*c*x**8/4 + c**2*x**10/10 + x**6*(a
*c/3 + b**2/6) + Integral(a**2, (x, x**2))/2
```

Mathematica [A] time = 0.0115952, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2\left(30a^2+10x^4\left(2ac+b^2\right)+30abx^2+15bcx^6+6c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6
*c^2*x^8))/60
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{\left(2\,ac+b^2\right)x^6}{6}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)^2/x,x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*
x^10
```

Maxima [A] time = 0.770447, size = 59, normalized size = 1.09

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,\left(b^2+2\,ac\right)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4
 + 1/2*a^2*x^2
```

Fricas [A] time = 0.253734, size = 59, normalized size = 1.09

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,\left(b^2+2\,ac\right)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4
 + 1/2*a^2*x^2
```

Sympy [A] time = 0.108159, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c
/3 + b**2/6)
```

GIAC/XCAS [A] time = 0.261069, size = 62, normalized size = 1.15

$$\frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b
*x^4 + 1/2*a^2*x^2
```

3.77 $\int \frac{(ax+bx^3+cx^5)^2}{x^2}\,dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Rubi [A] time = 0.0478899, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\left(\frac{2ac}{5}+\frac{b^2}{5}\right)+\int a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)**2/x**2,x)
```

```
[Out] 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/
5) + Integral(a**2, x)
```

Mathematica [A] time = 0.00798326, size = 49, normalized size = 1.

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Maple [A] time = 0.002, size = 42, normalized size = 0.9

$$a^{2}x+\frac{2\,ab x^{3}}{3}+\frac{\left(2\,ac+b^{2}\right)x^{5}}{5}+\frac{2\,bc x^{7}}{7}+\frac{c^{2}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)^2/x^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A] time = 0.779547, size = 55, normalized size = 1.12

$$\frac{1}{9}\,c^{2}x^{9}+\frac{2}{7}\,bc x^{7}+\frac{1}{5}\,\left(b^{2}+2\,ac\right)x^{5}+\frac{2}{3}\,ab x^{3}+a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 +
 a^2*x
```

Fricas [A] time = 0.271625, size = 55, normalized size = 1.12

$$\frac{1}{9}\,c^{2}x^{9}+\frac{2}{7}\,bc x^{7}+\frac{1}{5}\,\left(b^{2}+2\,ac\right)x^{5}+\frac{2}{3}\,ab x^{3}+a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 +
 a^2*x
```

Sympy [A] time = 0.111893, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)**2/x**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/
5 + b**2/5)
```

GIAC/XCAS [A] time = 0.248296, size = 58, normalized size = 1.18

$$\frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x
^3 + a^2*x
```

3.78 $\int \frac{x^8}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=100

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)
*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.240748, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)
*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\left(-3ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}}+\frac{\int^{x^2} x\,dx}{2c}-\frac{\int^{x^2} b\,dx}{2c^2}+\frac{\left(-ac+b^2\right)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**5+b*x**3+a*x),x)
```

```
[Out] b*(-3*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*
*3*sqrt(-4*a*c + b**2)) + Integral(x, (x, x**2))/(2*c) - Integral
(b, (x, x**2))/(2*c**2) + (-a*c + b**2)*log(a + b*x**2 + c*x**4)/
(4*c**3)
```

Mathematica [A] time = 0.170897, size = 93, normalized size = 0.93

$$\frac{-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(b^2-ac)\log\left(a+bx^2+cx^4\right)+cx^2\left(cx^2-2b\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^
2 + c*x^4])/(4*c^3)
```

Maple [A] time = 0.005, size = 142, normalized size = 1.4

$$\frac{x^4}{4\,c}-\frac{bx^2}{2\,c^2}-\frac{\ln\left(cx^4+bx^2+a\right)a}{4\,c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^2}{4\,c^3} \\ +\frac{3\,ab}{2\,c^2}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{b^3}{2\,c^3}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*x^4/c-1/2*b*x^2/c^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a+1/4/c^3*ln(c*
x^4+b*x^2+a)*b^2+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*a*b-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^4-2\,bx^2}{4\,c^2}-\frac{-\int\frac{(b^2-ac)x^3+abx}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 - integrate(-((b^2 - a*c)*x^3 + a*b*x)/
(c*x^4 + b*x^2 + a), x)/c^2
```

Fricas [A] time = 0.29014, size = 1, normalized size = 0.01

$$\left[-\frac{(b^3-3\,abc)\log\left(-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)x^2-(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(c^2x^4-2\,bcx^2+(b^2-ac)\log\left(cx^4+bx^2+a\right)\right)\sqrt{b^2-4\,ac}}{4\sqrt{b^2-4\,ac}c^3}, -\frac{2\,(b^3-3\,abc)\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(c^2x^4-2\,bcx^2+(b^2-ac)\log\left(cx^4+bx^2+a\right)\right)\sqrt{-b^2+4\,ac}}{4\sqrt{-b^2+4\,ac}c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3 - 3*a*b*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*
x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c
*x^4 + b*x^2 + a)) - (c^2*x^4 - 2*b*c*x^2 + (b^2 - a*c)*log(c*x^4
 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^3), -1/4*(
2*(b^3 - 3*a*b*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) - (c^2*x^4 - 2*b*c*x^2 + (b^2 - a*c)*log(c*x^4 + b*x^2 +
 a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)]
```

Sympy [A] time = 6.99187, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\,(3ac-b^2)}{4c^3\,(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right)+\left(\frac{b\sqrt{-4ac+b^2}\,(3ac-b^2)}{4c^3\,(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right)+\frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3
*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c -
a*b**2 + 8*a*c**3*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*
(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-
4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**
2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*c -
 b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(x**2
+ (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b
**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c*
*2*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2))
- (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + x**4/(4*c)
```

GIAC/XCAS [A] time = 0.254184, size = 124, normalized size = 1.24

$$\frac{cx^4 - 2\,bx^2}{4\,c^2} + \frac{\left(b^2 - ac\right)\ln\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*ln(c*x^4 + b*x^2 + a)
/c^3 - 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.79 $\int \frac{x^7}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c
+ (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi [A] time = 1.13711, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c
+ (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi in Sympy [A] time = 77.5825, size = 212, normalized size = 1.04

$$-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\sqrt{2}\left(b\left(-3ac+b^2\right)+\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(b\left(-3ac+b^2\right)-\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x/c**2 + x**3/(3*c) + sqrt(2)*(b*(-3*a*c + b**2) + sqrt(-4*a*c
 + b**2)*(-a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a
*c + b**2)))/(2*c**(5/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*
c + b**2)) - sqrt(2)*(b*(-3*a*c + b**2) - sqrt(-4*a*c + b**2)*(-a
*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))
/(2*c**(5/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.284121, size = 250, normalized size = 1.23

$$\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*
c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*
c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Maple [B] time = 0.03, size = 467, normalized size = 2.3

$$\frac{x^3}{3c} - \frac{bx}{c^2} - \frac{\sqrt{2}a}{2c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}b^2}{2c^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{3\sqrt{2}ab}{2c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}b^3}{2c^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}a}{2c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b^2}{2c^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{3\sqrt{2}ab}{2c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}b^3}{2c^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/3*x^3/c-b*x/c^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a+1/2/c^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
```

```
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^3+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c^
2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^3 - 3\,bx}{3\,c^2} - \frac{-\int \frac{(b^2-ac)\,x^2+ab}{cx^4+bx^2+a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x
^4 + b*x^2 + a), x)/c^2
```

Fricas [A] time = 0.29096, size = 2111, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*
c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7
- 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c^5 - 6*a*b^2*c
^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*
c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 -
```

```
4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(
b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c^5 - 6*a*b
^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(
b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^
5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sqrt(1/
2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6
*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)
))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqr
t(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5
 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 -
 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c
^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

Sympy [A] time = 8.06345, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2} + \mathrm{RootSum}\left(t^4\left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7\right) + a^5, \left(t \mapsto t\log\left(x + \frac{-64t^3a^2c^7}{}\right.\right.\right. + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b
**4*c**5) + _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b*
*5*c + 4*b**7) + a**5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7
 + 48*_t**3*a*b**2*c**6 - 8*_t**3*b**4*c**5 + 14*_t*a**3*b*c**3 -
 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c - 2*_t*b**7)/(a**4*c**2 -
3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

GIAC/XCAS [A] time = 0.80284, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] Done
```

3.80 $\int \frac{x^6}{ax+bx^3+cx^5} \, dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

Rubi [A] time = 0.16698, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

Rubi in Sympy [A] time = 32.5953, size = 73, normalized size = 0.9

$$-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*log(a + b*x**2 + c*x**4)/(4*c**2) + x**2/(2*c) - (-2*a*c + b**
2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**2*sqrt(-4*a*c
+ b**2))
```

Mathematica [A] time = 0.0753861, size = 78, normalized size = 0.96

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^2+cx^4\right)+2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.004, size = 111, normalized size = 1.4

$$\frac{x^2}{2c}-\frac{b\ln\left(cx^4+bx^2+a\right)}{4c^2}-\frac{a}{c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2}{2c^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x^2}{2c}-\frac{\int\frac{bx^3+ax}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [A] time = 0.267882, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(2\,cx^2-b\log\left(cx^4+bx^2+a\right)\right)\sqrt{b^2-4\,ac}}{4\,\sqrt{b^2-4\,ac}c^2},\ \frac{2\left(b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2
 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^
4 + b*x^2 + a)) - (2*c*x^2 - b*log(c*x^4 + b*x^2 + a))*sqrt(b^2 -
 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2), 1/4*(2*(b^2 - 2*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*x^2 - b*log(
c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 5.63033, size = 316, normalized size = 3.9

$$\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)}{2ac-b^2}\right)$$
$$+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
- b**2)))*log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c
+ b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4
```

```
*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2
))))/(2*a*c - b**2)) + (-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b - 8*a*c**2*(-b/
(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b*
*2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

GIAC/XCAS [A] time = 0.257856, size = 101, normalized size = 1.25

$$\frac{x^2}{2\,c} - \frac{b\ln\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*ln(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.81 $\int \frac{x^5}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.528439, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 55.5662, size = 189, normalized size = 1.06

$$\frac{x}{c}-\frac{\sqrt{2}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**5+b*x**3+a*x),x)
```

```
[Out] x/c - sqrt(2)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2
)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b + s
qrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(-2*a*c + b**2
 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4
*a*c + b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*
a*c + b**2))
```

Mathematica [A] time = 0.189434, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.004, size = 343, normalized size = 1.9

$$\frac{x}{c}-\frac{b\sqrt{2}}{2\,c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$+\sqrt{2}a\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$-\frac{\sqrt{2}b^{2}}{2\,c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$+\frac{b\sqrt{2}}{2\,c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$+\sqrt{2}aArtanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$-\frac{\sqrt{2}b^{2}}{2\,c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^5+b*x^3+a*x),x)
```

```
[Out] x/c-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c} - \frac{\int \frac{bx^2+a}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [A] time = 0.276631, size = 1430, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*
a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^
2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b
^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3
 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))
)/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2
*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c -
(b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/
2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*
c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] time = 5.89626, size = 129, normalized size = 0.72

$$\mathrm{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b^3c^3-4ta}{a^2c-ab}\right.\right.\right.$$
$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) +
_t**2*(48*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t,
_t*log(x + (32*_t**3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**
2 + 8*_t*a*b**2*c - 2*_t*b**4)/(a**2*c - a*b**2)))) + x/c
```

GIAC/XCAS [A] time = 0.774935, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] Done
```

3.82 $\int \frac{x^4}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*
c]) + Log[a + b*x^2 + c*x^4]/(4*c)
```

Rubi [A] time = 0.123116, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*
c]) + Log[a + b*x^2 + c*x^4]/(4*c)
```

Rubi in Sympy [A] time = 21.638, size = 54, normalized size = 0.86

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**5+b*x**3+a*x),x)
```

```
[Out] b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*sqrt(-4*a*c + b*
*2)) + log(a + b*x**2 + c*x**4)/(4*c)
```

Mathematica [A] time = 0.0378086, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^2+cx^4\right)-\frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c] + Log[a + b*x^2 + c*x^4])/(4*c)
```

Maple [A] time = 0.003, size = 60, normalized size = 1.

$$\frac{\ln\left(cx^4+bx^2+a\right)}{4\,c}-\frac{b}{2\,c}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278756, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\sqrt{b^2-4\,ac}\log\left(cx^4+bx^2+a\right)}{4\,\sqrt{b^2-4\,ac}c},\right.$$
$$\left.-\frac{2\,b\arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}\log\left(cx^4+bx^2+a\right)}{4\,\sqrt{-b^2+4\,ac}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4
 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) + sqrt(b^2 - 4*a*c)*log(c*x^4 + b*x^2 + a))/(sqrt(b^2 - 4*a*c)
*c), -1/4*(2*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)) - sqrt(-b^2 + 4*a*c)*log(c*x^4 + b*x^2 + a))/(sqrt(-b^2 + 4
*a*c)*c)]
```

Sympy [A] time = 2.70836, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2
+ (-8*a*c*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))
 + 2*a + 2*b**2*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/
(4*c)))/b) + (b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c
))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))
 + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b
**2)) + 1/(4*c)))/b)
```

GIAC/XCAS [A] time = 0.258247, size = 80, normalized size = 1.27

$$-\frac{b \arctan\left(\frac{2 \, cx^{2} + b}{\sqrt{-b^{2} + 4 \, ac}}\right)}{2 \, \sqrt{-b^{2} + 4 \, ac}c} + \frac{\ln\left(cx^{4} + bx^{2} + a\right)}{4 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*c) + 1/4*ln(c*x^4 + b*x^2 + a)/c
```

3.83 $\int \frac{x^3}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sq
rt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.215294, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sq
rt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 27.3728, size = 141, normalized size = 0.94

$$-\frac{\sqrt{2}\sqrt{b-\sqrt{-4ac+b^2}}\,\text{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\,\text{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -sqrt(2)*sqrt(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqr
t(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2)) + sqr
```

```
t(2)*sqrt(b + sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b
+ sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.165272, size = 165, normalized size = 1.1

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2
 - 4*a*c])
```

Maple [A] time = 0.021, size = 208, normalized size = 1.4

$$\frac{b\sqrt{2}}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{b\sqrt{2}}{2}\mathit{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}}{2}\mathit{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*b-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{cx^5 + bx^3 + ax}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [A] time = 0.269612, size = 755, normalized size = 5.03

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b + \frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c - 4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c - 4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2 - 4\,ac^3}} + x\right)$$
$$- \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b + \frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c - 4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c - 4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2 - 4\,ac^3}} + x\right)$$
$$- \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b - \frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c - 4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c - 4\,ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2 - 4\,ac^3}} + x\right)$$
$$+ \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b - \frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c - 4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c - 4\,ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2 - 4\,ac^3}} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3
))/(b^2*c - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b +
(b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqr
t(b^2*c^2 - 4*a*c^3) + x) - 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a
*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(-sqrt(1/2)*
(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a
*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2*sqrt
(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*
c - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c -
 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^
2 - 4*a*c^3) + x) + 1/2*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(-sqrt(1/2)*(b^2*c -
 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(
b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A] time = 2.56318, size = 75, normalized size = 0.5

$$\operatorname{RootSum}\left(t^{4}\left(256a^{2}c^{3}-128ab^{2}c^{2}+16b^{4}c\right)+t^{2}\left(-16abc+4b^{3}\right)+a,\left(t\mapsto t\log\left(64t^{3}ac^{2}-16t^{3}b^{2}c-2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t*
*2*(-16*a*b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 -
16*_t**3*b**2*c - 2*_t*b + x)))
```

GIAC/XCAS [A] time = 0.691112, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] Done
```

3.84 $\int \frac{x^2}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.0720432, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 12.4804, size = 34, normalized size = 0.94

$$-\frac{\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
```

Mathematica [A] time = 0.015515, size = 39, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.001, size = 36, normalized size = 1.

$$1\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262723, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2-(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}},\ \frac{\arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
/sqrt(b^2 - 4*a*c), arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c))/sqrt(-b^2 + 4*a*c)]
```

Sympy [A] time = 1.41003, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c
- b**2))*log(x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1
/(4*a*c - b**2)) + b)/(2*c))/2
```

GIAC/XCAS [A] time = 0.250465, size = 47, normalized size = 1.31

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.85 $\int \frac{x}{ax+bx^3+cx^5} \, dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.183197, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 19.8375, size = 138, normalized size = 0.92

$$-\frac{\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -sqrt(2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b*
*2)))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(
```

```
2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/
(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.133703, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[
b^2 - 4*a*c]
```

Maple [A] time = 0.018, size = 116, normalized size = 0.8

$$-c\sqrt{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-c\sqrt{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{cx^5 + bx^3 + ax}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [A] time = 0.283839, size = 828, normalized size = 5.52

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*
c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2
*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*
a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/
sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c
*x + sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 -
4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/s
qrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(
```

```
-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A] time = 3.0404, size = 87, normalized size = 0.58

$$\text{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t*
*2*(-16*a*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2
*b*c - 8*_t**3*a*b**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

GIAC/XCAS [A] time = 0.352038, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] Done
```

3.86 $\int \frac{1}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rubi [A] time = 0.142217, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rubi in Sympy [A] time = 28.9803, size = 63, normalized size = 0.91

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a\sqrt{-4ac+b^2}}+\frac{\log\left(x^2\right)}{2a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**5+b*x**3+a*x),x)
```

```
[Out] b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a*sqrt(-4*a*c + b*
*2)) + log(x**2)/(2*a) - log(a + b*x**2 + c*x**4)/(4*a)
```

Mathematica [A] time = 0.117639, size = 113, normalized size = 1.64

$$\frac{-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.006, size = 66, normalized size = 1.

$$-\frac{\ln\left(cx^4+bx^2+a\right)}{4\,a}-\frac{b}{2\,a}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -1/4*ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))+ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int\frac{cx^3+bx}{cx^4+bx^2+a}\,dx}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^3 + b*x)/(c*x^4 + b*x^2 + a), x)/a + log(x)/a
```

Fricas [A] time = 0.278983, size = 1, normalized size = 0.01

$$\left[\frac{b \log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\sqrt{b^2-4\,ac}\left(\log\left(cx^4+bx^2+a\right)-4\,\log\left(x\right)\right)}{4\,\sqrt{b^2-4\,ac}a},\; -\frac{2\,b\arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\sqrt{-b^2+4\,ac}\left(\log\left(cx^4+bx^2+a\right)-4\,\log\left(x\right)\right)}{4\,\sqrt{-b^2+4\,ac}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4
 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) - sqrt(b^2 - 4*a*c)*(log(c*x^4 + b*x^2 + a) - 4*log(x)))/(sqrt
(b^2 - 4*a*c)*a), -1/4*(2*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(log(c*x^4 + b*x^2 + a) -
 4*log(x)))/(sqrt(-b^2 + 4*a*c)*a)]
```

Sympy [A] time = 9.40503, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)$$
$$+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2
+ (-8*a**2*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*
a)) + 2*a*b**2*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(
4*a)) - 2*a*c + b**2)/(b*c)) + (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c
 - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2*c*(b*sqrt(-4*a*c + b**2)
/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqrt(-4*a*c + b**2
)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + log(x)
```

```
/a
```

GIAC/XCAS [A] time = 0.251115, size = 92, normalized size = 1.33

$$-\frac{b \arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a} - \frac{\ln\left(cx^4+bx^2+a\right)}{4\,a} + \frac{\ln\left(x^2\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a) - 1/4*ln(c*x^4 + b*x^2 + a)/a + 1/2*ln(x^2)/a
```

3.87 $\int \frac{1}{x(ax+bx^3+cx^5)}\,dx$

Optimal. Leaf size=174

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.427365, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 46.8162, size = 177, normalized size = 1.02

$$\frac{\sqrt{2}\sqrt{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] sqrt(2)*sqrt(c)*(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/
sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*sqrt(b + sqrt(-4*a*c + b**2))
*sqrt(-4*a*c + b**2)) - sqrt(2)*sqrt(c)*(b + sqrt(-4*a*c + b**2))
*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*a*sqrt(
b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - 1/(a*x)
```

Mathematica [A] time = 0.735514, size = 191, normalized size = 1.1

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

Maple [A] time = 0.023, size = 232, normalized size = 1.3

$$-\frac{c\sqrt{2}}{2\,a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}b}{2\,a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}b}{2\,a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}}{2\,a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^5+b*x^3+a*x),x)
```

```
[Out] -1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2*c/a/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b+1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)-1/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}\,dx}{a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [A] time = 0.307398, size = 1507, normalized size = 8.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c
+ 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c +
(a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4
*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*
b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x -
 sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*
c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqr
t(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
```

```
 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b
*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2
- 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))
/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^
3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2
)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5
*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt
(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

Sympy [A] time = 6.5407, size = 148, normalized size = 0.85

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2 - 128a^4b^2c + 16a^3b^4\right) + t^2\left(48a^2bc^2 - 28ab^3c + 4b^5\right) + c^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^5c^2 + 48t^3a^4b^2c - \right.\right.\right.$$

$$- \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) +
_t**2*(48*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t,
_t*log(x + (-64*_t**3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*
a**3*b**4 - 10*_t*a**2*b*c**2 + 10*_t*a*b**3*c - 2*_t*b**5)/(a*c*
*3 - b**2*c**2)))) - 1/(a*x)
```

GIAC/XCAS [A] time = 0.77669, size = 1, normalized size = 0.01

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)*x),x, algorithm="giac")
```

```
[Out] Done
```

3.88 $\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx$

Optimal. Leaf size=89

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*
a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*
x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.263068, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*
a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*
x^2 + c*x^4])/(4*a^2)
```

Rubi in Sympy [A] time = 45.2274, size = 87, normalized size = 0.98

$$-\frac{1}{2ax^2}-\frac{b\log\left(x^2\right)}{2a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x**2)/(2*a**2) + b*log(a + b*x**2 + c*x**4)
/(4*a**2) - (-2*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b*
*2))/(2*a**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.235116, size = 135, normalized size = 1.52

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{x^2}-4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*L
og[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 +
 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] time = 0.01, size = 119, normalized size = 1.3

$$\frac{b\ln\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{c}{a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{2\,a^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{1}{2\,ax^2}-\frac{b\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^5+b*x^3+a*x),x)
```

```
[Out] 1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*b^2-1/2/a/x^2-b*ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\log(x)}{a^2}+\frac{\int\frac{bcx^3+\left(b^2-ac\right)x}{cx^4+bx^2+a}\,dx}{a^2}-\frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)*x^2),x, algorithm="maxima")
```

```
[Out] -b*log(x)/a^2 + integrate((b*c*x^3 + (b^2 - a*c)*x)/(c*x^4 + b*x^
2 + a), x)/a^2 - 1/2/(a*x^2)
```

Fricas [A] time = 0.30246, size = 1, normalized size = 0.01

$$\left[-\frac{(b^2-2ac)x^2\log\left(\frac{b^3-4abc+2(b^2c-4ac^2)x^2+(2c^2x^4+2bcx^2+b^2-2ac)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-(bx^2\log(cx^4+bx^2+a)-4bx^2\log(x)-2a)}{4\sqrt{b^2-4ac}a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*x^2*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)
*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(
c*x^4 + b*x^2 + a)) - (b*x^2*log(c*x^4 + b*x^2 + a) - 4*b*x^2*log
(x) - 2*a)*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^2*x^2), 1/4*(2
*(b^2 - 2*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)) + (b*x^2*log(c*x^4 + b*x^2 + a) - 4*b*x^2*log(x) - 2*a)
*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*x^2)]
```

Sympy [A] time = 22.5222, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2} - \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2\,(4ac-b^2)}\right)\log\left(x^2+\frac{-8a^3c\left(\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+3abc-b^3}{2ac^2-b^2c}\right)$$
$$+\left(\frac{b}{4a^2} + \frac{\sqrt{-4ac+b^2}\,(2ac-b^2)}{4a^2\,(4ac-b^2)}\right)\log\left(x^2+\frac{-8a^3c\left(\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+3abc-b^3}{2ac^2-b^2c}\right)$$
$$-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c -
 b**2)))*log(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*
```

```
(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2)
 - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) +
3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (b/(4*a**2) + sqrt(-4*a*c
+ b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*log(x**2 + (-8*a*
*3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*
a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*
a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2
- b**2*c)) - 1/(2*a*x**2) - b*log(x)/a**2
```

GIAC/XCAS [A] time = 0.25636, size = 127, normalized size = 1.43

$$\frac{b\ln\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{b\ln\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)*x^2),x, algorithm="giac")
```

```
[Out] 1/4*b*ln(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*ln(x^2)/a^2 + 1/2*(b^2 -
2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a^2) + 1/2*(b*x^2 - a)/(a^2*x^2)
```

3.89 $\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=166

$$-\frac{(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}}+\frac{x^2(b^2-3ac)}{c^2(b^2-4ac)}-\frac{bx^4}{2c(b^2-4ac)}+\frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\log(a+bx^2+cx^4)}{2c^3}$$

```
[Out] ((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a
*c)) + (x^6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
- ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x^2 + c*x^4])/
(2*c^3)
```

Rubi [A] time = 0.422914, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$-\frac{(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3(b^2-4ac)^{3/2}}+\frac{x^2(b^2-3ac)}{c^2(b^2-4ac)}-\frac{bx^4}{2c(b^2-4ac)}+\frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\log(a+bx^2+cx^4)}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a
*c)) + (x^6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
- ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x^2 + c*x^4])/
(2*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\int^{x^2}x\,dx}{c(-4ac+b^2)}-\frac{b\log(a+bx^2+cx^4)}{2c^3}+\frac{x^6(2a+bx^2)}{2(-4ac+b^2)(a+bx^2+cx^4)}+\frac{x^2(-3ac+b^2)}{c^2(-4ac+b^2)}-\frac{(6a^2c^2-6ab^2c+b^4)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{c^3(-4ac+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -b*Integral(x, (x, x**2))/(c*(-4*a*c + b**2)) - b*log(a + b*x**2
+ c*x**4)/(2*c**3) + x**6*(2*a + b*x**2)/(2*(-4*a*c + b**2)*(a +
b*x**2 + c*x**4)) + x**2*(-3*a*c + b**2)/(c**2*(-4*a*c + b**2)) -
 (6*a**2*c**2 - 6*a*b**2*c + b**4)*atanh((b + 2*c*x**2)/sqrt(-4*a
*c + b**2))/(c**3*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.305693, size = 151, normalized size = 0.91

$$\frac{-\frac{2\left(6a^2c^2-6ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\frac{a^2c\left(3b-2cx^2\right)-ab^2\left(b-4cx^2\right)+b^4\left(-x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-b\log\left(a+bx^2+cx^4\right)+cx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (c*x^2 + (-(b^4*x^2) - a*b^2*(b - 4*c*x^2) + a^2*c*(3*b - 2*c*x^2
))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(b^4 - 6*a*b^2*c + 6*
a^2*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)
^(3/2) - b*Log[a + b*x^2 + c*x^4])/(2*c^3)
```

Maple [B] time = 0.018, size = 600, normalized size = 3.6

$$\frac{x^2}{2\,c^2}+\frac{a^2x^2}{c\,(cx^4+bx^2+a)\,(4\,ac-b^2)}-2\,\frac{ax^2b^2}{c^2\,(cx^4+bx^2+a)\,(4\,ac-b^2)}$$
$$+\frac{x^2b^4}{2\,c^3\,(cx^4+bx^2+a)\,(4\,ac-b^2)}-\frac{3\,a^2b}{2\,c^2\,(cx^4+bx^2+a)\,(4\,ac-b^2)}+\frac{ab^3}{2\,c^3\,(cx^4+bx^2+a)\,(4\,ac-b^2)}$$
$$-2\,\frac{\ln\left(\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)ab}{(4\,ac-b^2)\,c^2}+\frac{\ln\left(\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^3}{2\,c^3\,(4\,ac-b^2)}$$
$$-6\,\frac{a^2}{c\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$
$$+6\,\frac{ab^2}{c^2\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$
$$-\frac{b^4}{c^3}\arctan\left(\left(2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)\frac{1}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*x^2/c^2+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2-2/c^2/(c*x^4+
b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2
)*x^2*b^4-3/2/c^2/(c*x^4+b*x^2+a)*a^2*b/(4*a*c-b^2)+1/2/c^3/(c*x^
4+b*x^2+a)*a*b^3/(4*a*c-b^2)-2/c^2/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*
x^4+b*x^2+a))*a*b+1/2/c^3/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2
+a))*b^3-6/c/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arc
tan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^
2+12*a*b^4*c-b^6)^(1/2))*a^2+6/c^2/(64*a^3*c^3-48*a^2*b^2*c^2+12*
a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64
*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*a*b^2-1/c^3/(64*a^
3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*
c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(
1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ab^3 - 3\,a^2bc + \left(b^4 - 4\,ab^2c + 2\,a^2c^2\right)x^2}{2\,(ab^2c^3 - 4\,a^2c^4 + (b^2c^4 - 4\,ac^5)x^4 + (b^3c^3 - 4\,abc^4)x^2)} + \frac{x^2}{2\,c^2} + \frac{-2\int \frac{(b^3-4\,abc)\,x^3+(ab^2-3\,a^2c)\,x}{cx^4+bx^2+a}\,dx}{b^2c^2 - 4\,ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^3 - 3*a^2*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^2)/(a*b
^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c
^4)*x^2) + 1/2*x^2/c^2 + 2*integrate(-((b^3 - 4*a*b*c)*x^3 + (a*b
^2 - 3*a^2*c)*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Fricas [A] time = 0.299428, size = 1, normalized size = 0.01

$$\left[-\frac{\left(ab^4 - 6\,a^2b^2c + 6\,a^3c^2 + \left(b^4c - 6\,ab^2c^2 + 6\,a^2c^3\right)x^4 + \left(b^5 - 6\,ab^3c + 6\,a^2bc^2\right)x^2\right)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2}{cx^4+bx^2+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 +
6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2)*log((b^3 -
```

```
4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2
- 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^2*c^2 - 4*
a*c^3)*x^6 + (b^3*c - 4*a*b*c^2)*x^4 - a*b^3 + 3*a^2*b*c - (b^4 -
 5*a*b^2*c + 6*a^2*c^2)*x^2 - ((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 -
4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^2)*log(c*x^4 + b*x^2 + a))*sqrt(b
^2 - 4*a*c))/((a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 +
(b^3*c^3 - 4*a*b*c^4)*x^2)*sqrt(b^2 - 4*a*c)), 1/2*(2*(a*b^4 - 6*
a^2*b^2*c + 6*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (
b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b
^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*x^6 + (b^3*c -
4*a*b*c^2)*x^4 - a*b^3 + 3*a^2*b*c - (b^4 - 5*a*b^2*c + 6*a^2*c^2
)*x^2 - ((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a
*b^2*c)*x^2)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/((a*b^2*
c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)
*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 17.1066, size = 877, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c
 + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)))*log(x**2 + (-5*a**2*b*c - 16*a**2*c**4*(-b/(2*c**3) - s
qrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + a*b*
*3 + 8*a*b**2*c**3*(-b/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(6*a**
2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*
c**2 + 12*a*b**4*c - b**6))) - b**4*c**2*(-b/(2*c**3) - sqrt(-(4*
a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**
3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(6*a**2*c**2
- 6*a*b**2*c + b**4)) + (-b/(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(
6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*
b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 + (-5*a**2*b*c - 16*a*
*2*c**4*(-b/(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*
a*b**2*c + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a
*b**4*c - b**6))) + a*b**3 + 8*a*b**2*c**3*(-b/(2*c**3) + sqrt(-(
4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(2*c**3*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**4*c**2*
(-b/(2*c**3) + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c
 + b**4)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6))))/(6*a**2*c**2 - 6*a*b**2*c + b**4)) + (-3*a**2*b*c + a*
b**3 + x**2*(2*a**2*c**2 - 4*a*b**2*c + b**4))/(8*a**2*c**4 - 2*a
*b**2*c**3 + x**4*(8*a*c**5 - 2*b**2*c**4) + x**2*(8*a*b*c**4 - 2
*b**3*c**3)) + x**2/(2*c**2)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.90 $\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2-10ac\right)}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{2c\left(b^2-4ac\right)}+\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 -
4*a*c)) + (x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) - ((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rubi [A] time = 1.41993, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2-10ac\right)}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{2c\left(b^2-4ac\right)}+\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 -
4*a*c)) + (x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) - ((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[
```

```
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.10919, size = 327, normalized size = 0.99

$$\frac{-\frac{2\sqrt{c}x\left(2a^2c-ab\left(b-3cx^2\right)+b^3\left(-x^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\sqrt{2}\left(-20a^2c^2+19ab^2c-13abc\sqrt{b^2-4ac}+3b^3\sqrt{b^2-4ac}-3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(20a^2c^2-19ab^2c-13abc\sqrt{b^2\cdots}\right.}{(b^2-\cdots}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2
)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a
*b^2*c - 20*a^2*c^2 + 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(
3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*
b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/(4*c^(5/2))
```

Maple [B] time = 0.069, size = 2280, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] x/c^2+3/2/c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*
x^2+a)*b^3/(4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-
1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2-80*c^2/(-(4*a*c-b^2)^
3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2
))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*a^4+
116*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(
-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2
+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c
-b^2)*c)^(1/2))*a^3*b^2-55/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*a^2*b^4+43/4/c/(-(4*a*c-b^2)
^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)
/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*a*b
^6-3/4/c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+
b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*
a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(
4*a*c-b^2)*c)^(1/2))*b^8-13/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-
(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+
2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-
b^2)*c)^(1/2))*a^2*b+25/4/c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-
(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+
2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-
b^2)*c)^(1/2))*a*b^3-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(
-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2
+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c
-b^2)*c)^(1/2))*b^5+80*c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(
1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*a
rctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^4-116*c/(-(4*a*c-b^2)^3)^(1/2)
/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2
)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^3*b^2+55/(-(4*
a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*a^2*b^4-43/4/c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*
c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(
8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b
^2)^3)^(1/2)))^(1/2))*a*b^6+3/4/c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c
-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^8-13/(4*a*c-b^2)*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*ar
ctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+
(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^2*b+25/4/c/(4*a*c-b^2)*2^(1/2)/
((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan
```

```
(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4
*a*c-b^2)^3)^(1/2)))^(1/2))*a*b^3-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/
2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*
c-b^2)^3)^(1/2)))^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(b^3 - 3\,abc)\,x^3 + (ab^2 - 2\,a^2c)\,x}{2\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^4 + (b^3c^2 - 4\,abc^3)x^2)} + \frac{-\int \frac{3\,ab^2 - 10\,a^2c + (3\,b^3 - 13\,abc)\,x^2}{cx^4 + bx^2 + a}\,dx}{2\,(b^2c^2 - 4\,ac^3)} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^
2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/
2*integrate(-(3*a*b^2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4
 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3) + x/c^2
```

Fricas [A] time = 0.394613, size = 3856, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/
2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 -
4*a*b*c^3)*x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 42
0*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c
^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*
c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 250
0*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^
6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*
b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024
*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*
a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a
```

```
^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
64*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a
*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48
*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*
b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 4
8*a^2*b^2*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c +
 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 45
9*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c
^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 -
 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*
a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c
^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a
*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*
c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a
^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqr
t(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*
c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 -
918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/
(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6
*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(-(189*a^2
*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*x + 1/2*
sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^
4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3
- (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81
*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqr
t(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^
2 - 4*a*b*c^3)*x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2
- 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a
^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^
2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 +
 (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 +
1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2
550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b
^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 25
50*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
```

```
7 - 64*a^3*c^8))) + 2*(3*a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [A] time = 22.633, size = 450, normalized size = 1.36

$$\frac{x^3\left(3abc-b^3\right)+x\left(2a^2c-ab^2\right)}{8a^2c^3-2ab^2c^2+x^4\left(8ac^4-2b^2c^3\right)+x^2\left(8abc^3-2b^3c^2\right)}$$
$$+\text{RootSum}\left(t^4\left(1048576a^6c^{11}-1572864a^5b^2c^{10}+983040a^4b^4c^9-327680a^3b^6c^8+61440a^2b^8c^7-6144ab^{10}c^6+256b^{12}c^5\right)+\right.$$
$$+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (x**3*(3*a*b*c - b**3) + x*(2*a**2*c - a*b**2))/(8*a**2*c**3 - 2*
a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 -
2*b**3*c**2)) + RootSum(_t**4*(1048576*a**6*c**11 - 1572864*a**5*
b**2*c**10 + 983040*a**4*b**4*c**9 - 327680*a**3*b**6*c**8 + 6144
0*a**2*b**8*c**7 - 6144*a*b**10*c**6 + 256*b**12*c**5) + _t**2*(4
30080*a**6*b*c**6 - 716800*a**5*b**3*c**5 + 483840*a**4*b**5*c**4
 - 170496*a**3*b**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a*b**11*c
+ 144*b**13) + 10000*a**7*c**2 - 4200*a**6*b**2*c + 441*a**5*b**4
, Lambda(_t, _t*log(x + (65536*_t**3*a**4*b*c**9 - 61440*_t**3*a*
*3*b**3*c**8 + 21504*_t**3*a**2*b**5*c**7 - 3328*_t**3*a*b**7*c**
6 + 192*_t**3*b**9*c**5 - 8000*_t*a**5*c**5 + 36160*_t*a**4*b**2*
c**4 - 32476*_t*a**3*b**4*c**3 + 11592*_t*a**2*b**6*c**2 - 1836*_
t*a*b**8*c + 108*_t*b**10)/(2500*a**5*c**3 - 5625*a**4*b**2*c**2
+ 1971*a**3*b**4*c - 189*a**2*b**6)))) + x/c**2
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.91 $\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=132

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 +
 c*x^4]/(4*c^2)
```

Rubi [A] time = 0.297114, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}}-\frac{bx^2}{2c\left(b^2-4ac\right)}+\frac{x^4\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 +
 c*x^4]/(4*c^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\left(-6ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{x^4\left(2a+bx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{\int^{x^2} b\,dx}{2c\left(-4ac+b^2\right)}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] b*(-6*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*
*2*(-4*a*c + b**2)**(3/2)) + x**4*(2*a + b*x**2)/(2*(-4*a*c + b**
2)*(a + b*x**2 + c*x**4)) - Integral(b, (x, x**2))/(2*c*(-4*a*c +
```

```
b**2)) + log(a + b*x**2 + c*x**4)/(4*c**2)
```

Mathematica [A] time = 0.308738, size = 121, normalized size = 0.92

$$\frac{\frac{2\left(-2a^2c+ab\left(b-3cx^2\right)+b^3x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-
b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*
c^2)
```

Maple [B] time = 0.011, size = 342, normalized size = 2.6

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(\frac{b\left(3\,ac-b^2\right)x^2}{\left(4\,ac-b^2\right)c^2}+\frac{a\left(2\,ac-b^2\right)}{\left(4\,ac-b^2\right)c^2}\right)+\frac{\ln\left(c\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)}{4\,c^2}$$
$$-3\,\frac{ab}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\arctan\left(\frac{2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)$$
$$+\frac{b^3}{2\,c}\arctan\left(\left(2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/
c^2)/(c*x^4+b*x^2+a)+1/4/c^2*ln(c*(4*a*c-b^2)*(c*x^4+b*x^2+a))-3/
(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*
c^2*(4*a*c-b^2)*x^2+c*(4*a*c-b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+1
2*a*b^4*c^3-b^6*c^2)^(1/2))*a*b+1/2/(64*a^3*c^5-48*a^2*b^2*c^4+12
*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c-b^2)*x^2+c*(4*a*c-
b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2))*b
^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ab^2 - 2\,a^2c + \left(b^3 - 3\,abc\right)x^2}{2\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^4 + (b^3c^2 - 4\,abc^3)x^2)} - \frac{-\int \frac{(b^2-4\,ac)\,x^3+abx}{cx^4+bx^2+a}\,dx}{b^2c - 4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x^2)/(a*b^2*c^2 - 4*a^2*c^
3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - integr
ate(-((b^2 - 4*a*c)*x^3 + a*b*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c -
 4*a*c^2)
```

Fricas [A] time = 0.303848, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^3c - 6\,abc^2\right)x^4 + ab^3 - 6\,a^2bc + \left(b^4 - 6\,ab^2c\right)x^2\right)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) + \left(2\,ab^2 - 4\cdots\right.}{4\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^4 + (b^3c\cdots}\right.$$

$$-\frac{2\left(\left(b^3c - 6\,abc^2\right)x^4 + ab^3 - 6\,a^2bc + \left(b^4 - 6\,ab^2c\right)x^2\right)\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right) - \left(2\,ab^2 - 4\,a^2c + 2\left(b^3 - 3\,abc\right)x^2 + \cdots\right.}{4\,(ab^2c^2 - 4\,a^2c^3 + (b^2c^3 - 4\,ac^4)x^4 + (b^3c^2 - 4\,abc^3)x\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b
^2*c)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) + (2*a*b^2 - 4*a^2*c + 2*(b^3 - 3*a*b*c)*x^2 + ((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*log(c*x^4 + b
*x^2 + a))*sqrt(b^2 - 4*a*c))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3
- 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)), -
1/4*(2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*
b^2*c)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - (2*a*b^2 - 4*a^2*c + 2*(b^3 - 3*a*b*c)*x^2 + ((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*log(c*x^4 + b*x^
2 + a))*sqrt(-b^2 + 4*a*c))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 -
4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 12.8828, size = 745, normalized size = 5.64

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{4c^2}\right)+8a^2c+16ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{4c^2}\right)-ab}{6abc-b^3}\right.$$
$$+\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{4c^2}\right)+8a^2c+16ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{4c^2}\right)-ab^2-}{6abc-b^3}\right.$$
$$+\frac{2a^2c-ab^2+x^2\left(3abc-b^3\right)}{8a^2c^3-2ab^2c^2+x^4\left(8ac^4-2b^2c^3\right)+x^2\left(8abc^3-2b^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**
2 + (-32*a**2*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4
*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(-b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3))
+ (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x*
*2 + (-32*a**2*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4
*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) +
(2*a**2*c - a*b**2 + x**2*(3*a*b*c - b**3))/(8*a**2*c**3 - 2*a*b*
*2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b*
*3*c**2))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.92 $\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{bx}{2c(b^2-4ac)}$$

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2
)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.12177, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{bx}{2c(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2
)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 100.33, size = 267, normalized size = 0.99

$$-\frac{bx}{2c\left(-4ac+b^{2}\right)}+\frac{x^{3}\left(2a+bx^{2}\right)}{2\left(-4ac+b^{2}\right)\left(a+bx^{2}+cx^{4}\right)}+\frac{\sqrt{2}\left(-2abc+b\left(-6ac+b^{2}\right)+\left(-6ac+b^{2}\right)\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^{2}}}}\right)}{4c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(-abc+\frac{b\left(-6ac+b^{2}\right)}{2}-\frac{\left(-6ac+b^{2}\right)\sqrt{-4ac+b^{2}}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^{2}}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -b*x/(2*c*(-4*a*c + b**2)) + x**3*(2*a + b*x**2)/(2*(-4*a*c + b**
2)*(a + b*x**2 + c*x**4)) + sqrt(2)*(-2*a*b*c + b*(-6*a*c + b**2)
 + (-6*a*c + b**2)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sq
rt(b + sqrt(-4*a*c + b**2)))/(4*c**(3/2)*sqrt(b + sqrt(-4*a*c + b
**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*(-a*b*c + b*(-6*a*c + b**
2)/2 - (-6*a*c + b**2)*sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*sqrt(c
)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*a
*c + b**2))*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.92863, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}x\left(a\left(b-2cx^{2}\right)+b^{2}x^{2}\right)}{\left(b^{2}-4ac\right)\left(a+bx^{2}+cx^{4}\right)}+\frac{\sqrt{2}\left(b^{2}\sqrt{b^{2}-4ac}-6ac\sqrt{b^{2}-4ac}+8abc-b^{3}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^{2}-4ac}}}\right)}{\left(b^{2}-4ac\right)^{3/2}\sqrt{b-\sqrt{b^{2}-4ac}}}+\frac{\sqrt{2}\left(b^{2}\sqrt{b^{2}-4ac}-6ac\sqrt{b^{2}-4ac}-8abc+b^{3}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^{2}-4ac}+b}}\right)}{\left(b^{2}-4ac\right)^{3/2}\sqrt{\sqrt{b^{2}-4ac}+b}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c]
 - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^
(3/2))
```

Maple [B] time = 0.069, size = 2158, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x
^4+b*x^2+a)+32*c^3/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)
/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)
*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c
^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^3*b-20*c^2/(-c^2*(
4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b
^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2
*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)
^3)^(1/2)))^(1/2))*a^2*b^3+4*c/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-
b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(
1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b
^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*a*b^5-1/
4/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*
a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*
c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4
*a*c-b^2)^3)^(1/2)))^(1/2))*b^7+6*c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b
^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/
2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(
-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^2-5/2/(4*a*c-b^2)*2^(1/2)/((
4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*ar
ctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-
b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*a*b^2+1/4/c/(4*a*c-b^2)
*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)
))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*
(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^4-32*c^3/(
-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+
(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*
c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3
)^(1/2))*(4*a*c-b^2))^(1/2))*a^3*b+20*c^2/(-c^2*(4*a*c-b^2)^3)^(1
/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(
1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/
2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^
(1/2))*a^2*b^3-4*c/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)
/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2
)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c
+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*a*b^5+1/4/(-c^2*
(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2
*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2
*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2))^(1/2))*b^7+6*c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+
b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*
(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-
```

```
b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*a^2-5/2/(4*a*c-b^2)*2^(1/2)/((
-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*a
rctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-
c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*a*b^2+1/4/c/(4*a*c-
b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*
c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a
*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2-2\,ac\right)x^3+abx}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}-\frac{-\int\frac{\left(b^2-6\,ac\right)x^2+ab}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2
*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2
- 6*a*c)*x^2 + a*b)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [A] time = 0.324864, size = 3047, normalized size = 11.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c
^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2
*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12
*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*
b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2
*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*
c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6
*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + sqrt(1/2)*
((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
```

```
^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a
*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9
)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((
5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17
*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c
^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 -
 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c
^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a
*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^
6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
 (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2
- (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 6
4*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sq
rt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8
*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4
*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^
7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a
^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)
*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 4
8*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^
2*c^5 - 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^
2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*
c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*
a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4
*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^
3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144
*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*
b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 -
15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*
c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 -
 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^
4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [A] time = 16.1503, size = 379, normalized size = 1.4

$$-\frac{-abx + x^3\left(2ac - b^2\right)}{8a^2c^2 - 2ab^2c + x^4\left(8ac^3 - 2b^2c^2\right) + x^2\left(8abc^2 - 2b^3c\right)}$$

$$+\mathrm{RootSum}\left(t^4\left(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b^4c^7 - 327680a^3b^6c^6 + 61440a^2b^8c^5 - 6144ab^{10}c^4 + 256b^{12}c^3\right) + t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -(-a*b*x + x**3*(2*a*c - b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*
(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSu
m(_t**4*(1048576*a**6*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4
*b**4*c**7 - 327680*a**3*b**6*c**6 + 61440*a**2*b**8*c**5 - 6144*
a*b**10*c**4 + 256*b**12*c**3) + _t**2*(-61440*a**5*b*c**5 + 6144
0*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 4
32*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c + 25*a
**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_t
**3*a**3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b*
*6*c**4 + 64*_t**3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*
b**3*c**2 - 88*_t*a*b**5*c + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*
b**2*c + 5*a*b**4))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.93 $\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=78

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.129287, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 20.403, size = 70, normalized size = 0.9

$$\frac{2a\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{x^2\left(2a+bx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] 2*a*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3
/2) + x**2*(2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4
))
```

Mathematica [A] time = 0.150467, size = 93, normalized size = 1.19

$$\frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\frac{a\left(b-2cx^2\right)+b^2x^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^
4)) + (2*a*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*
c)^(3/2)
```

Maple [A] time = 0.005, size = 104, normalized size = 1.3

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(2\,ac-b^2\right)x^2}{(4\,ac-b^2)\,c}+\frac{ab}{(4\,ac-b^2)\,c}\right)+2\,\frac{a}{(4\,ac-b^2)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x
^2+a)+2*a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.322085, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(ac^{2}x^{4}+abcx^{2}+a^{2}c\right)\log\left(-\frac{b^{3}-4\,abc+2\left(b^{2}c-4\,ac^{2}\right)x^{2}-\left(2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)+\left(\left(b^{2}-2\,ac\right)x^{2}+ab\right)\sqrt{b^{2}-4\,ac}}{2\left(\left(b^{2}c^{2}-4\,ac^{3}\right)x^{4}+ab^{2}c-4\,a^{2}c^{2}+\left(b^{3}c-4\,abc^{2}\right)x^{2}\right)\sqrt{b^{2}-4\,ac}},\right.$$
$$\left.-\frac{4\left(ac^{2}x^{4}+abcx^{2}+a^{2}c\right)\arctan\left(-\frac{\left(2\,cx^{2}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)+\left(\left(b^{2}-2\,ac\right)x^{2}+ab\right)\sqrt{-b^{2}+4\,ac}}{2\left(\left(b^{2}c^{2}-4\,ac^{3}\right)x^{4}+ab^{2}c-4\,a^{2}c^{2}+\left(b^{3}c-4\,abc^{2}\right)x^{2}\right)\sqrt{-b^{2}+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*log(-(b^3 - 4*a*b*c + 2*
(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqr
t(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2 - 2*a*c)*x^2 + a*b)*
sqrt(b^2 - 4*a*c))/(((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^
2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)), -1/2*(4*(a*c^2*x
^4 + a*b*c*x^2 + a^2*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/
(b^2 - 4*a*c)) + ((b^2 - 2*a*c)*x^2 + a*b)*sqrt(-b^2 + 4*a*c))/((
(b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 6.46988, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}\log\left(x^{2}+\frac{-16a^{3}c^{2}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+8a^{2}b^{2}c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}-ab^{4}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+ab}{2ac}\right)$$
$$+a\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}\log\left(x^{2}+\frac{16a^{3}c^{2}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}-8a^{2}b^{2}c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+ab^{4}\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}+ab}{2ac}\right)$$
$$-\frac{-ab+x^{2}\left(2ac-b^{2}\right)}{8a^{2}c^{2}-2ab^{2}c+x^{4}\left(8ac^{3}-2b^{2}c^{2}\right)+x^{2}\left(8abc^{2}-2b^{3}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(
4*a*c - b**2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*
b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*
c - b**2)**3)*log(x**2 + (16*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3)
```

```
 - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) + a*b**4*sqrt(-1/(4*a
*c - b**2)**3) + a*b)/(2*a*c)) - (-a*b + x**2*(2*a*c - b**2))/(8*
a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*
a*b*c**2 - 2*b**3*c))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.94 $\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=237

$$\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (
b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sq
rt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.772554, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```


```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (
b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sq
rt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 59.3685, size = 218, normalized size = 0.92

$$\frac{x\left(2a+bx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(4ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(4ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] x*(2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) + sqrt
(2)*(4*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x
/sqrt(b + sqrt(-4*a*c + b**2)))/(4*sqrt(c)*sqrt(b + sqrt(-4*a*c +
 b**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*(4*a*c + b**2 - b*sqrt(
-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**
2)))/(4*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3
/2))
```

Mathematica [A] time = 0.721658, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

Maple [B] time = 0.052, size = 1641, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+16/(-(4*
a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*
c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c
)^(1/2))*a^3-4/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^2*2^(1/2)/((-
4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(
1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^
(1/2))*(4*a*c-b^2)*c)^(1/2))*a^2*b^2-c/(-(4*a*c-b^2)^3)^(1/2)/(4*
a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^
2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b
^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^4*a+1/4/(-(4*a
*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)
^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*
2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/
2))*b^6-c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)
/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b*a
+1/4/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(
4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4
*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^3-16/(
-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^3*2^(1/2)/((4*a*c-b^2)*c*(4*a
*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2
*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2))*a^3+4/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^2*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/
2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*
c-b^2)^3)^(1/2)))^(1/2))*a^2*b^2+c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-
b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))
^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a
*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^4*a-1/4/(-(4*a*c-b^2)^
3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c
-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^6-c/(4
*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1
/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b*a+1/4/(4*a*c-b^2)
*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/
2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c
-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^3 + 2\,ax}{2\,((b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2)} + \frac{\int \frac{bx^2-2\,a}{cx^4+bx^2+a}\,dx}{2\,(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 +
 a), x)/(b^2 - 4*a*c)
```

Fricas [A] time = 0.303173, size = 2252, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
 + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4
*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*
b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^
4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 6
4*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*
c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*
sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) + s
qrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*
c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^
4))*log((3*b^2 + 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c
^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqr
```

```
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b
^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*
sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log(
(3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*
a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c -
12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 11.9598, size = 294, normalized size = 1.24

$$-\frac{2ax + bx^3}{8a^2c - 2ab^2 + x^4\left(8ac^2 - 2b^2c\right) + x^2\left(8abc - 2b^3\right)}$$
$$+\operatorname{RootSum}\left(t^4\left(1048576a^6c^7 - 1572864a^5b^2c^6 + 983040a^4b^4c^5 - 327680a^3b^6c^4 + 61440a^2b^8c^3 - 6144ab^{10}c^2 + 256b^{12}c\right) + t^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -(2*a*x + b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*
c) + x**2*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7
- 1572864*a**5*b**2*c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b*
*6*c**4 + 61440*a**2*b**8*c**3 - 6144*a*b**10*c**2 + 256*b**12*c)
 + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*b**3*c**3 - 1536*a**2*b*
*5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c + 9*a*b**4, La
mbda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a**2*b
**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2
*c**2 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.95 $\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.119919, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 18.3567, size = 65, normalized size = 0.87

$$-\frac{b\,\text{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{2a+bx^2}{2(-4ac+b^2)(a+bx^2+cx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/
2) + (2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.122188, size = 79, normalized size = 1.05

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(
b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.005, size = 77, normalized size = 1.

$$\frac{-bx^2-2a}{\left(8ac-2b^2\right)\left(cx^4+bx^2+a\right)}-b\arctan\left(\left(2cx^2+b\right)\frac{1}{\sqrt{4ac-b^2}}\right)\left(4ac-b^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.307436, size = 1, normalized size = 0.01

$$\left[-\frac{\left(bcx^4+b^2x^2+ab\right)\log\left(\frac{b^3-4abc+2\left(b^2c-4ac^2\right)x^2+\left(2c^2x^4+2bcx^2+b^2-2ac\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-\left(bx^2+2a\right)\sqrt{b^2-4ac}}{2\left(\left(b^2c-4ac^2\right)x^4+ab^2-4a^2c+\left(b^3-4abc\right)x^2\right)\sqrt{b^2-4ac}},\ \frac{2\left(bcx^4+b^2x^2+a\right.}{2\left((b^2c-4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c*x^4 + b^2*x^2 + a*b)*log((b^3 - 4*a*b*c + 2*(b^2*c -
4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) - (b*x^2 + 2*a)*sqrt(b^2 - 4*a*c))/((
(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sq
rt(b^2 - 4*a*c)), 1/2*(2*(b*c*x^4 + b^2*x^2 + a*b)*arctan(-(2*c*x
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b*x^2 + 2*a)*sqrt(-b
^2 + 4*a*c))/(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 5.89395, size = 267, normalized size = 3.56

$$\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} - \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} - \frac{2a+bx^2}{8a^2c-2ab^2+x^4(8ac^2-2b^2c)+x^2(8abc-2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/
(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c
 - b**2)**3)*log(x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3
) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c -
 b**2)**3) + b**2)/(2*b*c))/2 - (2*a + b*x**2)/(8*a**2*c - 2*a*b*
*2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.96 $\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.52079, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 47.8794, size = 201, normalized size = 0.91

$$-\frac{\sqrt{2}\sqrt{c}\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(b-\frac{\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{x\left(b+2cx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*(b + sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*sqrt(c)
*x/sqrt(b + sqrt(-4*a*c + b**2)))/(sqrt(b + sqrt(-4*a*c + b**2))*
(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt(c)*(b - sqrt(-4*a*c + b**2
)/2)*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(sqrt(
b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) - x*(b + 2*c*x**
2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.80278, size = 222, normalized size = 1.

$$\frac{-bx-2cx^3}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[
c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A] time = 0.084, size = 342, normalized size = 1.6

$$\frac{x}{8\,ac-2\,b^{2}}\left(x^{2}+\frac{1}{2\,c}\sqrt{-4\,ac+b^{2}}+\frac{b}{2\,c}\right)^{-1}$$
$$+\frac{c\sqrt{2}b}{4\,ac-b^{2}}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$+\frac{c\sqrt{2}}{8\,ac-2\,b^{2}}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$+\frac{x}{8\,ac-2\,b^{2}}\left(x^{2}+\frac{b}{2\,c}-\frac{1}{2\,c}\sqrt{-4\,ac+b^{2}}\right)^{-1}$$
$$+\frac{c\sqrt{2}b}{4\,ac-b^{2}}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^{2}}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}$$
$$-\frac{c\sqrt{2}}{8\,ac-2\,b^{2}}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+c/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2*c/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2/(4*a*c-b^2)*x/(x^2+1/2*
b/c-1/2/c*(-4*a*c+b^2)^(1/2))+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,cx^{3}+bx}{2\left(\left(b^{2}c-4\,ac^{2}\right)x^{4}+ab^{2}-4\,a^{2}c+\left(b^{3}-4\,abc\right)x^{2}\right)}-\frac{\int\frac{2\,cx^{2}-b}{cx^{4}+bx^{2}+a}\,dx}{2\left(b^{2}-4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2
+ a), x)/(b^2 - 4*a*c)
```

<u>Fricas</u> [A]   time = 0.322846, size = 2268, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*
b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b
^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^
2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 4
8*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 6
4*a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
 + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b
^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2
*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2
*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
- 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48
*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64
*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*
c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5
 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*
c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*
a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 4
```

```
8*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^5
- 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c
^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a
^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a
^4*c^3))) + 2*b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^
3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 12.4669, size = 298, normalized size = 1.35

$$\frac{bx + 2cx^3}{8a^2c - 2ab^2 + x^4\left(8ac^2 - 2b^2c\right) + x^2\left(8abc - 2b^3\right)} + \text{RootSum}\left(t^4\left(1048576a^7c^6 - 1572864a^6b^2c^5 + 983040a^5b^4c^4 - 327680a^4b^6c^3 + 61440a^3b^8c^2 - 6144a^2b^{10}c + 256ab^{12}\right) + t^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c
) + x**2*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 -
 1572864*a**6*b**2*c**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**
6*c**3 + 61440*a**3*b**8*c**2 - 6144*a**2*b**10*c + 256*a*b**12)
+ _t**2*(-12288*a**4*b*c**4 + 8192*a**3*b**3*c**3 - 1536*a**2*b**
5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2 + 9*b**4*c, Lam
bda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*b**2*
c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*c*
*2 - 16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.97 $\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=74

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.119195, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 14.8741, size = 66, normalized size = 0.89

$$\frac{2c\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{b+2cx^2}{2(-4ac+b^2)(a+bx^2+cx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] 2*c*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3
/2) - (b + 2*c*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.144012, size = 79, normalized size = 1.07

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx^2}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c))
```

Maple [A] time = 0.005, size = 75, normalized size = 1.

$$\frac{2\,cx^2+b}{\left(8\,ac-2\,b^2\right)\left(cx^4+bx^2+a\right)}+2\,\frac{c}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284356, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(c^2x^4+bcx^2+ac\right)\log\left(-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2-\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{2\,((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)\sqrt{b^2-4\,ac}},\right.$$
$$\left.-\frac{4\left(c^2x^4+bcx^2+ac\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{2\,((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(c^2*x^4 + b*c*x^2 + a*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c
 - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2
- 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
*sqrt(b^2 - 4*a*c)), -1/2*(4*(c^2*x^4 + b*c*x^2 + a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*x^2 + b)*sqr
t(-b^2 + 4*a*c))/(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 5.74371, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)$$
$$+c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x^2+\frac{16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)$$
$$+\frac{b+2cx^2}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(
4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b*
*4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a
*c - b**2)**3)*log(x**2 + (16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3
) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*
```

```
a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/(8*a**2*c - 2*a
*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.98 $\int \frac{x^2}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c
- b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi [A] time = 0.925371, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c
- b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi in Sympy [A] time = 67.4129, size = 230, normalized size = 0.91

$$-\frac{\sqrt{2}\sqrt{c}\left(-12ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b+\sqrt{-4ac+b^2}}\,(-4ac+b^2)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(-12ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b-\sqrt{-4ac+b^2}}\,(-4ac+b^2)^{\frac{3}{2}}}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*(-12*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sq
rt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt(b + sqrt
(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt(c)*(-12*a
*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b
- sqrt(-4*a*c + b**2)))/(4*a*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*
c + b**2)**(3/2)) + x*(-2*a*c + b**2 + b*c*x**2)/(2*a*(-4*a*c + b
**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.752931, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(4*a)
```

Maple [B] time = 0.073, size = 733, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*b-c/(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2
*b/c)+1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b
^2)^(1/2)+1/2*b/c)*b^2-1/4*c/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b
^2)^(1/2))*b+c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/
c*(-4*a*c+b^2)^(1/2))-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2
+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*b^2+1/4*c/(4*a*c-b^2)/a*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b
^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^3 + (b^2 - 2\,ac)\,x}{2\,((ab^2c - 4\,a^2c^2)x^4 + a^2b^2 - 4\,a^3c + (ab^3 - 4\,a^2bc)x^2)} + \frac{\int \frac{bcx^2 + b^2 - 6\,ac}{cx^4 + bx^2 + a}\,dx}{2\,(ab^2 - 4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2
 + b^2 - 6*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [A] time = 0.339881, size = 3117, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 -
 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3
)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4
)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3
*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^
2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*
((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((
5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 -
23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a
^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a
^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c +
60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
+ a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*
b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 +
324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^
2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7*c + 144
*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*
b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64
*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c
 + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
 sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18
*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1
/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a
^4*c^4 + (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*
c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
```

```
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2
)
```

Sympy [A] time = 15.714, size = 394, normalized size = 1.56

$$-\frac{bcx^3 + x\left(-2ac + b^2\right)}{8a^3c - 2a^2b^2 + x^4\left(8a^2c^2 - 2ab^2c\right) + x^2\left(8a^2bc - 2ab^3\right)}$$
$$+\mathrm{RootSum}\left(t^4\left(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4 - 327680a^6b^6c^3 + 61440a^5b^8c^2 - 6144a^4b^{10}c + 256a^3b^{12}\right) + t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -(b*c*x**3 + x*(-2*a*c + b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8
*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSu
m(_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7
*b**4*c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*
a**4*b**10*c + 256*a**3*b**12) + _t**2*(-61440*a**5*b*c**5 + 6144
0*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 4
32*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 + 25*b
**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*
_t**3*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**
4*b**7*c + 64*_t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*
b**2*c**3 + 740*_t*a**2*b**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(
324*a**2*c**4 - 81*a*b**2*c**3 + 5*b**4*c**2))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.99 $\int \frac{x}{(ax+bx^3+cx^5)^2} \, dx$

Optimal. Leaf size=122

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^
2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a
^2)
```

Rubi [A] time = 0.37126, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^
2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a
^2)
```

Rubi in Sympy [A] time = 52.5295, size = 116, normalized size = 0.95

$$\frac{-2ac+b^2+bcx^2}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{b\left(-6ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\log\left(x^2\right)}{2a^2}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(2*a*(-4*a*c + b**2)*(a + b*x**2 + c*x
**4)) + b*(-6*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2
))/(2*a**2*(-4*a*c + b**2)**(3/2)) + log(x**2)/(2*a**2) - log(a +
```

```
b*x**2 + c*x**4)/(4*a**2)
```

Mathematica [A] time = 0.685109, size = 207, normalized size = 1.7

$$\frac{\frac{2a(-2ac+b^2+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2cx\right.}{(b^2-4ac)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + 4*Log[x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sq
rt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a
*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)
^(3/2))/(4*a^2)
```

Maple [B] time = 0.012, size = 405, normalized size = 3.3

$$-\frac{bcx^2}{2a(cx^4+bx^2+a)(4ac-b^2)}+\frac{c}{(4ac-b^2)(cx^4+bx^2+a)}-\frac{b^2}{2a(cx^4+bx^2+a)(4ac-b^2)}$$
$$-\frac{c\ln\left(\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)\right)}{a(4ac-b^2)}+\frac{\ln\left(\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^2}{4\,(4ac-b^2)\,a^2}$$
$$-3\frac{bc}{a\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx^2+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)$$
$$+\frac{b^3}{2a^2}\arctan\left((2\left(4ac-b^2\right)cx^2+\left(4ac-b^2\right)b)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}$$
$$+\frac{\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2+1/(c*x^4+b*x^2+a)/(4*a
*c-b^2)*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2-1/a/(4*a*c-b^2)*c
*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))+1/4/a^2/(4*a*c-b^2)*ln((4*a*c-b^
2)*(c*x^4+b*x^2+a))*b^2-3/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^
3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*c+1/2/a^2/(64*a^3*c^3-4
8*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(
```

```
4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b
^3+ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^2+b^2-2\,ac}{2\,((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2)}+\frac{-\int\frac{(b^2c-4\,ac^2)x^3+(b^3-5\,abc)x}{cx^4+bx^2+a}\,dx}{a^2b^2-4\,a^3c}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2
- 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-((b^2*c - 4*a*c
^2)*x^3 + (b^3 - 5*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4
*a^3*c) + log(x)/a^2
```

Fricas [A] time = 0.360782, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(2\,abcx^2+\cdots\right.}{4\,(a^3b^2-4\,a^4c+\cdots}\right.$$

$$-\frac{2\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(2\,abcx^2+2\,ab^2-4\,a^2c-\left(\left(b^2c-4\cdots\right.\right.\right.}{4\,(a^3b^2-4\,a^4c+(a^2b^2c-4\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b
^2*c)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) + (2*a*b*c*x^2 + 2*a*b^2 - 4*a^2*c - ((b^2*c - 4*a*c^2)*x^4
 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*log(c*x^4 + b*x^2 + a)
+ 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^
2)*log(x))*sqrt(b^2 - 4*a*c))/((a^3*b^2 - 4*a^4*c + (a^2*b^2*c -
4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(b^2 - 4*a*c)), -
1/4*(2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*
b^2*c)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - (2*a*b*c*x^2 + 2*a*b^2 - 4*a^2*c - ((b^2*c - 4*a*c^2)*x^4 +
```

```
a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*
log(x))*sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*
a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 111.429, size = 772, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2))*log(x**
2 + (-32*a**4*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4
*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
1/(4*a**2)) + 16*a**3*b**2*c*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(-b*sqrt(-(4*a*c - b**2)**3)*
(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 + 7*a*b**2*c - b**4)/(
6*a*b*c**2 - b**3*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2
)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
) - 1/(4*a**2))*log(x**2 + (-32*a**4*c**2*(b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - 1/(4*a**2)) + 16*a**3*b**2*c*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(b*sqrt(
-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 +
7*a*b**2*c - b**4)/(6*a*b*c**2 - b**3*c)) - (-2*a*c + b**2 + b*c*
x**2)/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) +
 x**2*(8*a**2*b*c - 2*a*b**3)) + log(x)/a**2
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^5 + b*x^3 + a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.100 $\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2x\,(b^2-4ac)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{2ax\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^
2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 -
16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3 - 16*a*b*
c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.58842, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{3b^2-10ac}{2a^2x\,(b^2-4ac)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{2ax\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)^(-2),x]
```


```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^
2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 -
16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3 - 16*a*b*
c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 140.539, size = 282, normalized size = 0.92

$$\frac{-2ac+b^2+bcx^2}{2ax\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-16abc+3b^3-\left(-10ac+3b^2\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a^2\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} - \frac{\sqrt{2}\sqrt{c}\left(-16abc+3b^3+\left(-10ac+3b^2\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a^2\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} - \frac{-10ac+3b^2}{2a^2x\left(-4ac+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(2*a*x*(-4*a*c + b**2)*(a + b*x**2 + c
*x**4)) + sqrt(2)*sqrt(c)*(-16*a*b*c + 3*b**3 - (-10*a*c + 3*b**2
)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*
c + b**2)))/(4*a**2*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)
**(3/2)) - sqrt(2)*sqrt(c)*(-16*a*b*c + 3*b**3 + (-10*a*c + 3*b**
2)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a
*c + b**2)))/(4*a**2*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2
)**(3/2)) - (-10*a*c + 3*b**2)/(2*a**2*x*(-4*a*c + b**2))
```

Mathematica [A] time = 1.0898, size = 302, normalized size = 0.98

$$\frac{-\frac{2x\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4a}}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)^(-2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c
- 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 - 16*a*b
*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.059, size = 2012, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*x^4+b*x^2+a)*
c/(4*a*c-b^2)*x^3*b^2-3/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x+1/2
/a^2/(c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)*x-64*a/(-(4*a*c-b^2)^3)^(1/2
)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a
*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*
b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b*c^4+44/(-
(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*b^3*c^3-10/a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/(
(-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctan
h(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3
)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^5*c^2+3/4/a^2*c/(-(4*a*c-b^2)^3)
^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))
*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((
-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^7-10
/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*
c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*c^3+11/2/a/(
4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-
b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c
+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*c^2*b^2-3/4/a^
2*c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4
*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*
a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^4+64*a/
(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)
*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2))*b*c^4-44/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c
-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8
*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^
2)^3)^(1/2)))^(1/2))*b^3*c^3+10/a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b
^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^
(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*
b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^5*c^2-3/4/a^2*c/(-(4*a*
c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b
^7-10/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^
2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*
c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*c^3+11/2/a/
(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^
```

```
(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*c^2*b^2-3/4/a^2*c
/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2
)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^4-1/a^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\,b^2c-10\,ac^2\right)x^4+2\,ab^2-8\,a^2c+\left(3\,b^3-11\,abc\right)x^2}{2\left((a^2b^2c-4\,a^3c^2)x^5+(a^2b^3-4\,a^3bc)x^3+(a^3b^2-4\,a^4c)x\right)}+\frac{-\int\frac{3\,b^3-13\,abc+\left(3\,b^2c-10\,ac^2\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(-2),x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*
a*b*c)*x^2)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*
x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c +
 (3*b^2*c - 10*a*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a
^3*c)
```

Fricas [A] time = 0.418164, size = 3931, normalized size = 12.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(-2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3
- 11*a*b*c)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b
^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 305
1*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4
*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^
4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7
*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*s
qrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^
13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
```

b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sq
rt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)))
 + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)
*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 -
 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2
*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 520
0*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344
*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(
-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/2)*((
a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2
 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c
^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c
^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500
*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (
3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 +
2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 305
1*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a
^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*
sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^
4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*
b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1
/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^
3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10 - 55
*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^
4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*

```
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 -
 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4
*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [A] time = 24.5584, size = 481, normalized size = 1.56

$$\mathrm{RootSum}\Big(t^4\left(1048576a^{11}c^6 - 1572864a^{10}b^2c^5 + 983040a^9b^4c^4 - 327680a^8b^6c^3 + 61440a^7b^8c^2 - 6144a^6b^{10}c + 256a^5b^{12}\right) +$$

$$-\frac{8a^2c - 2ab^2 + x^4\left(10ac^2 - 3b^2c\right) + x^2\left(11abc - 3b^3\right)}{x^5\left(8a^3c^2 - 2a^2b^2c\right) + x^3\left(8a^3bc - 2a^2b^3\right) + x\left(8a^4c - 2a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] RootSum(_t**4*(1048576*a**11*c**6 - 1572864*a**10*b**2*c**5 + 983
040*a**9*b**4*c**4 - 327680*a**8*b**6*c**3 + 61440*a**7*b**8*c**2
 - 6144*a**6*b**10*c + 256*a**5*b**12) + _t**2*(430080*a**6*b*c**
6 - 716800*a**5*b**3*c**5 + 483840*a**4*b**5*c**4 - 170496*a**3*b
**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a*b**11*c + 144*b**13) + 1
0000*a**2*c**7 - 4200*a*b**2*c**6 + 441*b**4*c**5, Lambda(_t, _t*
log(x + (-81920*_t**3*a**10*c**5 + 139264*_t**3*a**9*b**2*c**4 -
86016*_t**3*a**8*b**4*c**3 + 25088*_t**3*a**7*b**6*c**2 - 3520*_t
**3*a**6*b**8*c + 192*_t**3*a**5*b**10 - 27200*_t*a**5*b*c**5 + 6
0176*_t*a**4*b**3*c**4 - 42448*_t*a**3*b**5*c**3 + 13320*_t*a**2*
b**7*c**2 - 1944*_t*a*b**9*c + 108*_t*b**11)/(2500*a**3*c**6 - 56
25*a**2*b**2*c**5 + 1971*a*b**4*c**4 - 189*b**6*c**3)))) - (8*a**
2*c - 2*a*b**2 + x**4*(10*a*c**2 - 3*b**2*c) + x**2*(11*a*b*c - 3
*b**3))/(x**5*(8*a**3*c**2 - 2*a**2*b**2*c) + x**3*(8*a**3*b*c -
2*a**2*b**3) + x*(8*a**4*c - 2*a**3*b**2))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(-2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.101 $\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2
)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
 + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2
 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2 + c*x^4])/
(2*a^3)
```

Rubi [A] time = 0.493281, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4

$$\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2
)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
 + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2
 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2 + c*x^4])/
(2*a^3)
```

Rubi in Sympy [A] time = 83.3631, size = 153, normalized size = 0.94

$$\frac{-2ac+b^2+bcx^2}{2ax^2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{-3ac+b^2}{a^2x^2\left(-4ac+b^2\right)}-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{a^3\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(2*a*x**2*(-4*a*c + b**2)*(a + b*x**2
+ c*x**4)) - (-3*a*c + b**2)/(a**2*x**2*(-4*a*c + b**2)) - b*log(
x**2)/a**3 + b*log(a + b*x**2 + c*x**4)/(2*a**3) - (6*a**2*c**2 -
 6*a*b**2*c + b**4)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(a*
*3*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.466427, size = 248, normalized size = 1.53

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}-\cdots}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6
*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 +
 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2
 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3
/2))/(2*a^3)
```

Maple [B] time = 0.015, size = 569, normalized size = 3.5

$$-\frac{c^2x^2}{a\,(cx^4+bx^2+a)\,(4\,ac-b^2)}+\frac{x^2b^2c}{2\,a^2\,(cx^4+bx^2+a)\,(4\,ac-b^2)}-\frac{3\,bc}{2\,a\,(cx^4+bx^2+a)\,(4\,ac-b^2)}+\frac{b^3}{2\,a^2\,(cx^4+bx^2+a)\,(4\,ac-b^2)}+2\,\frac{c\ln\left(\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b}{(4\,ac-b^2)\,a^2}-\frac{\ln\left(\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^3}{2\,a^3\,(4\,ac-b^2)}-6\,\frac{c^2}{a\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)+6\,\frac{b^2c}{a^2\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)-\frac{b^4}{a^3}\arctan\left(\left(2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)\frac{1}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}-\frac{1}{2\,a^2x^2}-2\,\frac{b\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2+1/2/a^2/(c*x^4+b*x^2+a)*
c/(4*a*c-b^2)*x^2*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*c+1/2/a
^2/(c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)+2/a^2/(4*a*c-b^2)*c*ln((4*a*c-
b^2)*(c*x^4+b*x^2+a))*b-1/2/a^3/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4
+b*x^2+a))*b^3-6/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/
2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)^(1/2))*c^2+6/a^2/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*
b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^2*c-1/a^3/
(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c
-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-
b^6)^(1/2))*b^4-1/2/a^2/x^2-2*b*ln(x)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(b^2c-3\,ac^2\right)x^4+ab^2-4\,a^2c+\left(2\,b^3-7\,abc\right)x^2}{2\left((a^2b^2c-4\,a^3c^2)x^6+(a^2b^3-4\,a^3bc)x^4+(a^3b^2-4\,a^4c)x^2\right)}-\frac{-2\int\frac{\left(b^3c-4\,abc^2\right)x^3+\left(b^4-5\,ab^2c+3\,a^2c^2\right)x}{cx^4+bx^2+a}\,dx}{a^3b^2-4\,a^4c}-\frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*
c)*x^2)/((a^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4
+ (a^3*b^2 - 4*a^4*c)*x^2) - 2*integrate(-((b^3*c - 4*a*b*c^2)*x^
3 + (b^4 - 5*a*b^2*c + 3*a^2*c^2)*x)/(c*x^4 + b*x^2 + a), x)/(a^3
*b^2 - 4*a^4*c) - 2*b*log(x)/a^3
```

Fricas [A] time = 0.425886, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x),x, algorithm="fricas")
```

```
[Out] [-1/2*(((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c
+ 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*log((
b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*(a*b^
2*c - 3*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (2*a*b^3 - 7*a^2*b*c)*
x^2 - ((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 -
 4*a^2*b*c)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^3*c - 4*a*b*c^2)*
x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b*c)*x^2)*log(x))*sq
rt(b^2 - 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4
*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(b^2 - 4*a*c)), 1/2*(2*(
(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*
b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*arctan(-(2*c*
x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*(a*b^2*c - 3*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (2*a*b^3 - 7*a^2*b*c)*x^2 - ((b^3*
c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b*c)*
x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 -
 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b*c)*x^2)*log(x))*sqrt(-b^2 + 4*
a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 +
 (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.102 $\int \frac{1}{x^2(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=361

$$\frac{b\left(5b^2-19ac\right)}{2a^3x\left(b^2-4ac\right)}-\frac{5b^2-14ac}{6a^2x^3\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c)
)/(2*a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4
*a*c)*x^3*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 2
8*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4
*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4 - 29*a
*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^
3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 5.66413, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b\left(5b^2-19ac\right)}{2a^3x\left(b^2-4ac\right)}-\frac{5b^2-14ac}{6a^2x^3\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b\left(5b^2-19ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -(5*b^2 - 14*a*c)/(6*a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c)
)/(2*a^3*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4
*a*c)*x^3*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 2
8*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4
*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4 - 29*a
*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^
3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.23494, size = 344, normalized size = 0.95

$$\frac{\frac{6x\left(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-28a^2c^2+29ab^2c-19\cdots\right.}{\cdots}}{12a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] ((-4*a)/x^3 + (24*b)/x + (6*x*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*
c*x^2 - 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*
Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2
 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c - 28*a
^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [B] time = 0.07, size = 2349, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 3/2/a^2/(c*x^4+b*x^2+a)*b*c^2/(4*a*c-b^2)*x^3-1/2/a^3/(c*x^4+b*x^
2+a)*b^3*c/(4*a*c-b^2)*x^3-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c^2+
2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*c-1/2/a^3/(c*x^4+b*x^2+a)
/(4*a*c-b^2)*x*b^4-112*a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/
2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*ar
ctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^
2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*c^5+172/(-(4*a*c-b^2)^3)^(1/2)
/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*
c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*c^4*b^2-85/a
/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a
*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^
2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)
*c)^(1/2))*b^4*c^3+69/4/a^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^
(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)
*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^6*c^2-5/4/a^3*c/(-(4*a*c-
b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)
^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))
*b^8+19/a/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)
/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b*c
^3-39/4/a^2/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(
1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/
2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b
^3*c^2+5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)
^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*
2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/
2))*b^5+112*a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-
b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*
a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2
)^3)^(1/2)))^(1/2))*c^5-172/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*
arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^
3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*c^4*b^2+85/a/(-(4*a*c-b^2)^3)^(
1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2
)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c
-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^4*c^3-69/4
/a^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4
*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b
^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2
)))^(1/2))*b^6*c^2+5/4/a^3*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2
^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)
```

```
*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b
^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^8+19/a/(4*a*c-b^2)*2^(1/2)/(
(4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(
1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*
a*c-b^2)^3)^(1/2)))^(1/2))*b*c^3-39/4/a^2/(4*a*c-b^2)*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/
2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*
c-b^2)^3)^(1/2)))^(1/2))*b^3*c^2+5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((
4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1
/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a
*c-b^2)^3)^(1/2)))^(1/2))*b^5-1/3/a^2/x^3+2/a^3*b/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(5\,b^3c-19\,abc^2\right)x^6+\left(15\,b^4-62\,ab^2c+14\,a^2c^2\right)x^4-2\,a^2b^2+8\,a^3c+10\left(ab^3-4\,a^2bc\right)x^2}{6\left((a^3b^2c-4\,a^4c^2)x^7+(a^3b^3-4\,a^4bc)x^5+(a^4b^2-4\,a^5c)x^3\right)}-\frac{-\int\frac{5\,b^4-24\,ab^2c+14\,a^2c^2+\left(5\,b^3c-19\,abc^2\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(a^3b^2-4\,a^4c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2
*c^2)*x^4 - 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*a^2*b*c)*x^2)/((a
^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2
- 4*a^5*c)*x^3) - 1/2*integrate(-(5*b^4 - 24*a*b^2*c + 14*a^2*c^2
 + (5*b^3*c - 19*a*b*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 -
 4*a^4*c)
```

Fricas [A] time = 0.573563, size = 4637, normalized size = 12.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6 + 2*(15*b^4 - 62*a*b^2*c + 14*
a^2*c^2)*x^4 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*x^2
+ 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c
)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(25*b^9 - 315*a*b^7*c + 13
86*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 1
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250
```

*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*
c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48
*a^9*b^2*c^2 - 64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5
+ 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*s
qrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a
^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*
b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^
7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*
sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^
6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(
25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260
*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c
^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^
3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1
/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a
^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*
c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c
+ 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 241
08*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c
^2 - 64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^
2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(
125*b^14 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3
 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 -
10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 25
76*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76
686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 1
2*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 3
15*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4
 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((
625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3
+ 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6
 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12
*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4
*a^5*c)*x^3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 241
5*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^
2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2
*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2
- 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6
- 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 -
 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*
a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*
c^7 + (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^
5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*b^12 - 825
0*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^

```
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4
*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c
 + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^
6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 -
 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^
4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*
a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^
3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*
c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2
- 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 240
1*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*l
og((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^
3*b^2*c^7 + 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^
12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^
4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a
^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 46
72*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*b^12 - 8250*a*b^10*c
 + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24
108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^
2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8
*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^
10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4
- 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*
b^2*c^2 - 64*a^10*c^3))))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3
 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
```

Sympy [A] time = 43.1683, size = 566, normalized size = 1.57

$$\mathrm{RootSum}\left(t^4\left(1048576a^{13}c^6 - 1572864a^{12}b^2c^5 + 983040a^{11}b^4c^4 - 327680a^{10}b^6c^3 + 61440a^9b^8c^2 - 6144a^8b^{10}c + 256a^7b^{12}\right)\right.$$

$$+\frac{-8a^3c + 2a^2b^2 + x^6\left(57abc^2 - 15b^3c\right) + x^4\left(-14a^2c^2 + 62ab^2c - 15b^4\right) + x^2\left(40a^2bc - 10ab^3\right)}{x^7\left(24a^4c^2 - 6a^3b^2c\right) + x^5\left(24a^4bc - 6a^3b^3\right) + x^3\left(24a^5c - 6a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] RootSum(_t**4*(1048576*a**13*c**6 - 1572864*a**12*b**2*c**5 + 983
040*a**11*b**4*c**4 - 327680*a**10*b**6*c**3 + 61440*a**9*b**8*c*
*2 - 6144*a**8*b**10*c + 256*a**7*b**12) + _t**2*(-1290240*a**7*b
*c**7 + 3440640*a**6*b**3*c**6 - 3515904*a**5*b**5*c**5 + 1870848
*a**4*b**7*c**4 - 572272*a**3*b**9*c**3 + 101856*a**2*b**11*c**2
- 9840*a*b**13*c + 400*b**15) + 38416*a**2*c**9 - 17640*a*b**2*c*
```

```
*8 + 2025*b**4*c**7, Lambda(_t, _t*log(x + (-212992*_t**3*a**12*b
*c**5 + 299008*_t**3*a**11*b**3*c**4 - 164864*_t**3*a**10*b**5*c*
*3 + 44800*_t**3*a**9*b**7*c**2 - 6016*_t**3*a**8*b**9*c + 320*_t
**3*a**7*b**11 - 21952*_t*a**7*c**7 + 289856*_t*a**6*b**2*c**6 -
682820*_t*a**5*b**4*c**5 + 642828*_t*a**4*b**6*c**4 - 302316*_t*a
**3*b**8*c**3 + 75760*_t*a**2*b**10*c**2 - 9700*_t*a*b**12*c + 50
0*_t*b**14)/(9604*a**4*c**8 - 50421*a**3*b**2*c**7 + 43410*a**2*b
**4*c**6 - 12325*a*b**6*c**5 + 1125*b**8*c**4)))) + (-8*a**3*c +
2*a**2*b**2 + x**6*(57*a*b*c**2 - 15*b**3*c) + x**4*(-14*a**2*c**
2 + 62*a*b**2*c - 15*b**4) + x**2*(40*a**2*b*c - 10*a*b**3))/(x**
7*(24*a**4*c**2 - 6*a**3*b**2*c) + x**5*(24*a**4*b*c - 6*a**3*b**
3) + x**3*(24*a**5*c - 6*a**4*b**2))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.103 $\int \frac{1}{x^3(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=219

$$-\frac{(3b^2-2ac)\log(a+bx^2+cx^4)}{4a^4}+\frac{\log(x)(3b^2-2ac)}{a^4}+\frac{b(3b^2-11ac)}{2a^3x^2(b^2-4ac)}-\frac{3b^2-8ac}{4a^2x^4(b^2-4ac)}+\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^4(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] -(3*b^2 - 8*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))
/(2*a^3*(b^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 -
4*a*c)*x^4*(a + b*x^2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2
*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a
*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[
a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi [A] time = 0.626969, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{(3b^2-2ac)\log(a+bx^2+cx^4)}{4a^4}+\frac{\log(x)(3b^2-2ac)}{a^4}+\frac{b(3b^2-11ac)}{2a^3x^2(b^2-4ac)}-\frac{3b^2-8ac}{4a^2x^4(b^2-4ac)}+\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^4(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -(3*b^2 - 8*a*c)/(4*a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))
/(2*a^3*(b^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 -
4*a*c)*x^4*(a + b*x^2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2
*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a
*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2 - 2*a*c)*Log[
a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi in Sympy [A] time = 96.2513, size = 209, normalized size = 0.95

$$\frac{-2ac+b^2+bcx^2}{2ax^4\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{-8ac+3b^2}{4a^2x^4\left(-4ac+b^2\right)}+\frac{b\left(-11ac+3b^2\right)}{2a^3x^2\left(-4ac+b^2\right)}+\frac{b\left(30a^2c^2-20ab^2c+3b^4\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^4\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\left(-2ac+3b^2\right)\log\left(x^2\right)}{2a^4}-\frac{\left(-2ac+3b^2\right)\log\left(a+bx^2+cx^4\right)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(2*a*x**4*(-4*a*c + b**2)*(a + b*x**2
+ c*x**4)) - (-8*a*c + 3*b**2)/(4*a**2*x**4*(-4*a*c + b**2)) + b*
(-11*a*c + 3*b**2)/(2*a**3*x**2*(-4*a*c + b**2)) + b*(30*a**2*c**
2 - 20*a*b**2*c + 3*b**4)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2
))/(2*a**4*(-4*a*c + b**2)**(3/2)) + (-2*a*c + 3*b**2)*log(x**2)/
(2*a**4) - (-2*a*c + 3*b**2)*log(a + b*x**2 + c*x**4)/(4*a**4)
```

Mathematica [A] time = 0.625627, size = 328, normalized size = 1.5

$$\frac{\frac{2a\left(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(8a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-20ab^3c-14ab^2c\sqrt{b^2-4ac}+3b^4\sqrt{b^2-4ac}+3b^5\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(-8a^2c^2\sqrt{b \cdots}{4a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a^2/x^4) + (4*a*b)/x^2 + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b
^3*c*x^2 - 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
4*(3*b^2 - 2*a*c)*Log[x] - ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2 +
3*b^4*Sqrt[b^2 - 4*a*c] - 14*a*b^2*c*Sqrt[b^2 - 4*a*c] + 8*a^2*c^
2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 -
 4*a*c)^(3/2) + ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2 - 3*b^4*Sqrt[
b^2 - 4*a*c] + 14*a*b^2*c*Sqrt[b^2 - 4*a*c] - 8*a^2*c^2*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/
2))/(4*a^4)
```

Maple [B] time = 0.027, size = 671, normalized size = 3.1

$$
\begin{aligned}
&\frac{3c^2bx^2}{2a^2(cx^4+bx^2+a)(4ac-b^2)}-\frac{b^3cx^2}{2a^3(cx^4+bx^2+a)(4ac-b^2)}-\frac{c^2}{a(cx^4+bx^2+a)(4ac-b^2)}\\
&+2\frac{b^2c}{a^2(cx^4+bx^2+a)(4ac-b^2)}-\frac{b^4}{2a^3(cx^4+bx^2+a)(4ac-b^2)}+2\frac{c^2\ln\left(\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)\right)}{(4ac-b^2)a^2}\\
&-\frac{7c\ln\left(\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^2}{2a^3(4ac-b^2)}+\frac{3\ln\left(\left(4ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^4}{4a^4(4ac-b^2)}\\
&+15\frac{c^2b}{a^2\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx^2+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)\\
&-10\frac{b^3c}{a^3\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\arctan\left(\frac{2\left(4ac-b^2\right)cx^2+\left(4ac-b^2\right)b}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)\\
&+\frac{3b^5}{2a^4}\arctan\left(\left(2\left(4ac-b^2\right)cx^2+\left(4ac-b^2\right)b\right)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\right)\frac{1}{\sqrt{64a^3c^3-48a^2b^2c^2+12ab^4c-b^6}}\\
&-\frac{1}{4a^2x^4}-2\frac{\ln(x)c}{a^3}+3\frac{b^2\ln(x)}{a^4}+\frac{b}{x^2a^3}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^5+b*x^3+a*x)^2,x)
```

```
[Out] 3/2/a^2/(c*x^4+b*x^2+a)*b*c^2/(4*a*c-b^2)*x^2-1/2/a^3/(c*x^4+b*x^
2+a)*b^3*c/(4*a*c-b^2)*x^2-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c^2+2/
a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*c-1/2/a^3/(c*x^4+b*x^2+a)/(4*
a*c-b^2)*b^4+2/a^2/(4*a*c-b^2)*c^2*ln((4*a*c-b^2)*(c*x^4+b*x^2+a)
)-7/2/a^3/(4*a*c-b^2)*c*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^2+3/4/a
^4/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))*b^4+15/a^2/(64*a^3
*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c
*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1
/2))*b*c^2-10/a^3/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2
)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b
^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^3*c+3/2/a^4/(64*a^3*c^3-48*a^2*b^
2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^
2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^5-1/4/a
^2/x^4-2/a^3*ln(x)*c+3/a^4*b^2*ln(x)+1/a^3*b/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\frac{2\left(3b^3c-11abc^2\right)x^6+\left(6b^4-25ab^2c+8a^2c^2\right)x^4-a^2b^2+4a^3c+3\left(ab^3-4a^2bc\right)x^2}{4\left((a^3b^2c-4a^4c^2)x^8+(a^3b^3-4a^4bc)x^6+(a^4b^2-4a^5c)x^4\right)}
-\frac{\int\frac{\left(3b^4c-14ab^2c^2+8a^2c^3\right)x^3+\left(3b^5-17ab^3c+19a^2bc^2\right)x}{cx^4+bx^2+a}\,dx}{a^4b^2-4a^5c}+\frac{\left(3b^2-2ac\right)\log(x)}{a^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*b^3*c - 11*a*b*c^2)*x^6 + (6*b^4 - 25*a*b^2*c + 8*a^2*c
^2)*x^4 - a^2*b^2 + 4*a^3*c + 3*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^
2*c - 4*a^4*c^2)*x^8 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a
^5*c)*x^4) - integrate(((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^3
+ (3*b^5 - 17*a*b^3*c + 19*a^2*b*c^2)*x)/(c*x^4 + b*x^2 + a), x)/
(a^4*b^2 - 4*a^5*c) + (3*b^2 - 2*a*c)*log(x)/a^4
```

Fricas [A] time = 0.527704, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20
*a*b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3
*b*c^2)*x^4)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c
^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x
^2 + a)) - (2*(3*a*b^3*c - 11*a^2*b*c^2)*x^6 - a^3*b^2 + 4*a^4*c
+ (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^4 + 3*(a^2*b^3 - 4*a^3*b
*c)*x^2 - ((3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^8 + (3*b^5 - 14
*a*b^3*c + 8*a^2*b*c^2)*x^6 + (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2
)*x^4)*log(c*x^4 + b*x^2 + a) + 4*((3*b^4*c - 14*a*b^2*c^2 + 8*a^
2*c^3)*x^8 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*x^6 + (3*a*b^4 -
14*a^2*b^2*c + 8*a^3*c^2)*x^4)*log(x))*sqrt(b^2 - 4*a*c))/(((a^4*
b^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 - 4*a^5*b*c)*x^6 + (a^5*b^2 - 4
*a^6*c)*x^4)*sqrt(b^2 - 4*a*c)), -1/4*(2*((3*b^5*c - 20*a*b^3*c^2
 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6
+ (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*arctan(-(2*c*x^2 +
 b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*(3*a*b^3*c - 11*a^2*b*
c^2)*x^6 - a^3*b^2 + 4*a^4*c + (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^
2)*x^4 + 3*(a^2*b^3 - 4*a^3*b*c)*x^2 - ((3*b^4*c - 14*a*b^2*c^2 +
 8*a^2*c^3)*x^8 + (3*b^5 - 14*a*b^3*c + 8*a^2*b*c^2)*x^6 + (3*a*b
^4 - 14*a^2*b^2*c + 8*a^3*c^2)*x^4)*log(c*x^4 + b*x^2 + a) + 4*((
3*b^4*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^8 + (3*b^5 - 14*a*b^3*c + 8
*a^2*b*c^2)*x^6 + (3*a*b^4 - 14*a^2*b^2*c + 8*a^3*c^2)*x^4)*log(x
))*sqrt(-b^2 + 4*a*c))/(((a^4*b^2*c - 4*a^5*c^2)*x^8 + (a^4*b^3 -
 4*a^5*b*c)*x^6 + (a^5*b^2 - 4*a^6*c)*x^4)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^2*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.104 $\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=142

$$\frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(3*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.479121, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(3*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 42.4287, size = 138, normalized size = 0.97

$$\frac{2x^2\left(a+bx^2+cx^4\right)\operatorname{appellf}_1\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3a\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] 2*x**2*(a + b*x**2 + c*x**4)*appellf1(3/4, 1/2, 1/2, 7/4, -2*c*x*
*2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2))
```

```
)/(3*a*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2
/(b + sqrt(-4*a*c + b**2)) + 1)*sqrt(a*x + b*x**3 + c*x**5))
```

Mathematica [B] time = 0.309689, size = 383, normalized size = 2.7

$$-\frac{14a^2x^3\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{3\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(x\left(a+bx^2+cx^4\right)\right)^{3/2}\left(x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (-14*a^2*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*(b - Sqrt
[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(x*(a + b*x^2 + c*x^4))^(3
/2)*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^
2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
- 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{cx^5 + bx^3 + ax}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x\left(a + bx^2 + cx^4\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^5 + bx^3 + ax}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.105 $\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=380

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt[4]{a}\sqrt{x}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{ax+bx^3+cx^5}}-\frac{2x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}$$

```
[Out] (-2*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(15*c^(3/2)*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b + 3*c*
x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(15*c) + (2*a^(1/4)*(b^2 - 3*a*c)
*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5])
- (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(30*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.527865, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt[4]{a}\sqrt{x}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{ax+bx^3+cx^5}}-\frac{2x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(15*c^(3/2)*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b + 3*c*
```

```
x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(15*c) + (2*a^(1/4)*(b^2 - 3*a*c)
*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5])
- (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]
))/4])/(30*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 71.5908, size = 350, normalized size = 0.92

$$\frac{2\sqrt[4]{a}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-3ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{15c^{\frac{7}{4}}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{a}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{30c^{\frac{7}{4}}\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c} - \frac{2x^{\frac{3}{2}}\left(-3ac+b^2\right)\left(a+bx^2+cx^4\right)}{15c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] 2*a**(1/4)*sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*
x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(-3*a*c + b**2)*elliptic_e(2*a
tan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(15*c**(7/
4)*sqrt(a*x + b*x**3 + c*x**5)) - a**(1/4)*sqrt(x)*sqrt((a + b*x*
*2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2
)*(sqrt(a)*b*sqrt(c) - 6*a*c + 2*b**2)*elliptic_f(2*atan(c**(1/4)
*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(30*c**(7/4)*sqrt(a*x
+ b*x**3 + c*x**5)) + sqrt(x)*(b + 3*c*x**2)*sqrt(a*x + b*x**3 +
c*x**5)/(15*c) - 2*x**(3/2)*(-3*a*c + b**2)*(a + b*x**2 + c*x**4)
/(15*c**(3/2)*(sqrt(a) + sqrt(c)*x**2)*sqrt(a*x + b*x**3 + c*x**5
))
```

Mathematica [C] time = 2.61868, size = 486, normalized size = 1.28

$$\sqrt{x}\left(-i\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+2cx\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a
+ b*x^2 + c*x^4) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(
2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ell
ipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*S
qrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(30*c^2*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.048, size = 1042, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] -1/30/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)*(-6*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*c^2-6*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*x^7*b*c^2-8*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+
b^2)^(1/2)*x^5*b*c-8*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5*b^2*c-
6*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*a*c-2*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b^2-6*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*a*b*c-2*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*x^3*b^3+b*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a
)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/
2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4
*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)+12*(-2*
(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1
/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/
c)^(1/2))*a^2*c-3*b^2*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)
^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*
a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))-12*(-2*(x^2*(-4*a*c+b^2)^(1
/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1
/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*c+4*(-
2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^
(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/
a/c)^(1/2))*a*b^2-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2
)^(1/2)*x*a*b-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b^2)/(c*x^4
```

```
+b*x^2+a)/c/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^5 + bx^3 + ax}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^5 + bx^3 + ax}x^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\sqrt{x\left(a + bx^2 + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^5 + bx^3 + ax}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

3.106 $\int \sqrt{x}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=129

$$\frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 -
 4*a*c)*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 +
 c*x^5])
```

Rubi [A] time = 0.167005, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 -
 4*a*c)*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 +
 c*x^5])
```

Rubi in Sympy [A] time = 22.4492, size = 117, normalized size = 0.91

$$\frac{\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{\frac{3}{2}}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] (b + 2*c*x**2)*sqrt(a*x + b*x**3 + c*x**5)/(8*c*sqrt(x)) - sqrt(x
)*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)*atanh((b + 2*c*x**2)/
(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(16*c**(3/2)*sqrt(a*x + b*
```

```
x**3 + c*x**5))
```

Mathematica [A] time = 0.134639, size = 122, normalized size = 0.95

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}-\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)\right)}{16c^{3/2}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a
+ b*x^2 + c*x^4] - (b^2 - 4*a*c)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt
[a + b*x^2 + c*x^4]]))/(16*c^(3/2)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.014, size = 157, normalized size = 1.2

$$\frac{1}{16}\sqrt{x\left(cx^4+bx^2+a\right)}\left(4x^2c^{3/2}\sqrt{cx^4+bx^2+a}+4\ln\left(1/2\,\frac{2\,cx^2+2\sqrt{cx^4+bx^2+a}\sqrt{c}+b}{\sqrt{c}}\right)ac-\ln\left(\frac{1}{2}\left(2\,cx^2+2\sqrt{cx^4+bx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/16*(x*(c*x^4+b*x^2+a))^(1/2)/c^(3/2)*(4*x^2*c^(3/2)*(c*x^4+b*x^
2+a)^(1/2)+4*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c
^(1/2))*a*c-ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^
(1/2))*b^2+2*b*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^
2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288967, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-4\,ac\right)x\log\left(-\frac{4\,\sqrt{cx^5+bx^3+ax}\left(2\,c^2x^2+bc\right)\sqrt{x}+\left(8\,c^2x^5+8\,bcx^3+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right)-4\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{c}\sqrt{x}}{32\,c^{\frac{3}{2}}x},\ -\frac{\left(b^2-4\,ac\right)x\arctan\left(\frac{\left(2\,cx^3+bx\right)\sqrt{-c}}{2\,\sqrt{cx^5+bx^3+ax}c\sqrt{x}}\right)-2\,\sqrt{cx^5+bx^3+ax}\left(2\,cx^2+b\right)\sqrt{-c}\sqrt{x}}{16\,\sqrt{-c}cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*x*log(-(4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*
x^2 + b*c)*sqrt(x) + (8*c^2*x^5 + 8*b*c*x^3 + (b^2 + 4*a*c)*x)*sq
rt(c))/x) - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqr
t(x))/(c^(3/2)*x), -1/16*((b^2 - 4*a*c)*x*arctan(1/2*(2*c*x^3 + b
*x)*sqrt(-c)/(sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))) - 2*sqrt(c*x^
5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x))/(sqrt(-c)*c*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.301127, size = 171, normalized size = 1.33

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2\,x^2+\frac{b}{c}\right)+\frac{\left(b^2-4\,ac\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{3}{2}}}-\frac{b^2\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)-4\,ac\ln\left(\left|-b+2\,\sqrt{a}\sqrt{c}\right|\right)+2\,\sqrt{a}b\sqrt{c}}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*ln
(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(
3/2) - 1/16*(b^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*ln(abs(-
b + 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*b*sqrt(c))/c^(3/2)
```

3.107 $\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$

Optimal. Leaf size=347

$$\frac{\sqrt[4]{a}\sqrt{x}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{ax+bx^3+cx^5}} + \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5} + \frac{bx^{3/2}\left(a+bx^2+cx^4\right)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]
)/3 - (a^(1/4)*b*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(3/4)*Sqrt[a*x + b
*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*Sqrt[x]*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.433454, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{\sqrt[4]{a}\sqrt{x}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{a}b\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{ax+bx^3+cx^5}} + \frac{1}{3}\sqrt{x}\sqrt{ax+bx^3+cx^5} + \frac{bx^{3/2}\left(a+bx^2+cx^4\right)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]
)/3 - (a^(1/4)*b*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
```

```
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(3/4)*Sqrt[a*x + b
*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*Sqrt[x]*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 63.4335, size = 314, normalized size = 0.9

$$-\frac{\sqrt[4]{a}b\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3c^{\frac{3}{4}}\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt[4]{a}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6c^{\frac{3}{4}}\sqrt{ax+bx^3+cx^5}}+\frac{bx^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\sqrt{ax+bx^3+cx^5}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(1/2),x)
```

```
[Out] -a**(1/4)*b*sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)
*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/
a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(3*c**(3/4)*sqrt(a*x + b*
x**3 + c*x**5)) + a**(1/4)*sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sq
rt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(2*sqrt(a)*sqr
t(c) + b)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt
(a)*sqrt(c)))/(6*c**(3/4)*sqrt(a*x + b*x**3 + c*x**5)) + b*x**(3/
2)*(a + b*x**2 + c*x**4)/(3*sqrt(c)*(sqrt(a) + sqrt(c)*x**2)*sqrt
(a*x + b*x**3 + c*x**5)) + sqrt(x)*sqrt(a*x + b*x**3 + c*x**5)/3
```

Mathematica [C] time = 1.87102, size = 452, normalized size = 1.3

$$\frac{\sqrt{x}\left(-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+ib\left(\sqrt{b\right.\right.}{12c\sqrt{\frac{c}{\sqrt{b^2-4ac+}}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^
4) + I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c]
+ 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(
2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ell
ipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
 Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(12*c*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.025, size = 508, normalized size = 1.5

$$\frac{1}{3\,c x^4+3\,b x^2+3\,a}\sqrt{x\left(c x^4+b x^2+a\right)}\left(\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}\sqrt{-4\,ac+b^2}x^5c+\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}x^5bc+\sqrt{\frac{1}{a}\left(-b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x)
```

```
[Out] 1/3*(x*(c*x^4+b*x^2+a))^(1/2)/x^(1/2)*(((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(-4*a*c+b^2)^(1/2)*x^5*c+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*x^5*b*c+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^
3*b+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*b^2+a*(-2*(x^2*(-4*a*c+
b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a
)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4
*a*c+b^2)^(1/2)+b*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/
2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+
b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(-4*a*c+b^2)^(1/2)*x*a+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b
)/(c*x^4+b*x^2+a)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5+bx^3+ax}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5 + bx^3 + ax}}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(a + bx^2 + cx^4\right)}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/sqrt(x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5 + bx^3 + ax}}{\sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

3.108 $\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a +
 b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
 c*x^4])])/(2*Sqrt[a*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*
x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*
x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.444921, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```


```
[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a +
 b*x^2 + c*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
 c*x^4])])/(2*Sqrt[a*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*
x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*
x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 46.3928, size = 177, normalized size = 0.91

$$-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\,\mathrm{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2),x)
```

```
[Out] -sqrt(a)*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*atanh((2*a + b*x**2)/(
2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(2*sqrt(a*x + b*x**3 + c*x*
*5)) + b*sqrt(x)*sqrt(a + b*x**2 + c*x**4)*atanh((b + 2*c*x**2)/(
2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(4*sqrt(c)*sqrt(a*x + b*x**
3 + c*x**5)) + sqrt(a*x + b*x**3 + c*x**5)/(2*sqrt(x))
```

Mathematica [A] time = 0.325108, size = 165, normalized size = 0.85

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}-2\sqrt{a}\sqrt{c}\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)+b\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2c\right.\right.}{4\sqrt{c}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4] + 4*Sqrt[a]*Sqrt[c]*Log[x] - 2*Sqrt[a]*Sqrt[c]*Log[2*a + b*x^2
 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]] + b*Log[b + 2*c*x^2 + 2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4]]))/(4*Sqrt[c]*Sqrt[x*(a + b*x^2 + c*
x^4)])
```

Maple [A] time = 0.015, size = 136, normalized size = 0.7

$$-\frac{1}{4}\sqrt{x\left(cx^4+bx^2+a\right)}\left(2\sqrt{a}\ln\left(\frac{2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)\sqrt{c}-2\sqrt{cx^4+bx^2+a}\sqrt{c}-b\ln\left(\frac{1}{2}\left(2cx^2+2\sqrt{cx^4+bx^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x)
```

```
[Out] -1/4*(x*(c*x^4+b*x^2+a))^(1/2)/x^(1/2)*(2*a^(1/2)*ln((2*a+b*x^2+2
*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*c^(1/2)-2*(c*x^4+b*x^2+a)^(1
/2)*c^(1/2)-b*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/
c^(1/2)))/(c*x^4+b*x^2+a)^(1/2)/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.323933, size = 1, normalized size = 0.01

$$\left[\frac{bx\log\left(-\frac{4\sqrt{cx^5+bx^3+ax}\left(2c^2x^2+bc\right)\sqrt{x}+\left(8c^2x^5+8bcx^3+\left(b^2+4ac\right)x\right)\sqrt{c}}{x}\right)+2\sqrt{a}\sqrt{c}x\log\left(-\frac{\left(b^2+4ac\right)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}\left(bx^2+2\right.}{x^5}\right.}{8\sqrt{c}x}\right.$$

$$-\frac{4\sqrt{-a}\sqrt{c}x\arctan\left(\frac{bx^3+2ax}{2\sqrt{cx^5+bx^3+ax}\sqrt{-a}\sqrt{x}}\right)-bx\log\left(-\frac{4\sqrt{cx^5+bx^3+ax}\left(2c^2x^2+bc\right)\sqrt{x}+\left(8c^2x^5+8bcx^3+\left(b^2+4ac\right)x\right)\sqrt{c}}{x}\right)-4\sqrt{cx^5+bx^3}}{8\sqrt{c}x}$$

$$\left.-\frac{2\sqrt{-a}\sqrt{-c}x\arctan\left(\frac{bx^3+2ax}{2\sqrt{cx^5+bx^3+ax}\sqrt{-a}\sqrt{x}}\right)-bx\arctan\left(\frac{\left(2cx^3+bx\right)\sqrt{-c}}{2\sqrt{cx^5+bx^3+ax}c\sqrt{x}}\right)-2\sqrt{cx^5+bx^3+ax}\sqrt{-c}\sqrt{x}}{4\sqrt{-c}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*x*log(-(4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqr
t(x) + (8*c^2*x^5 + 8*b*c*x^3 + (b^2 + 4*a*c)*x)*sqrt(c))/x) + 2*
sqrt(a)*sqrt(c)*x*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x -
 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)
+ 4*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(c)*sqrt(x))/(sqrt(c)*x), 1/4*(
b*x*arctan(1/2*(2*c*x^3 + b*x)*sqrt(-c)/(sqrt(c*x^5 + b*x^3 + a*x
)*c*sqrt(x))) + sqrt(a)*sqrt(-c)*x*log(-((b^2 + 4*a*c)*x^5 + 8*a*
```

```
b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(
a)*sqrt(x))/x^5) + 2*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(-c)*sqrt(x))/
(sqrt(-c)*x), -1/8*(4*sqrt(-a)*sqrt(c)*x*arctan(1/2*(b*x^3 + 2*a*
x)/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(-a)*sqrt(x))) - b*x*log(-(4*sq
rt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x) + (8*c^2*x^5 +
8*b*c*x^3 + (b^2 + 4*a*c)*x)*sqrt(c))/x) - 4*sqrt(c*x^5 + b*x^3 +
 a*x)*sqrt(c)*sqrt(x))/(sqrt(c)*x), -1/4*(2*sqrt(-a)*sqrt(-c)*x*a
rctan(1/2*(b*x^3 + 2*a*x)/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(-a)*sqr
t(x))) - b*x*arctan(1/2*(2*c*x^3 + b*x)*sqrt(-c)/(sqrt(c*x^5 + b*
x^3 + a*x)*c*sqrt(x))) - 2*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(-c)*sqr
t(x))/(sqrt(-c)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x\left(a+bx^2+cx^4\right)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/x**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^5+bx^3+ax}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)
```

3.109 $\int x^{3/2} \left(ax + bx^3 + cx^5\right)^{3/2} dx$

Optimal. Leaf size=244

$$\frac{\left(128a^2c^2 - 100ab^2c + 15b^4\right)\sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{3b\sqrt{x}\left(b^2 - 4ac\right)^2\sqrt{a + bx^2 + cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax + bx^3 + cx^5}} - \frac{x^{3/2}\left(4cx^2\left(5b^2 - 16ac\right) + b\left(5b^2 - 4ac\right)\right)\sqrt{ax + bx^3 + cx^5}}{640c^2} + \frac{\sqrt{x}\left(3b + 8cx^2\right)\left(ax + bx^3 + cx^5\right)^{3/2}}{80c}$$

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/
(1280*c^3*Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 1
6*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b
+ 8*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*
c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[a*x + b*x^3 + c
*x^5])
```

Rubi [A] time = 0.583595, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{\left(128a^2c^2 - 100ab^2c + 15b^4\right)\sqrt{ax + bx^3 + cx^5}}{1280c^3\sqrt{x}} - \frac{3b\sqrt{x}\left(b^2 - 4ac\right)^2\sqrt{a + bx^2 + cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax + bx^3 + cx^5}} - \frac{x^{3/2}\left(4cx^2\left(5b^2 - 16ac\right) + b\left(5b^2 - 4ac\right)\right)\sqrt{ax + bx^3 + cx^5}}{640c^2} + \frac{\sqrt{x}\left(3b + 8cx^2\right)\left(ax + bx^3 + cx^5\right)^{3/2}}{80c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/
(1280*c^3*Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 1
6*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b
+ 8*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*
c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[a*x + b*x^3 + c
*x^5])
```

Rubi in Sympy [A] time = 63.7888, size = 230, normalized size = 0.94

$$-\frac{3b\sqrt{x}\left(-4ac+b^2\right)^2\sqrt{a+bx^2+cx^4}\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{\frac{7}{2}}\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{\frac{3}{2}}}{80c}-\frac{x^{\frac{3}{2}}\left(b\left(-4ac+5b^2\right)+4cx^2\left(-16ac+5b^2\right)\right)\sqrt{ax+bx^3+cx^5}}{640c^2}+\frac{\sqrt{ax+bx^3+cx^5}\left(128a^2c^2-100ab^2c+15b^4\right)}{1280c^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] -3*b*sqrt(x)*(-4*a*c + b**2)**2*sqrt(a + b*x**2 + c*x**4)*atanh((
b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(512*c**(7/2
)*sqrt(a*x + b*x**3 + c*x**5)) + sqrt(x)*(3*b + 8*c*x**2)*(a*x +
b*x**3 + c*x**5)**(3/2)/(80*c) - x**(3/2)*(b*(-4*a*c + 5*b**2) +
4*c*x**2*(-16*a*c + 5*b**2))*sqrt(a*x + b*x**3 + c*x**5)/(640*c**
2) + sqrt(a*x + b*x**3 + c*x**5)*(128*a**2*c**2 - 100*a*b**2*c +
15*b**4)/(1280*c**3*sqrt(x))
```

Mathematica [A] time = 0.246137, size = 181, normalized size = 0.74

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4b^2c\left(2cx^4-25a\right)+8bc^2x^2\left(7a+22cx^4\right)+128c^2\left(a+cx^4\right)^2+15b^4-10b^3cx^2\right)-\right.}{2560c^{7/2}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4]*(15*b^4 - 10*b^3*c*x^2 + 128*c^2*(a + c*x^4)^2 + 4*b^2*c*(-25*
a + 2*c*x^4) + 8*b*c^2*x^2*(7*a + 22*c*x^4)) - 15*b*(b^2 - 4*a*c)
^2*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(2560*c
^(7/2)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.019, size = 369, normalized size = 1.5

$$-\frac{1}{2560}\sqrt{x\left(cx^4+bx^2+a\right)}\left(-256\,x^8c^{9/2}\sqrt{cx^4+bx^2+a}-352\,x^6bc^{7/2}\sqrt{cx^4+bx^2+a}-512\,x^4ac^{7/2}\sqrt{cx^4+bx^2+a}-16\,x^4b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/2560*(x*(c*x^4+b*x^2+a))^(1/2)/c^(7/2)*(-256*x^8*c^(9/2)*(c*x^
4+b*x^2+a)^(1/2)-352*x^6*b*c^(7/2)*(c*x^4+b*x^2+a)^(1/2)-512*x^4*
a*c^(7/2)*(c*x^4+b*x^2+a)^(1/2)-16*x^4*b^2*c^(5/2)*(c*x^4+b*x^2+a
)^(1/2)-112*x^2*a*b*c^(5/2)*(c*x^4+b*x^2+a)^(1/2)+20*x^2*b^3*c^(3
/2)*(c*x^4+b*x^2+a)^(1/2)+240*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(
1/2)*c^(1/2)+b)/c^(1/2))*a^2*b*c^2-120*ln(1/2*(2*c*x^2+2*(c*x^4+b
*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*b^3*c+15*ln(1/2*(2*c*x^2+2*(c
*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*b^5-256*a^2*c^(5/2)*(c*x^
4+b*x^2+a)^(1/2)+200*a*b^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)-30*b^4*c
^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.30706, size = 1, normalized size = 0.

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x\log\left(\frac{4\,\sqrt{cx^5+bx^3+ax}\left(2\,c^2x^2+bc\right)\sqrt{x}-\left(8\,c^2x^5+8\,bcx^3+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right)+4\left(128\,c^4x^8+176\,bc^3x^6+8\left(b^2c^2\right.\right.}{5120\,c^{\frac{7}{2}}x}\right.$$

$$-\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x\arctan\left(\frac{\left(2\,cx^3+bx\right)\sqrt{-c}}{2\,\sqrt{cx^5+bx^3+ax}\,c\sqrt{x}}\right)-2\left(128\,c^4x^8+176\,bc^3x^6+8\left(b^2c^2+32\,ac^3\right)x^4+15\,b^4-100\,ab^2c\right.}{2560\,\sqrt{-c}c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x*log((4*sqrt(c*x^5
+ b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x) - (8*c^2*x^5 + 8*b*c*x^3
 + (b^2 + 4*a*c)*x)*sqrt(c))/x) + 4*(128*c^4*x^8 + 176*b*c^3*x^6
```

```
+ 8*(b^2*c^2 + 32*a*c^3)*x^4 + 15*b^4 - 100*a*b^2*c + 128*a^2*c^2
 - 2*(5*b^3*c - 28*a*b*c^2)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(c
)*sqrt(x))/(c^(7/2)*x), -1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*x*arctan(1/2*(2*c*x^3 + b*x)*sqrt(-c)/(sqrt(c*x^5 + b*x^3 + a
*x)*c*sqrt(x))) - 2*(128*c^4*x^8 + 176*b*c^3*x^6 + 8*(b^2*c^2 + 3
2*a*c^3)*x^4 + 15*b^4 - 100*a*b^2*c + 128*a^2*c^2 - 2*(5*b^3*c -
28*a*b*c^2)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(-c)*sqrt(x))/(sqr
t(-c)*c^3*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)
```

3.110 $\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx$

Optimal. Leaf size=487

$$\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{x^{3/2}\left(84a^2c^2-57ab^2c+8b^4\right)\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt{x}\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}$$

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^(3/2)*(a + b*x^2 + c*x^4))/(
315*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) -
(Sqrt[x]*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a*x +
 b*x^3 + c*x^5])/(315*c^2) + ((3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^
5)^(3/2))/(63*c*Sqrt[x]) - (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*
c^2)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a*x + b*x^3 + c*
x^5]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*S
qrt[c]*(b^2 - 6*a*c))*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11/4)*Sqrt
[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.847472, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{x^{3/2}\left(84a^2c^2-57ab^2c+8b^4\right)\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$
$$-\frac{\sqrt{x}\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^(3/2)*(a + b*x^2 + c*x^4))/(
315*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) -
(Sqrt[x]*(b*(4*b^2 - 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a*x +
 b*x^3 + c*x^5])/(315*c^2) + ((3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^
5)^(3/2))/(63*c*Sqrt[x]) - (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*
c^2)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
(2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a*x + b*x^3 + c*
x^5]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*S
qrt[c]*(b^2 - 6*a*c))*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11/4)*Sqrt
[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 103.107, size = 454, normalized size = 0.93

$$-\frac{\sqrt[4]{a}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(84a^2c^2-57ab^2c+8b^4\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{315c^{\frac{11}{4}}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\sqrt[4]{a}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(4\sqrt{a}b\sqrt{c}\left(-6ac+b^2\right)+84a^2c^2-57ab^2c+8b^4\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{630c^{\frac{11}{4}}\sqrt{ax+bx^3+cx^5}}$$
$$+\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{\frac{3}{2}}}{63c\sqrt{x}}-\frac{\sqrt{x}\left(b\left(-9ac+4b^2\right)+6cx^2\left(-7ac+2b^2\right)\right)\sqrt{ax+bx^3+cx^5}}{315c^2}$$
$$+\frac{x^{\frac{3}{2}}\left(a+bx^2+cx^4\right)\left(84a^2c^2-57ab^2c+8b^4\right)}{315c^{\frac{5}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)**(3/2)*x**(1/2),x)
```

```
[Out] -a**(1/4)*sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x
**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(84*a**2*c**2 - 57*a*b**2*c + 8
*b**4)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)
*sqrt(c)))/(315*c**(11/4)*sqrt(a*x + b*x**3 + c*x**5)) + a**(1/4)
*sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*
(sqrt(a) + sqrt(c)*x**2)*(4*sqrt(a)*b*sqrt(c)*(-6*a*c + b**2) + 8
4*a**2*c**2 - 57*a*b**2*c + 8*b**4)*elliptic_f(2*atan(c**(1/4)*x/
a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(630*c**(11/4)*sqrt(a*x +
 b*x**3 + c*x**5)) + (3*b + 7*c*x**2)*(a*x + b*x**3 + c*x**5)**(3
/2)/(63*c*sqrt(x)) - sqrt(x)*(b*(-9*a*c + 4*b**2) + 6*c*x**2*(-7*
```

```
a*c + 2*b**2))*sqrt(a*x + b*x**3 + c*x**5)/(315*c**2) + x**(3/2)*
(a + b*x**2 + c*x**4)*(84*a**2*c**2 - 57*a*b**2*c + 8*b**4)/(315*
c**(5/2)*(sqrt(a) + sqrt(c)*x**2)*sqrt(a*x + b*x**3 + c*x**5))
```

Mathematica [C] time = 3.98863, size = 609, normalized size = 1.25

$$\frac{\sqrt{x}\left(i\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2}}{b-\sqrt{b^2}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3
*c*x^4 + 53*b^2*c^2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*
b + 77*c*x^2) + a*(-4*b^3 + 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^
3*x^6)) + I*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*(-b + Sqrt[b^2 - 4*
a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b
^5 + 65*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*
b^2*c*Sqrt[b^2 - 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Ellipti
cF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqr
t[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(1260*c^3*Sqrt[c/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.033, size = 1880, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x)
```

```
[Out] 1/315*(x*(c*x^4+b*x^2+a))^(1/2)/x^(1/2)*(2*(-2*(x^2*(-4*a*c+b^2)^
(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^
(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+
b^2)^(1/2)*a*b^3-12*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/
```

```
2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+
b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*b*c-((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^5*b^3*c-4*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*b^5-6*(-2*(x^2*(-4*a*c+b^2)^(1/
2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^4+84*(-
2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^
(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/
a/c)^(1/2))*a^3*c^2+8*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(
1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*
c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^4+85*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^9*b*c^3+112*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*a*c^3+53*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*b^2*c^2+112*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*x^7*a*b*c^3+151*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*x^5*a*b^2*c^2+77*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+
b^2)^(1/2)*x^3*a^2*c^2+77*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*a
^2*b*c^2+27*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*a*b^3*c-4*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x*a*b^3+24*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a^2*b^2*c-84*(-2*(x^2*(-4*a*c+b^2)^
(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^
(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^3*c^2+
35*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^11*c^4+
35*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^11*b*c^4+85*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*x^9*b^2*c^3+53*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*x^7*b^3*c^2-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5*b^4*c-4*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b^4-4*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b^4+45*(-2*(x^2*(-4*a*c+b^2)^(1/
2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*b^2*c-5
7*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b
^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b
^2)/a/c)^(1/2))*a^2*b^2*c+24*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-
4*a*c+b^2)^(1/2)*x*a^2*b*c+151*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(-4*a*c+b^2)^(1/2)*x^5*a*b*c^2+27*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(-4*a*c+b^2)^(1/2)*x^3*a*b^2*c)/(c*x^4+b*x^2+a)/c^2/((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}\sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x),x, algorithm="fricas")
```

```
[Out] integral((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x}\left(x\left(a + bx^2 + cx^4\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}\sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

3.111 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=177

$$\frac{3\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}} - \frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(128*c
^2*Sqrt[x]) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x
^(3/2)) + (3*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5
/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.245809, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{3\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}} - \frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{16cx^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(128*c
^2*Sqrt[x]) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x
^(3/2)) + (3*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5
/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 32.1498, size = 167, normalized size = 0.94

$$\frac{\left(b+2cx^2\right)\left(ax+bx^3+cx^5\right)^{\frac{3}{2}}}{16cx^{\frac{3}{2}}} - \frac{3\left(b+2cx^2\right)\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{3\sqrt{x}\left(-4ac+b^2\right)^2\sqrt{a+bx^2+cx^4}\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{\frac{5}{2}}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

```
[Out] (b + 2*c*x**2)*(a*x + b*x**3 + c*x**5)**(3/2)/(16*c*x**(3/2)) - 3
*(b + 2*c*x**2)*(-4*a*c + b**2)*sqrt(a*x + b*x**3 + c*x**5)/(128*
c**2*sqrt(x)) + 3*sqrt(x)*(-4*a*c + b**2)**2*sqrt(a + b*x**2 + c*
x**4)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))
/(256*c**(5/2)*sqrt(a*x + b*x**3 + c*x**5))
```

Mathematica [A] time = 0.198028, size = 150, normalized size = 0.85

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}\right.\right.}{256c^{5/2}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a
+ b*x^2 + c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(
b^2 - 4*a*c)^2*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4
]]))/(256*c^(5/2)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.015, size = 295, normalized size = 1.7

$$\frac{1}{256}\sqrt{x\left(cx^4+bx^2+a\right)}\left(32\,x^6c^{7/2}\sqrt{cx^4+bx^2+a}+48\,x^4bc^{5/2}\sqrt{cx^4+bx^2+a}+80\,x^2ac^{5/2}\sqrt{cx^4+bx^2+a}+4\,x^2b^2c^{3/2}\sqrt{cx^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)
```

```
[Out] 1/256*(x*(c*x^4+b*x^2+a))^(1/2)/c^(5/2)*(32*x^6*c^(7/2)*(c*x^4+b*
x^2+a)^(1/2)+48*x^4*b*c^(5/2)*(c*x^4+b*x^2+a)^(1/2)+80*x^2*a*c^(5
/2)*(c*x^4+b*x^2+a)^(1/2)+4*x^2*b^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)
+48*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a
^2*c^2-24*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1
/2))*a*b^2*c+3*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)
/c^(1/2))*b^4+40*a*b*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)-6*b^3*c^(1/2)*
(c*x^4+b*x^2+a)^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.322915, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)x\log\left(-\frac{4\sqrt{cx^5+bx^3+ax}\left(2\,c^2x^2+bc\right)\sqrt{x}+\left(8\,c^2x^5+8\,bcx^3+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right)+4\left(16\,c^3x^6+24\,bc^2x^4-3\,b^3+20\,c\right.}{512\,c^{\frac{5}{2}}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x),x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*x*log(-(4*sqrt(c*x^5 + b
*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x) + (8*c^2*x^5 + 8*b*c*x^3 +
(b^2 + 4*a*c)*x)*sqrt(c))/x) + 4*(16*c^3*x^6 + 24*b*c^2*x^4 - 3*b
^3 + 20*a*b*c + 2*(b^2*c + 20*a*c^2)*x^2)*sqrt(c*x^5 + b*x^3 + a*
x)*sqrt(c)*sqrt(x))/(c^(5/2)*x), 1/256*(3*(b^4 - 8*a*b^2*c + 16*a
^2*c^2)*x*arctan(1/2*(2*c*x^3 + b*x)*sqrt(-c)/(sqrt(c*x^5 + b*x^3
 + a*x)*c*sqrt(x))) + 2*(16*c^3*x^6 + 24*b*c^2*x^4 - 3*b^3 + 20*a
*b*c + 2*(b^2*c + 20*a*c^2)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(-
c)*sqrt(x))/(sqrt(-c)*c^2*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.411531, size = 624, normalized size = 3.53

$$\frac{1}{16}\left(2\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{(b^2-4ac)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}-\frac{b^2\ln\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-4ac\ln\left(\right.}{c^{\frac{3}{2}}}\right.$$
$$+\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3(b^3-4abc)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}+\frac{3b^3\ln\left(\right.}{}\right.$$
$$+\frac{1}{384}\left(\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c^7-12ac^8}{c^9}\right)x^2+\frac{15b^3c^6-52abc^7}{c^9}\right)-\frac{15\sqrt{a}b^3-52a^{\frac{3}{2}}bc}{c^3}\right)c$$
$$+\frac{(5b^4c^6-24ab^2c^7+16a^2c^8)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{256c^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*ln(
abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3
/2) - (b^2*ln(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*ln(abs(-b + 2*
sqrt(a)*sqrt(c))) + 2*sqrt(a)*b*sqrt(c))/c^(3/2))*a + 1/96*(2*sqr
t(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2)
- 3*(b^3 - 4*a*b*c)*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 +
 a))*sqrt(c) - b))/c^(5/2) + (3*b^3*ln(abs(-b + 2*sqrt(a)*sqrt(c)
)) - 12*a*b*c*ln(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqr
t(c) - 16*a^(3/2)*c^(3/2))/c^(5/2))*b + 1/384*(sqrt(c*x^4 + b*x^2
 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c^7 - 12*a*c^8)/c^9)*x^2 +
 (15*b^3*c^6 - 52*a*b*c^7)/c^9) - (15*sqrt(a)*b^3 - 52*a^(3/2)*b*
c)/c^3)*c + 1/256*(5*b^4*c^6 - 24*a*b^2*c^7 + 16*a^2*c^8)*ln(abs(
-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(17/2)
```

3.112 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=425

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt[4]{a}b\sqrt{x}\left(b^2-8ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{ax+bx^3+cx^5}}-\frac{2bx^{3/2}\left(b^2-8ac\right)\left(a+bx^2+cx^4\right)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\left(10ac+b^2+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c}+\frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}}$$

```
[Out] (-2*b*(b^2 - 8*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(35*c^(3/2)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b^2 +
10*a*c + 3*b*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*
x^3 + c*x^5)^(3/2)/(7*Sqrt[x]) + (2*a^(1/4)*b*(b^2 - 8*a*c)*Sqrt[
x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5]) - (a^(
1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*Sqrt[x]
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(70*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.796389, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt[4]{a}b\sqrt{x}\left(b^2-8ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{ax+bx^3+cx^5}}-\frac{2bx^{3/2}\left(b^2-8ac\right)\left(a+bx^2+cx^4\right)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\left(10ac+b^2+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c}+\frac{\left(ax+bx^3+cx^5\right)^{3/2}}{7\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*b*(b^2 - 8*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(35*c^(3/2)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b^2 +
10*a*c + 3*b*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*
x^3 + c*x^5)^(3/2)/(7*Sqrt[x]) + (2*a^(1/4)*b*(b^2 - 8*a*c)*Sqrt[
x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5]) - (a^(
1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*Sqrt[x]
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(70*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 96.3897, size = 394, normalized size = 0.93

$$\frac{2\sqrt[4]{a}b\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-8ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{35c^{\frac{7}{4}}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{a}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\sqrt{c}\left(-20ac+b^2\right)+2b\left(-8ac+b^2\right)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{70c^{\frac{7}{4}}\sqrt{ax+bx^3+cx^5}} - \frac{2bx^{\frac{3}{2}}\left(-8ac+b^2\right)\left(a+bx^2+cx^4\right)}{35c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\left(ax+bx^3+cx^5\right)^{\frac{3}{2}}}{7\sqrt{x}} + \frac{\sqrt{x}\left(10ac+b^2+3bcx^2\right)\sqrt{ax+bx^3+cx^5}}{35c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(3/2),x)
```

```
[Out] 2*a**(1/4)*b*sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c
)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(-8*a*c + b**2)*elliptic_e(2
*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(35*c**(
7/4)*sqrt(a*x + b*x**3 + c*x**5)) - a**(1/4)*sqrt(x)*sqrt((a + b*
x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x*
*2)*(sqrt(a)*sqrt(c)*(-20*a*c + b**2) + 2*b*(-8*a*c + b**2))*elli
ptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/
(70*c**(7/4)*sqrt(a*x + b*x**3 + c*x**5)) - 2*b*x**(3/2)*(-8*a*c
+ b**2)*(a + b*x**2 + c*x**4)/(35*c**(3/2)*(sqrt(a) + sqrt(c)*x**
2)*sqrt(a*x + b*x**3 + c*x**5)) + (a*x + b*x**3 + c*x**5)**(3/2)/
(7*sqrt(x)) + sqrt(x)*(10*a*c + b**2 + 3*b*c*x**2)*sqrt(a*x + b*x
**3 + c*x**5)/(35*c)
```

Mathematica [C] time = 3.09902, size = 540, normalized size = 1.27

$$\sqrt{x}\left(2cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(15a^2c+a\left(b^2+23bcx^2+20c^2x^4\right)+x^2\left(b^3+9b^2cx^2+13bc^2x^4+5c^3x^6\right)\right)+i\left(-20a^2c^2+9ab^2c-8abc\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^
2 + 23*b*c*x^2 + 20*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*
x^4 + 5*c^3*x^6)) - I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (
b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b
^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*
a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(70*c^2
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.029, size = 1394, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x)
```

```
[Out] -1/70*(x*(c*x^4+b*x^2+a))^(1/2)*(-10*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*x^9*b*c^3-10*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)
^(1/2)*x^9*c^3-26*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^7*b^2*c^2-2
6*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^7*b*c^2-
40*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^5*a*b*c^2-40*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^5*a*c^2-18*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*x^5*b^3*c-18*((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(-4*a*c+b^2)^(1/2)*x^5*b^2*c-46*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*x^3*a*b^2*c-46*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2
)^(1/2)*x^3*a*b*c-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3*b^4-2*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b^3+12*(-
2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^
(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/
```

```
a/c)^(1/2))*a^2*b*c-20*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^
(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a
*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*c-3*(
-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)
^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)
/a/c)^(1/2))*a*b^3+(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2
)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b
^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a*b^2-32*(-2*
(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1
/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/
c)^(1/2))*a^2*b*c+4*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/
2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+
b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^3-30*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*x*a^2*b*c-30*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*
c+b^2)^(1/2)*x*a^2*c-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b^3-
2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x*a*b^2)/x
^(1/2)/(c*x^4+b*x^2+a)/c/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4
*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^5+bx^3+ax}\left(cx^4+bx^2+a\right)}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*(c*x^4 + b*x^2 + a)/sqrt(x), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

3.113 $\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.109748, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 14.876, size = 75, normalized size = 0.91

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(a + b*x**2 + c*x**4)*atanh((b + 2*c*x**2)/(2*sqrt(c)
*sqrt(a + b*x**2 + c*x**4)))/(2*sqrt(c)*sqrt(a*x + b*x**3 + c*x**
5))
```

Mathematica [A] time = 0.0579355, size = 80, normalized size = 0.98

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{2\sqrt{c}\sqrt{x\,(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt
[a + b*x^2 + c*x^4]])/(2*Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.013, size = 72, normalized size = 0.9

$$\frac{1}{2}\sqrt{x\,(cx^4+bx^2+a)}\ln\left(\frac{1}{2}\left(2\,cx^2+2\,\sqrt{cx^4+bx^2+a}\sqrt{c}+b\right)\frac{1}{\sqrt{c}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{cx^4+bx^2+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*ln(1/
2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.297037, size = 1, normalized size = 0.01

$$\left[\frac{\log\left(-\frac{4\,\sqrt{cx^5+bx^3+ax}\left(2\,c^2x^2+bc\right)\sqrt{x}+\left(8\,c^2x^5+8\,bcx^3+\left(b^2+4\,ac\right)x\right)\sqrt{c}}{x}\right)}{4\,\sqrt{c}},\ \frac{\arctan\left(\frac{\left(2\,cx^3+bx\right)\sqrt{-c}}{2\,\sqrt{cx^5+bx^3+ax}c\sqrt{x}}\right)}{2\,\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x)
+ (8*c^2*x^5 + 8*b*c*x^3 + (b^2 + 4*a*c)*x)*sqrt(c))/x)/sqrt(c),
1/2*arctan(1/2*(2*c*x^3 + b*x)*sqrt(-c)/(sqrt(c*x^5 + b*x^3 + a*x
)*c*sqrt(x)))/sqrt(-c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.336757, size = 54, normalized size = 0.66

$$-\frac{\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] -1/1024*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c)
 - b))/c^(3/2)
```

3.114 $\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c
*x^5])
```

Rubi [A] time = 0.0984117, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c
*x^5])
```

Rubi in Sympy [A] time = 17.618, size = 109, normalized size = 0.9

$$\frac{\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(
sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1
```

```
/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(1/4)*c**(1/4)*sqrt(a*x + b*x**
3 + c*x**5))
```

Mathematica [C] time = 0.235926, size = 193, normalized size = 1.6

$$-\frac{i\sqrt{x}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\mid\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((-I)*Sqrt[x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.024, size = 177, normalized size = 1.5

$$\frac{1}{2cx^4+2bx^2+2a}\sqrt{x\left(cx^4+bx^2+a\right)}\sqrt{-2\frac{x^2\sqrt{-4ac+b^2}-bx^2-2a}{a}}\sqrt{\frac{1}{a}\left(x^2\sqrt{-4ac+b^2}+bx^2+2a\right)}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^
(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a
*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^5 + bx^3 + ax}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x\left(a + bx^2 + cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^5 + bx^3 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.115 $\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*
x^5])]/(2*Sqrt[a])
```

Rubi [A] time = 0.0524891, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*
x^5])]/(2*Sqrt[a])
```

Rubi in Sympy [A] time = 20.1797, size = 76, normalized size = 1.49

$$-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] -sqrt(x)*sqrt(a + b*x**2 + c*x**4)*atanh((2*a + b*x**2)/(2*sqrt(a
)*sqrt(a + b*x**2 + c*x**4)))/(2*sqrt(a)*sqrt(a*x + b*x**3 + c*x*
*5))
```

Mathematica [A] time = 0.151655, size = 89, normalized size = 1.75

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)\right)}{2\sqrt{a}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(Log[x^2] - Log[2*a + b*x^2 + 2*
Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]]))/(2*Sqrt[a]*Sqrt[x*(a + b*x^2
 + c*x^4)])
```

Maple [A] time = 0.016, size = 72, normalized size = 1.4

$$-\frac{1}{2}\sqrt{x\left(cx^4+bx^2+a\right)}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{cx^4+bx^2+a}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] -1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/a^(1
/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288419, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\sqrt{cx^5+bx^3+ax}\left(abx^2+2\,a^2\right)\sqrt{x}-\left(\left(b^2+4\,ac\right)x^5+8\,abx^3+8\,a^2x\right)\sqrt{a}}{x^5}\right)}{4\sqrt{a}}, -\frac{\arctan\left(\frac{\left(bx^3+2\,ax\right)\sqrt{-a}}{2\sqrt{cx^5+bx^3+ax}a\sqrt{x}}\right)}{2\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] [1/4*log((4*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*x^2 + 2*a^2)*sqrt(x) -
 ((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x)*sqrt(a))/x^5)/sqrt(a),
 -1/2*arctan(1/2*(b*x^3 + 2*a*x)*sqrt(-a)/(sqrt(c*x^5 + b*x^3 + a
*x)*a*sqrt(x)))/sqrt(-a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5+bx^3+ax}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)
```

3.116 $\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=330

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{\sqrt{c}x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[a*x + b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)
) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a*x + b*x^3 +
 c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x
+ b*x^3 + c*x^5])
```

Rubi [A] time = 0.379692, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}} + \frac{\sqrt{c}x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[a*x + b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)
) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
```

```
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a*x + b*x^3 +
 c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x
+ b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 63.6706, size = 296, normalized size = 0.9

$$\frac{\sqrt{c}x^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{\frac{3}{2}}}-\frac{\sqrt[4]{c}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{a^{\frac{3}{4}}\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt[4]{c}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{\frac{3}{4}}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] sqrt(c)*x**(3/2)*(a + b*x**2 + c*x**4)/(a*(sqrt(a) + sqrt(c)*x**2
)*sqrt(a*x + b*x**3 + c*x**5)) - sqrt(a*x + b*x**3 + c*x**5)/(a*x
**(3/2)) - c**(1/4)*sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) +
 sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**
(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(a**(3/4)*sqrt(a*
x + b*x**3 + c*x**5)) + c**(1/4)*sqrt(x)*sqrt((a + b*x**2 + c*x**
4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic
_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a
**(3/4)*sqrt(a*x + b*x**3 + c*x**5))
```

Mathematica [C] time = 0.952771, size = 303, normalized size = 0.92

$$\frac{-4\left(a+bx^2+cx^4\right)+\frac{i\sqrt{2}x\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\right.\right.}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{4a\sqrt{x}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (-4*(a + b*x^2 + c*x^4) + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*x*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sq
rt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[x]*Sqrt[x
*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.028, size = 508, normalized size = 1.5

$$\frac{1}{(cx^4+bx^2+a)\,a}\left(-\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}\sqrt{-4\,ac+b^2}x^4c-\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}x^4bc-c\sqrt{-2\,\frac{x^2\sqrt{-4\,ac+b^2}-bx^2}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] (-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^4*c-((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^4*b*c-c*(-2*(x^2*(-4*a*c+b^2)^(1/
2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/
2)*x*a*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))+c*(-2*(
x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/
2)+b*x^2+2*a)/a)^(1/2)*x*a*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)
/a/c)^(1/2))-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)
*x^2*b-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^2*b^2-((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a-((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*a*b)/x^(3/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)/a/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5+bx^3+ax}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^5+bx^3+ax}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

3.117 $\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal. Leaf size=391

$$\frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}} + \frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{b\sqrt{c}x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x
^3 + c*x^5]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 -
4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*Sqrt[a*x +
b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b
 - 2*Sqrt[a]*Sqrt[c])*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.512995, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{b\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}} + \frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{b\sqrt{c}x^{3/2}\left(a+bx^2+cx^4\right)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x
^3 + c*x^5]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 -
```

```
4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*Sqrt[a*x +
b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b
 - 2*Sqrt[a]*Sqrt[c])*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 75.4271, size = 362, normalized size = 0.93

$$-\frac{b\sqrt{c}x^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{x^{\frac{3}{2}}\left(-2ac+b^2+bcx^2\right)}{a\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt[4]{c}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{a^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] -b*sqrt(c)*x**(3/2)*(a + b*x**2 + c*x**4)/(a*(sqrt(a) + sqrt(c)*x
**2)*(-4*a*c + b**2)*sqrt(a*x + b*x**3 + c*x**5)) + x**(3/2)*(-2*
a*c + b**2 + b*c*x**2)/(a*(-4*a*c + b**2)*sqrt(a*x + b*x**3 + c*x
**5)) + b*c**(1/4)*sqrt(x)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) +
sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(
1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(a**(3/4)*(-4*a*c
+ b**2)*sqrt(a*x + b*x**3 + c*x**5)) - c**(1/4)*sqrt(x)*sqrt((a +
 b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)
*x**2)*(2*sqrt(a)*sqrt(c) + b)*elliptic_f(2*atan(c**(1/4)*x/a**(1
/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(3/4)*(-4*a*c + b**2)*sq
rt(a*x + b*x**3 + c*x**5))
```

Mathematica [C] time = 1.85435, size = 463, normalized size = 1.18

$$-\frac{\sqrt{x}\left(-4x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-2ac+b^2+bcx^2\right)-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\cdots\right.\right.\right.}{4a\left(b^2-4ac\right)\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] -(Sqrt[x]*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*
c*x^2) + I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a
*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(4*a*(b^2 - 4*
a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.028, size = 533, normalized size = 1.4

$$\frac{1}{(4\,ac-b^2)\,(cx^4+bx^2+a)\,a}\sqrt{x\,(cx^4+bx^2+a)}\left(-\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}x^3b^2c-\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}\sqrt{-4\,ac+b^2}x^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] (x*(c*x^4+b*x^2+a))^(1/2)*(-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^3
*b^2*c-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^3*b
*c+b*c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*
a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*EllipticE(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-
2*a*c+b^2)/a/c)^(1/2))+c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a
)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/
2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4
*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*(-4*a*c+b^2)^(1/2)+2*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x*a*b*c+2*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x*a*c-((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*x*b^3-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x*
b^2)/x^(1/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(cx^5+bx^3+ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^5+bx^3+ax}(cx^4+bx^2+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/(sqrt(c*x^5 + b*x^3 + a*x)*(c*x^4 + b*x^2 + a)),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.118 $\int \frac{\sqrt{x}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x
^3 + c*x^5]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*
x + b*x^3 + c*x^5])]/(2*a^(3/2))
```

Rubi [A] time = 0.128364, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```


```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x
^3 + c*x^5]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*
x + b*x^3 + c*x^5])]/(2*a^(3/2))
```

Rubi in Sympy [A] time = 28.5493, size = 122, normalized size = 1.18

$$\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{a\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{\frac{3}{2}}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] sqrt(x)*(-2*a*c + b**2 + b*c*x**2)/(a*(-4*a*c + b**2)*sqrt(a*x +
b*x**3 + c*x**5)) - sqrt(x)*sqrt(a + b*x**2 + c*x**4)*atanh((2*a
+ b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(2*a**(3/2)*sqrt
(a*x + b*x**3 + c*x**5))
```

Mathematica [A] time = 0.369491, size = 154, normalized size = 1.5

$$\frac{\sqrt{x}\left(-2\sqrt{a}\left(-2ac+b^2+bcx^2\right)-\log\left(x^2\right)\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}+\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}\right.\right.}{2a^{3/2}\left(4ac-b^2\right)\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(-2*Sqrt[a]*(b^2 - 2*a*c + b*c*x^2) - (b^2 - 4*a*c)*Sqrt
[a + b*x^2 + c*x^4]*Log[x^2] + (b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x
^4]*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/(2*a^(
3/2)*(-b^2 + 4*a*c)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.018, size = 179, normalized size = 1.7

$$-\frac{1}{(2\,cx^4+2\,bx^2+2\,a)\,(4\,ac-b^2)}\sqrt{x\,(cx^4+bx^2+a)}\left(2\,x^2bc\sqrt{a}+4\,\ln\left(\frac{2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)ac\sqrt{cx^4+bx^2+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] -1/2*(x*(c*x^4+b*x^2+a))^(1/2)/a^(3/2)*(2*x^2*b*c*a^(1/2)+4*ln((2
*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*a*c*(c*x^4+b*x^2+a
)^(1/2)-ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^2*(
c*x^4+b*x^2+a)^(1/2)-4*a^(3/2)*c+2*b^2*a^(1/2))/x^(1/2)/(c*x^4+b*
x^2+a)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x}}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Fricas [A] time = 0.310398, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{cx^5+bx^3+ax}\left(bcx^2+b^2-2\,ac\right)\sqrt{a}\sqrt{x}+\left(\left(b^2c-4\,ac^2\right)x^5+\left(b^3-4\,abc\right)x^3+\left(ab^2-4\,a^2c\right)x\right)\log\left(\frac{4\sqrt{cx^5+bx^3+ax}\left(abx^2+\cdots\right.}{\cdots}\right.}{4\left((ab^2c-4\,a^2c^2)x^5+(ab^3-4\,a^2bc)x^3+(a^2b^2-4\,a^3c)x\right)\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^5 + b*x^3 + a*x)*(b*c*x^2 + b^2 - 2*a*c)*sqrt(a)
*sqrt(x) + ((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2
- 4*a^2*c)*x)*log((4*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*x^2 + 2*a^2)*
sqrt(x) - ((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x)*sqrt(a))/x^5)
)/(((a*b^2*c - 4*a^2*c^2)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3 + (a^2*b^
2 - 4*a^3*c)*x)*sqrt(a)), 1/2*(2*sqrt(c*x^5 + b*x^3 + a*x)*(b*c*x
^2 + b^2 - 2*a*c)*sqrt(-a)*sqrt(x) - ((b^2*c - 4*a*c^2)*x^5 + (b^
3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*arctan(1/2*(b*x^3 + 2*a*x
)*sqrt(-a)/(sqrt(c*x^5 + b*x^3 + a*x)*a*sqrt(x))))/(((a*b^2*c - 4
*a^2*c^2)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3 + (a^2*b^2 - 4*a^3*c)*x)*
sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

3.119 $\int \frac{1}{\sqrt{x}(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal. Leaf size=468

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{2\sqrt[4]{c}\sqrt{x}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2x^{3/2}\left(b^2-4ac\right)} + \frac{2\sqrt{c}x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3
 + c*x^5]) + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4)
)/(a^2*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c
*x^5]) - (2*(b^2 - 3*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(a^2*(b^2 -
4*a*c)*x^(3/2)) - (2*c^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(
2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^
(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi [A] time = 0.788604, antiderivative size = 468, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{2\sqrt[4]{c}\sqrt{x}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}} - \frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2x^{3/2}\left(b^2-4ac\right)} + \frac{2\sqrt{c}x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3
 + c*x^5]) + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4)
)/(a^2*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c
*x^5]) - (2*(b^2 - 3*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(a^2*(b^2 -
4*a*c)*x^(3/2)) - (2*c^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(
2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^
(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rubi in Sympy [A] time = 108.185, size = 434, normalized size = 0.93

$$\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt{c}x^{\frac{3}{2}}\left(-3ac+b^2\right)\left(a+bx^2+cx^4\right)}{a^2\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{2\left(-3ac+b^2\right)\sqrt{ax+bx^3+cx^5}}{a^2x^{\frac{3}{2}}\left(-4ac+b^2\right)}-\frac{2\sqrt[4]{c}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-3ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{a^{\frac{7}{4}}\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt[4]{c}\sqrt{x}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{\frac{7}{4}}\left(-4ac+b^2\right)\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(a*sqrt(x)*(-4*a*c + b**2)*sqrt(a*x +
b*x**3 + c*x**5)) + 2*sqrt(c)*x**(3/2)*(-3*a*c + b**2)*(a + b*x**
2 + c*x**4)/(a**2*(sqrt(a) + sqrt(c)*x**2)*(-4*a*c + b**2)*sqrt(a
*x + b*x**3 + c*x**5)) - 2*(-3*a*c + b**2)*sqrt(a*x + b*x**3 + c*
x**5)/(a**2*x**(3/2)*(-4*a*c + b**2)) - 2*c**(1/4)*sqrt(x)*sqrt((
a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt
(c)*x**2)*(-3*a*c + b**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)),
 1/2 - b/(4*sqrt(a)*sqrt(c)))/(a**(7/4)*(-4*a*c + b**2)*sqrt(a*x
+ b*x**3 + c*x**5)) + c**(1/4)*sqrt(x)*sqrt((a + b*x**2 + c*x**4)
/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*b
*sqrt(c) - 6*a*c + 2*b**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4))
```

```
, 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(7/4)*(-4*a*c + b**2)*sqrt(a
*x + b*x**3 + c*x**5))
```

Mathematica [C] time = 2.40704, size = 519, normalized size = 1.11

$$-\frac{2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-4a^2c+a\left(b^2-7bcx^2-6c^2x^4\right)+2b^2x^2\left(b+cx^2\right)\right)-ix\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\cdots}{\cdots}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] -(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*
x^2) + a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + S
qrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2
 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(
2*a^2*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x]*Sqrt[
x*(a + b*x^2 + c*x^4)])
```

Maple [B] time = 0.032, size = 1136, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x)
```

```
[Out] -1/2*(x*(c*x^4+b*x^2+a))^(1/2)/x^(3/2)*(12*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^4*a*c^2-4*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*x^4*b^2*c+12*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*x^4*a*b*c^2-4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^4*
b^3*c+a*b*c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2
*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/
2)-2*a*c+b^2)/a/c)^(1/2))*x*(-4*a*c+b^2)^(1/2)+12*EllipticF(1/2*x
```

```
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*
c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b
*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*x
*a^2*c^2-3*a*b^2*c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2
)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b
^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x-12*EllipticE(1/2*x*2^(1/2)*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)
-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)
^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*x*a^2*c^2+4*E
llipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1
/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-2*(x^2*(-4*a*
c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2
*a)/a)^(1/2)*x*a*b^2*c+14*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a
*c+b^2)^(1/2)*x^2*a*b*c-4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a
*c+b^2)^(1/2)*x^2*b^3+14*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^2*a*
b^2*c-4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x^2*b^4+8*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a^2*c-2*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a*b^2+8*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*a^2*b*c-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^3)/(
c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/
(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(cx^5 + bx^3 + ax\right)^{\frac{3}{2}} \sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] integral(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)

[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.120 $\int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}-\frac{(3b^2-8ac)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3
 + c*x^5]) - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(
b^2 - 4*a*c)*x^(5/2)) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*S
qrt[a]*Sqrt[a*x + b*x^3 + c*x^5])])/(4*a^(5/2))
```

Rubi [A] time = 0.295247, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}-\frac{(3b^2-8ac)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3
 + c*x^5]) - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(
b^2 - 4*a*c)*x^(5/2)) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*S
qrt[a]*Sqrt[a*x + b*x^3 + c*x^5])])/(4*a^(5/2))
```

Rubi in Sympy [A] time = 54.2542, size = 172, normalized size = 1.12

$$\frac{-2ac+b^2+bcx^2}{ax^{\frac{3}{2}}(-4ac+b^2)\sqrt{ax+bx^3+cx^5}}-\frac{(-8ac+3b^2)\sqrt{ax+bx^3+cx^5}}{2a^2x^{\frac{5}{2}}(-4ac+b^2)}+\frac{3b\sqrt{x}\sqrt{a+bx^2+cx^4}\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{\frac{5}{2}}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(a*x**(3/2)*(-4*a*c + b**2)*sqrt(a*x +
 b*x**3 + c*x**5)) - (-8*a*c + 3*b**2)*sqrt(a*x + b*x**3 + c*x**5
)/(2*a**2*x**(5/2)*(-4*a*c + b**2)) + 3*b*sqrt(x)*sqrt(a + b*x**2
 + c*x**4)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x*
*4)))/(4*a**(5/2)*sqrt(a*x + b*x**3 + c*x**5))
```

Mathematica [A] time = 0.393025, size = 192, normalized size = 1.25

$$\frac{2\sqrt{a}\left(-4a^2c+a\left(b^2-10bcx^2-8c^2x^4\right)+3b^2x^2\left(b+cx^2\right)\right)+3bx^2\log\left(x^2\right)\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}-3bx^2\left(b^2-4ac\right)\sqrt{a}}{4a^{5/2}x^{3/2}\left(4ac-b^2\right)\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (2*Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^
2 - 8*c^2*x^4)) + 3*b*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]*L
og[x^2] - 3*b*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]*Log[2*a +
 b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]])/(4*a^(5/2)*(-b^2 + 4
*a*c)*x^(3/2)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A] time = 0.019, size = 220, normalized size = 1.4

$$\frac{1}{\left(4\,cx^4+4\,bx^2+4\,a\right)\left(4\,ac-b^2\right)}\sqrt{x\left(cx^4+bx^2+a\right)}\left(-16\,x^4a^{3/2}c^2+6\,x^4b^2c\sqrt{a}+12\,\ln\left(\frac{2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}}{x^2}\right)x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x)
```

```
[Out] 1/4*(x*(c*x^4+b*x^2+a))^(1/2)/a^(5/2)*(-16*x^4*a^(3/2)*c^2+6*x^4*
b^2*c*a^(1/2)+12*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x
^2)*x^2*a*b*c*(c*x^4+b*x^2+a)^(1/2)-3*ln((2*a+b*x^2+2*a^(1/2)*(c*
x^4+b*x^2+a)^(1/2))/x^2)*x^2*b^3*(c*x^4+b*x^2+a)^(1/2)-20*a^(3/2)
*x^2*b*c+6*x^2*b^3*a^(1/2)-8*a^(5/2)*c+2*a^(3/2)*b^2)/x^(5/2)/(c*
x^4+b*x^2+a)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^5+bx^3+ax\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

Fricas [A] time = 0.3226, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{cx^5+bx^3+ax}\left(\left(3b^2c-8ac^2\right)x^4+ab^2-4a^2c+\left(3b^3-10abc\right)x^2\right)\sqrt{a}\sqrt{x}-3\left(\left(b^3c-4abc^2\right)x^7+\left(b^4-4ab^2c\right)x^5+\right.}{8\left(\left(a^2b^2c-4a^3c^2\right)x^7+\left(a^2b^3-4a^3bc\right)x^5+\left(a^3b^2-4\right.\right.}\right.$$

$$-\frac{2\sqrt{cx^5+bx^3+ax}\left(\left(3b^2c-8ac^2\right)x^4+ab^2-4a^2c+\left(3b^3-10abc\right)x^2\right)\sqrt{-a}\sqrt{x}-3\left(\left(b^3c-4abc^2\right)x^7+\left(b^4-4ab^2c\right)x^5+\right.}{4\left(\left(a^2b^2c-4a^3c^2\right)x^7+\left(a^2b^3-4a^3bc\right)x^5+\left(a^3b^2-4a^4c\right)x^3\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*sqrt(c*x^5 + b*x^3 + a*x)*((3*b^2*c - 8*a*c^2)*x^4 + a*b
^2 - 4*a^2*c + (3*b^3 - 10*a*b*c)*x^2)*sqrt(a)*sqrt(x) - 3*((b^3*
c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*
x^3)*log(-(4*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*x^2 + 2*a^2)*sqrt(x)
+ ((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x)*sqrt(a))/x^5))/(((a^2
*b^2*c - 4*a^3*c^2)*x^7 + (a^2*b^3 - 4*a^3*b*c)*x^5 + (a^3*b^2 -
4*a^4*c)*x^3)*sqrt(a)), -1/4*(2*sqrt(c*x^5 + b*x^3 + a*x)*((3*b^2
*c - 8*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (3*b^3 - 10*a*b*c)*x^2)*sqr
t(-a)*sqrt(x) - 3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^
5 + (a*b^3 - 4*a^2*b*c)*x^3)*arctan(1/2*(b*x^3 + 2*a*x)*sqrt(-a)/
(sqrt(c*x^5 + b*x^3 + a*x)*a*sqrt(x))))/(((a^2*b^2*c - 4*a^3*c^2)
*x^7 + (a^2*b^3 - 4*a^3*b*c)*x^5 + (a^3*b^2 - 4*a^4*c)*x^3)*sqrt(
-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^5 + bx^3 + ax)^{\frac{3}{2}} x^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

3.121 $\int \frac{x^{\frac{3}{2}(-1+n)}}{(ax^{-1+n}+bx^n+cx^{1+n})^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^(-1 + n)
+ b*x^n + c*x^(1 + n)])
```

Rubi [A] time = 0.0705383, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^(-1 + n)
+ b*x^n + c*x^(1 + n)])
```

Rubi in Sympy [A] time = 8.14377, size = 48, normalized size = 0.94

$$-\frac{2x^{\frac{n}{2}-\frac{1}{2}}(b+2cx)}{(-4ac+b^2)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2),x)
```

```
[Out] -2*x**(n/2 - 1/2)*(b + 2*c*x)/((-4*a*c + b**2)*sqrt(a*x**(n - 1)
+ b*x**n + c*x**(n + 1)))
```

Mathematica [A] time = 0.127814, size = 46, normalized size = 0.9

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{x^{n-1}(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^(-1 + n)*(a
 + x*(b + c*x))])
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int 1x^{-\frac{3}{2}+\frac{3n}{2}} \left(ax^{-1+n} + bx^{n} + cx^{1+n}\right)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

```
[Out] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}n-\frac{3}{2}}}{\left(cx^{n+1} + ax^{n-1} + bx^{n}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2),x, algorithm='
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/
2), x)
```

Fricas [A] time = 0.353622, size = 112, normalized size = 2.2

$$-\frac{2\left(2\,cx^{2} + bx\right)\sqrt{\frac{(cx^{2}+bx+a)x^{n+1}}{x^{2}}}}{(ab^{2} - 4\,a^{2}c + (b^{2}c - 4\,ac^{2})x^{2} + (b^{3} - 4\,abc)x)x^{\frac{1}{2}n+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2),x, algorithm='
```

```
[Out] -2*(2*c*x^2 + b*x)*sqrt((c*x^2 + b*x + a)*x^(n + 1)/x^2)/((a*b^2
- 4*a^2*c + (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*x^(1/2*n +
 1/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}\,n-\frac{3}{2}}}{\left(cx^{n+1}+ax^{n-1}+bx^{n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2),x, algorithm='
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/
2), x)
```

3.122 $\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=287

$$\frac{2dx^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}$$

```
[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/(3*Sqrt[a*x + b*x^3 + c*x^5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3 + c*
x^5])
```

Rubi [A] time = 1.0874, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{2dx^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In]  Int[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/(3*Sqrt[a*x + b*x^3 + c*x^5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3 + c*
x^5])
```

Rubi in Sympy [A] time = 91.4364, size = 280, normalized size = 0.98

$$\frac{2dx^2\left(a+bx^2+cx^4\right)\text{appellf}_1\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3a\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\left(a+bx^2+cx^4\right)\text{appellf}_1\left(\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{7a\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(e*x**2+d)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] 2*d*x**2*(a + b*x**2 + c*x**4)*appellf1(3/4, 1/2, 1/2, 7/4, -2*c*
x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2
)))/(3*a*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x*
*2/(b + sqrt(-4*a*c + b**2)) + 1)*sqrt(a*x + b*x**3 + c*x**5)) +
2*e*x**4*(a + b*x**2 + c*x**4)*appellf1(7/4, 1/2, 1/2, 11/4, -2*c
*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**
2)))/(7*a*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt(2*c*x
**2/(b + sqrt(-4*a*c + b**2)) + 1)*sqrt(a*x + b*x**3 + c*x**5))
```

Mathematica [B] time = 0.754711, size = 639, normalized size = 2.23

$$\frac{ax^3\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-\frac{49dF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\cdots\right.}{x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{3}{2},\frac{1}{2};\cdots\right.\right.}\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (a*x^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)*((-49*d*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-7*a*Appe
llF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*Ap
pellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appe
llF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) - (33*e*x^2*AppellF1[7/4, 1/2,
 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])/(-11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] +
```

```
 x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
]))))/(42*c*(x*(a + b*x^2 + c*x^4))^(3/2))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int x\left(ex^{2}+d\right)\frac{1}{\sqrt{cx^{5}+bx^{3}+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^{2}+d\right)x}{\sqrt{cx^{5}+bx^{3}+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex^{3}+dx}{\sqrt{cx^{5}+bx^{3}+ax}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="fricas")
```

```
[Out] integral((e*x^3 + d*x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(d+ex^2\right)}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(e*x**2+d)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x*(d + e*x**2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)x}{\sqrt{cx^5+bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

3.123 $\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(
2*Sqrt[3])
```

Rubi [A] time = 0.0211058, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x(2-x^2)}{2\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -ArcTanh[(Sqrt[3]*x*(2 - x^2))/(2*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*
Sqrt[3])
```

Rubi in Sympy [A] time = 17.9455, size = 68, normalized size = 1.51

$$-\frac{\sqrt{3}x\sqrt{x^4-3x^2+3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(-3x^2+6)}{6\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] -sqrt(3)*x*sqrt(x**4 - 3*x**2 + 3)*atanh(sqrt(3)*(-3*x**2 + 6)/(6
*sqrt(x**4 - 3*x**2 + 3)))/(6*sqrt(x**6 - 3*x**4 + 3*x**2))
```

Mathematica [A] time = 0.0744703, size = 78, normalized size = 1.73

$$\frac{x\sqrt{x^4-3x^2+3}\left(\log\left(x^2\right)-\log\left(-3x^2+2\sqrt{3}\sqrt{x^4-3x^2+3}+6\right)\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*(Log[x^2] - Log[6 - 3*x^2 + 2*Sqrt[3]*Sq
rt[3 - 3*x^2 + x^4]]))/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.017, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3\,x^2+3}Artanh\left(\frac{\left(x^2-2\right)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3\,x^2+3}}\right)\frac{1}{\sqrt{x^6-3\,x^4+3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^6-3*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh
(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [A] time = 0.878521, size = 27, normalized size = 0.6

$$-\frac{1}{6}\sqrt{3}\,\mathrm{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

Fricas [A] time = 0.287875, size = 124, normalized size = 2.76

$$\frac{1}{6}\,\sqrt{3}\log\left(\frac{6\,x^{3}+\sqrt{3}\left(2\,x^{5}-3\,x^{3}+6\,x\right)-2\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}\left(\sqrt{3}x^{2}+3\right)}{2\,x^{5}-3\,x^{3}-2\,\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((6*x^3 + sqrt(3)*(2*x^5 - 3*x^3 + 6*x) - 2*sqrt(x
^6 - 3*x^4 + 3*x^2)*(sqrt(3)*x^2 + 3))/(2*x^5 - 3*x^3 - 2*sqrt(x^
6 - 3*x^4 + 3*x^2)*x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^{6}-3x^{4}+3x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^{6}-3\,x^{4}+3\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

3.124 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}} \, dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(
2*Sqrt[3])
```

Rubi [A] time = 0.0259301, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x(2-x^2)}{2\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -ArcTanh[(Sqrt[3]*x*(2 - x^2))/(2*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*
Sqrt[3])
```

Rubi in Sympy [A] time = 18.191, size = 68, normalized size = 1.51

$$-\frac{\sqrt{3}x\sqrt{x^4-3x^2+3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(-3x^2+6)}{6\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] -sqrt(3)*x*sqrt(x**4 - 3*x**2 + 3)*atanh(sqrt(3)*(-3*x**2 + 6)/(6
*sqrt(x**4 - 3*x**2 + 3)))/(6*sqrt(x**6 - 3*x**4 + 3*x**2))
```

Mathematica [A] time = 0.0300454, size = 78, normalized size = 1.73

$$\frac{x\sqrt{x^4-3x^2+3}\left(\log\left(x^2\right)-\log\left(-3x^2+2\sqrt{3}\sqrt{x^4-3x^2+3}+6\right)\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*(Log[x^2] - Log[6 - 3*x^2 + 2*Sqrt[3]*Sq
rt[3 - 3*x^2 + x^4]]))/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.011, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3\,x^2+3}Artanh\left(\frac{\left(x^2-2\right)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3\,x^2+3}}\right)\frac{1}{\sqrt{x^2\left(x^4-3\,x^2+3\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2*(x^4-3*x^2+3))^(1/2),x)
```

```
[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arcta
nh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [A] time = 0.865981, size = 27, normalized size = 0.6

$$-\frac{1}{6}\sqrt{3}\operatorname{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

Fricas [A] time = 0.299546, size = 124, normalized size = 2.76

$$\frac{1}{6}\sqrt{3}\log\left(\frac{6x^3+\sqrt{3}\left(2x^5-3x^3+6x\right)-2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}x^2+3\right)}{2x^5-3x^3-2\sqrt{x^6-3x^4+3x^2}x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((6*x^3 + sqrt(3)*(2*x^5 - 3*x^3 + 6*x) - 2*sqrt(x
^6 - 3*x^4 + 3*x^2)*(sqrt(3)*x^2 + 3))/(2*x^5 - 3*x^3 - 2*sqrt(x^
6 - 3*x^4 + 3*x^2)*x^2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^4-3x^2+3)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

3.125 $\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(
2*Sqrt[3])
```

Rubi [A] time = 0.0270069, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x(2-x^2)}{2\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -ArcTanh[(Sqrt[3]*x*(2 - x^2))/(2*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*
Sqrt[3])
```

Rubi in Sympy [A] time = 18.3713, size = 68, normalized size = 1.51

$$-\frac{\sqrt{3}x\sqrt{x^4-3x^2+3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(-3x^2+6)}{6\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] -sqrt(3)*x*sqrt(x**4 - 3*x**2 + 3)*atanh(sqrt(3)*(-3*x**2 + 6)/(6
*sqrt(x**4 - 3*x**2 + 3)))/(6*sqrt(x**6 - 3*x**4 + 3*x**2))
```

Mathematica [A] time = 0.0300083, size = 78, normalized size = 1.73

$$\frac{x\sqrt{x^4-3x^2+3}\left(\log\left(x^2\right)-\log\left(-3x^2+2\sqrt{3}\sqrt{x^4-3x^2+3}+6\right)\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*(Log[x^2] - Log[6 - 3*x^2 + 2*Sqrt[3]*Sq
rt[3 - 3*x^2 + x^4]]))/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.007, size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3\,x^2+3}Artanh\left(\frac{\left(x^2-2\right)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3\,x^2+3}}\right)\frac{1}{\sqrt{x^6-3\,x^4+3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1-(-x^2+1)^3)^(1/2),x)
```

```
[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh
(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [A] time = 0.861839, size = 27, normalized size = 0.6

$$-\frac{1}{6}\sqrt{3}\,\operatorname{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^2 - 1)^3 + 1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

Fricas [A] time = 0.295055, size = 124, normalized size = 2.76

$$\frac{1}{6}\sqrt{3}\log\left(\frac{6x^3+\sqrt{3}\left(2x^5-3x^3+6x\right)-2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}x^2+3\right)}{2x^5-3x^3-2\sqrt{x^6-3x^4+3x^2}x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^2 - 1)^3 + 1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((6*x^3 + sqrt(3)*(2*x^5 - 3*x^3 + 6*x) - 2*sqrt(x
^6 - 3*x^4 + 3*x^2)*(sqrt(3)*x^2 + 3))/(2*x^5 - 3*x^3 - 2*sqrt(x^
6 - 3*x^4 + 3*x^2)*x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\left(-x^2+1\right)^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-(-x**2 + 1)**3 + 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\left(x^2-1\right)^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^2 - 1)^3 + 1),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)
```

3.126 $\int \sqrt{3x^2 - 3x^4 + x^6}\, dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2}\,(3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 -
3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 +
 x^4])
```

Rubi [A] time = 0.0789696, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$-\frac{\sqrt{x^6 - 3x^4 + 3x^2}\,(3 - 2x^2)}{8x} - \frac{3\sqrt{x^6 - 3x^4 + 3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 -
3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 +
 x^4])
```

Rubi in Sympy [A] time = 14.9335, size = 85, normalized size = 0.99

$$-\frac{(-2x^2 + 3)\sqrt{x^6 - 3x^4 + 3x^2}}{8x} + \frac{3\sqrt{x^6 - 3x^4 + 3x^2}\,\operatorname{atanh}\left(\frac{2x^2-3}{2\sqrt{x^4-3x^2+3}}\right)}{16x\sqrt{x^4 - 3x^2 + 3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] -(-2*x**2 + 3)*sqrt(x**6 - 3*x**4 + 3*x**2)/(8*x) + 3*sqrt(x**6 -
 3*x**4 + 3*x**2)*atanh((2*x**2 - 3)/(2*sqrt(x**4 - 3*x**2 + 3)))
/(16*x*sqrt(x**4 - 3*x**2 + 3))
```

Mathematica [A] time = 0.0546125, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSi
nh[(-3 + 2*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.013, size = 81, normalized size = 0.9

$$\frac{1}{16\,x}\sqrt{x^6-3\,x^4+3\,x^2}\left(4\,\sqrt{x^4-3\,x^2+3}x^2+3\,Arcsinh\left(1/3\,\sqrt{3}\left(2\,x^2-3\right)\right)-6\,\sqrt{x^4-3\,x^2+3}\right)\frac{1}{\sqrt{x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^6-3*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsinh
(1/3*3^(1/2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1
/2)
```

Maxima [A] time = 0.880689, size = 63, normalized size = 0.73

$$\frac{1}{4}\sqrt{x^4-3\,x^2+3}x^2-\frac{3}{8}\sqrt{x^4-3\,x^2+3}+\frac{3}{16}\,\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}\left(2\,x^2-3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^6 - 3*x^4 + 3*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(x^4 - 3*x^2 + 3)*x^2 - 3/8*sqrt(x^4 - 3*x^2 + 3) + 3/16*
arcsinh(1/3*sqrt(3)*(2*x^2 - 3))
```

Fricas [A] time = 0.28617, size = 95, normalized size = 1.1

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\left(2\,x^2-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^6 - 3*x^4 + 3*x^2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) -
 8*sqrt(x^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^6-3x^4+3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

GIAC/XCAS [A] time = 0.264947, size = 93, normalized size = 1.08

$$\frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}\left(2\,x^2-3\right)-3\ln\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sign}\left(x\right)$$
$$+\frac{3}{16}\left(2\,\sqrt{3}+\ln\left(2\,\sqrt{3}+3\right)\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^6 - 3*x^4 + 3*x^2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*ln(-2*x^2 + 2*sqrt(
x^4 - 3*x^2 + 3) + 3))*sign(x) + 3/16*(2*sqrt(3) + ln(2*sqrt(3) +
 3))*sign(x)
```

3.127 $\int \sqrt{x^2 \left(3-3x^2+x^4\right)}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,\left(3-2x^2\right)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 -
3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 +
 x^4])
```

Rubi [A] time = 0.0812002, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,\left(3-2x^2\right)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 -
3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 +
 x^4])
```

Rubi in Sympy [A] time = 15.1702, size = 85, normalized size = 0.99

$$-\frac{\left(-2x^2+3\right)\sqrt{x^6-3x^4+3x^2}}{8x}+\frac{3\sqrt{x^6-3x^4+3x^2}\,\operatorname{atanh}\left(\frac{2x^2-3}{2\sqrt{x^4-3x^2+3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] -(-2*x**2 + 3)*sqrt(x**6 - 3*x**4 + 3*x**2)/(8*x) + 3*sqrt(x**6 -
 3*x**4 + 3*x**2)*atanh((2*x**2 - 3)/(2*sqrt(x**4 - 3*x**2 + 3)))
/(16*x*sqrt(x**4 - 3*x**2 + 3))
```

Mathematica [A] time = 0.00890353, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSi
nh[(-3 + 2*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.006, size = 81, normalized size = 0.9

$$\frac{1}{16\,x}\sqrt{x^2\left(x^4-3\,x^2+3\right)}\left(4\,\sqrt{x^4-3\,x^2+3}x^2+3\,Arcsinh\left(1/3\,\sqrt{3}\left(2\,x^2-3\right)\right)-6\,\sqrt{x^4-3\,x^2+3}\right)\frac{1}{\sqrt{x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2*(x^4-3*x^2+3))^(1/2),x)
```

```
[Out] 1/16*(x^2*(x^4-3*x^2+3))^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsi
nh(1/3*3^(1/2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^
(1/2)
```

Maxima [A] time = 0.891268, size = 63, normalized size = 0.73

$$\frac{1}{4}\,\sqrt{x^4-3\,x^2+3}x^2-\frac{3}{8}\,\sqrt{x^4-3\,x^2+3}+\frac{3}{16}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(x^4 - 3*x^2 + 3)*x^2 - 3/8*sqrt(x^4 - 3*x^2 + 3) + 3/16*
arcsinh(1/3*sqrt(3)*(2*x^2 - 3))
```

Fricas [A] time = 0.287179, size = 95, normalized size = 1.1

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\left(2\,x^2-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) -
 8*sqrt(x^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265665, size = 93, normalized size = 1.08

$$\frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}\left(2\,x^2-3\right)-3\ln\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sign}\left(x\right)$$
$$+\frac{3}{16}\left(2\,\sqrt{3}+\ln\left(2\,\sqrt{3}+3\right)\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*ln(-2*x^2 + 2*sqrt(
x^4 - 3*x^2 + 3) + 3))*sign(x) + 3/16*(2*sqrt(3) + ln(2*sqrt(3) +
 3))*sign(x)
```

3.128 $\int \sqrt{1-\left(1-x^2\right)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{x^6-3x^4+3x^2}\left(3-2x^2\right)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 -
3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 +
 x^4])
```

Rubi [A] time = 0.0796927, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353

$$-\frac{\sqrt{x^6-3x^4+3x^2}\left(3-2x^2\right)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/(8*x) - (3*Sqrt[3*x^2 -
3*x^4 + x^6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 +
 x^4])
```

Rubi in Sympy [A] time = 15.3607, size = 85, normalized size = 0.99

$$-\frac{\left(-2x^2+3\right)\sqrt{x^6-3x^4+3x^2}}{8x}+\frac{3\sqrt{x^6-3x^4+3x^2}\operatorname{atanh}\left(\frac{2x^2-3}{2\sqrt{x^4-3x^2+3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] -(-2*x**2 + 3)*sqrt(x**6 - 3*x**4 + 3*x**2)/(8*x) + 3*sqrt(x**6 -
 3*x**4 + 3*x**2)*atanh((2*x**2 - 3)/(2*sqrt(x**4 - 3*x**2 + 3)))
/(16*x*sqrt(x**4 - 3*x**2 + 3))
```

Mathematica [A] time = 0.00917743, size = 70, normalized size = 0.81

$$\frac{x\left(4x^6-18x^4+30x^2+3\sqrt{x^4-3x^2+3}\sinh^{-1}\left(\frac{2x^2-3}{\sqrt{3}}\right)-18\right)}{16\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 + 3*Sqrt[3 - 3*x^2 + x^4]*ArcSi
nh[(-3 + 2*x^2)/Sqrt[3]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0.006, size = 81, normalized size = 0.9

$$\frac{1}{16\,x}\sqrt{x^6-3\,x^4+3\,x^2}\left(4\,\sqrt{x^4-3\,x^2+3}x^2+3\,Arcsinh\left(1/3\,\sqrt{3}\left(2\,x^2-3\right)\right)-6\,\sqrt{x^4-3\,x^2+3}\right)\frac{1}{\sqrt{x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1-(-x^2+1)^3)^(1/2),x)
```

```
[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsinh
(1/3*3^(1/2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1
/2)
```

Maxima [A] time = 0.86638, size = 63, normalized size = 0.73

$$\frac{1}{4}\,\sqrt{x^4-3\,x^2+3}x^2-\frac{3}{8}\,\sqrt{x^4-3\,x^2+3}+\frac{3}{16}\,\operatorname{arsinh}\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 - 1)^3 + 1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(x^4 - 3*x^2 + 3)*x^2 - 3/8*sqrt(x^4 - 3*x^2 + 3) + 3/16*
arcsinh(1/3*sqrt(3)*(2*x^2 - 3))
```

Fricas [A] time = 0.275742, size = 95, normalized size = 1.1

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\left(2\,x^2-3\right)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 - 1)^3 + 1),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) -
 8*sqrt(x^6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\left(-x^2+1\right)^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(-(-x**2 + 1)**3 + 1), x)
```

GIAC/XCAS [A] time = 0.264022, size = 93, normalized size = 1.08

$$\frac{1}{16}\left(2\,\sqrt{x^4-3\,x^2+3}\left(2\,x^2-3\right)-3\ln\left(-2\,x^2+2\,\sqrt{x^4-3\,x^2+3}+3\right)\right)\mathrm{sign}\left(x\right)+\frac{3}{16}\left(2\,\sqrt{3}+\ln\left(2\,\sqrt{3}+3\right)\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((x^2 - 1)^3 + 1),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*ln(-2*x^2 + 2*sqrt(
x^4 - 3*x^2 + 3) + 3))*sign(x) + 3/16*(2*sqrt(3) + ln(2*sqrt(3) +
 3))*sign(x)
```

3.129 $\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

Rubi [A] time = 0.0428515, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x + c*x^2]),x]

[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

Rubi in Sympy [A] time = 6.56539, size = 34, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**2+b*x+a)**(1/2),x)

[Out] -atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b*x + c*x**2)))/sqrt(a)
```

Mathematica [A] time = 0.055654, size = 39, normalized size = 1.03

$$\frac{\log(x)-\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (Log[x] - Log[2*a + b*x + 2*Sqrt[a]*Sqrt[a + x*(b + c*x)]])/Sqrt[
a]
```

Maple [A] time = 0.005, size = 35, normalized size = 0.9

$$-1\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29034, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{4\,(abx+2\,a^2)\sqrt{cx^2+bx+a}-(8\,abx+(b^2+4\,ac)\,x^2+8\,a^2)\sqrt{a}}{x^2}\right)}{2\,\sqrt{a}},-\frac{\arctan\left(\frac{(bx+2\,a)\sqrt{-a}}{2\,\sqrt{cx^2+bx+a}a}\right)}{\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*x),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*(a*b*x + 2*a^2)*sqrt(c*x^2 + b*x + a) - (8*a*b*x + (b
^2 + 4*a*c)*x^2 + 8*a^2)*sqrt(a))/x^2)/sqrt(a), -arctan(1/2*(b*x
+ 2*a)*sqrt(-a)/(sqrt(c*x^2 + b*x + a)*a))/sqrt(-a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.270821, size = 47, normalized size = 1.24

$$\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^2 + b*x + a)*x),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

3.130 $\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
]/Sqrt[a])
```

Rubi [A] time = 0.0427059, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
]/Sqrt[a])
```

Rubi in Sympy [A] time = 19.4967, size = 68, normalized size = 1.51

$$-\frac{x\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] -x*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt(a + b
*x + c*x**2)))/(sqrt(a)*sqrt(a*x**2 + b*x**3 + c*x**4))
```

Mathematica [A] time = 0.0558866, size = 70, normalized size = 1.56

$$\frac{x\sqrt{a+x(b+cx)}\left(\log(x)-\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(Log[x] - Log[2*a + b*x + 2*Sqrt[a]*Sqrt
[a + x*(b + c*x)]]))/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A] time = 0.012, size = 64, normalized size = 1.4

$$-x\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{x^2\,(cx^2+bx+a)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -1/(x^2*(c*x^2+b*x+a))^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*
a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((c*x^2 + b*x + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283968, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\,\sqrt{cx^4+bx^3+ax^2}\left(abx+2\,a^2\right)-\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\left(bx^2+2\,ax\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^3+ax^2}\,a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((c*x^2 + b*x + a)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2) - (8*a*b*
x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3)/sqrt(a), sqrt(-a
)*arctan(1/2*(b*x^2 + 2*a*x)*sqrt(-a)/(sqrt(c*x^4 + b*x^3 + a*x^2
)*a))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274126, size = 80, normalized size = 1.78

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sign}(x)}{\sqrt{-a}}+\frac{2\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sign}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((c*x^2 + b*x + a)*x^2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sign(x)/sqrt(-a) + 2*arctan(-(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sign(x))
```

3.131 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x
^3])]/Sqrt[a])
```

Rubi [A] time = 0.117825, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx)}{2\sqrt{a}\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x
^3])]/Sqrt[a])
```

Rubi in Sympy [A] time = 19.1881, size = 70, normalized size = 1.49

$$-\frac{\sqrt{x}\sqrt{a+bx+cx^2}\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{ax+bx^2+cx^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] -sqrt(x)*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqrt
(a + b*x + c*x**2)))/(sqrt(a)*sqrt(a*x + b*x**2 + c*x**3))
```

Mathematica [A] time = 0.0682834, size = 72, normalized size = 1.53

$$\frac{\sqrt{x}\sqrt{a+x(b+cx)}\left(\log(x)-\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)\right)}{\sqrt{a}\sqrt{x(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + x*(b + c*x)]*(Log[x] - Log[2*a + b*x + 2*Sqrt[a
]*Sqrt[a + x*(b + c*x)]]))/(Sqrt[a]*Sqrt[x*(a + x*(b + c*x))])
```

Maple [A] time = 0.016, size = 64, normalized size = 1.4

$$-1\sqrt{x}\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{x\,(cx^2+bx+a)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -x^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((
2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.348107, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4\,\sqrt{cx^3+bx^2+ax}\left(abx+2\,a^2\right)\sqrt{x}-\left(8\,abx^2+\left(b^2+4\,ac\right)x^3+8\,a^2x\right)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\arctan\left(\frac{bx^2+2\,ax}{2\,\sqrt{cx^3+bx^2+ax}\sqrt{-a}\sqrt{x}}\right)}{\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(4*sqrt(c*x^3 + b*x^2 + a*x)*(a*b*x + 2*a^2)*sqrt(x) -
(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x)*sqrt(a))/x^3)/sqrt(a),
arctan(1/2*(b*x^2 + 2*a*x)/(sqrt(c*x^3 + b*x^2 + a*x)*sqrt(-a)*sq
rt(x)))/sqrt(-a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.277979, size = 47, normalized size = 1.

$$\frac{2 \arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

3.132 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c
*x^5])]/Sqrt[a])
```

Rubi [A] time = 0.146853, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c
*x^5])]/Sqrt[a])
```

Rubi in Sympy [A] time = 17.595, size = 71, normalized size = 1.45

$$-\frac{x^{\frac{3}{2}}\sqrt{a+bx+cx^2}\,\operatorname{atanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] -x**(3/2)*sqrt(a + b*x + c*x**2)*atanh((2*a + b*x)/(2*sqrt(a)*sqr
t(a + b*x + c*x**2)))/(sqrt(a)*sqrt(a*x**3 + b*x**4 + c*x**5))
```

Mathematica [A] time = 0.0608748, size = 74, normalized size = 1.51

$$\frac{x^{3/2}\sqrt{a+x(b+cx)}\left(\log(x)-\log\left(2\sqrt{a}\sqrt{a+x(b+cx)}+2a+bx\right)\right)}{\sqrt{a}\sqrt{x^3(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] (x^(3/2)*Sqrt[a + x*(b + c*x)]*(Log[x] - Log[2*a + b*x + 2*Sqrt[a
]*Sqrt[a + x*(b + c*x)]]))/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))])
```

Maple [A] time = 0.012, size = 66, normalized size = 1.4

$$-1x^{\frac{3}{2}}\sqrt{cx^2+bx+a}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{x^3\,(cx^2+bx+a)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x)
```

```
[Out] -1/(x^3*(c*x^2+b*x+a))^(1/2)*x^(3/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*
ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.33202, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-\frac{4\,\sqrt{cx^5+bx^4+ax^3}\left(abx+2\,a^2\right)\sqrt{x}-\left(8\,abx^3+\left(b^2+4\,ac\right)x^4+8\,a^2x^2\right)\sqrt{a}}{x^4}\right)}{2\,\sqrt{a}},-\frac{\sqrt{-a}\arctan\left(\frac{bx^3+2\,ax^2}{2\,\sqrt{cx^5+bx^4+ax^3}\sqrt{-a}\sqrt{x}}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(4*sqrt(c*x^5 + b*x^4 + a*x^3)*(a*b*x + 2*a^2)*sqrt(x)
- (8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2)*sqrt(a))/x^4)/sqrt(
a), -sqrt(-a)*arctan(1/2*(b*x^3 + 2*a*x^2)/(sqrt(c*x^5 + b*x^4 +
a*x^3)*sqrt(-a)*sqrt(x)))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28359, size = 47, normalized size = 0.96

$$\frac{2 \arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

3.133 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sq
rt[a])
```

Rubi [A] time = 0.0865967, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sq
rt[a])
```

Rubi in Sympy [A] time = 12.5317, size = 39, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(2*s
qrt(a))
```

Mathematica [A] time = 0.0822935, size = 50, normalized size = 1.14

$$\frac{\log\left(x^2\right) - \log\left(2\sqrt{a}\sqrt{a + x^2\left(b + cx^2\right)} + 2a + bx^2\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]
])/(2*Sqrt[a])
```

Maple [A] time = 0.006, size = 39, normalized size = 0.9

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(2\,a + bx^2 + 2\,\sqrt{a}\sqrt{cx^4 + bx^2 + a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.309025, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\,\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x^4+8\,abx^2+8\,a^2\right)\sqrt{a}}{x^4}\right)}{4\,\sqrt{a}}, -\frac{\arctan\left(\frac{\left(bx^2+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^2+a}a}\right)}{2\,\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4
*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4)/sqrt(a), -1/2*arctan
(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a))/sqrt(-a)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x), x)
```

3.134 $\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6]
)]/(2*Sqrt[a])
```

Rubi [A] time = 0.0321583, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6]
)]/(2*Sqrt[a])
```

Rubi in Sympy [A] time = 26.1205, size = 75, normalized size = 1.53

$$-\frac{x\sqrt{a+bx^2+cx^4}\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] -x*sqrt(a + b*x**2 + c*x**4)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt
(a + b*x**2 + c*x**4)))/(2*sqrt(a)*sqrt(a*x**2 + b*x**4 + c*x**6)
)
```

Mathematica [A] time = 0.0796575, size = 87, normalized size = 1.78

$$\frac{x\sqrt{a+bx^2+cx^4}\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)\right)}{2\sqrt{a}\sqrt{x^2\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4]*(Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a
]*Sqrt[a + x^2*(b + c*x^2)]]))/(2*Sqrt[a]*Sqrt[x^2*(a + b*x^2 + c
*x^4)])
```

Maple [A] time = 0.011, size = 72, normalized size = 1.5

$$-\frac{x}{2}\sqrt{cx^4+bx^2+a}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x^2\left(cx^4+bx^2+a\right)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x)
```

```
[Out] -1/2/(x^2*(c*x^4+b*x^2+a))^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*
ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299648, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\sqrt{cx^6+bx^4+ax^2}\,(abx^2+2\,a^2)-\left((b^2+4\,ac)\,x^5+8\,abx^3+8\,a^2x\right)\sqrt{a}}{x^5}\right)}{4\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{(bx^3+2\,ax)\sqrt{-a}}{2\sqrt{cx^6+bx^4+ax^2}\,a}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*log((4*sqrt(c*x^6 + b*x^4 + a*x^2)*(a*b*x^2 + 2*a^2) - ((b^2
 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x)*sqrt(a))/x^5)/sqrt(a), 1/2*s
qrt(-a)*arctan(1/2*(b*x^3 + 2*a*x)*sqrt(-a)/(sqrt(c*x^6 + b*x^4 +
 a*x^2)*a))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4+bx^2+a)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)
```

3.135 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*
x^5])]/(2*Sqrt[a])
```

Rubi [A] time = 0.105695, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] -ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*
x^5])]/(2*Sqrt[a])
```

Rubi in Sympy [A] time = 22.7385, size = 76, normalized size = 1.49

$$-\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] -sqrt(x)*sqrt(a + b*x**2 + c*x**4)*atanh((2*a + b*x**2)/(2*sqrt(a
)*sqrt(a + b*x**2 + c*x**4)))/(2*sqrt(a)*sqrt(a*x + b*x**3 + c*x*
*5))
```

Mathematica [A] time = 0.0862469, size = 89, normalized size = 1.75

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)\right)}{2\sqrt{a}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(Log[x^2] - Log[2*a + b*x^2 + 2*
Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]]))/(2*Sqrt[a]*Sqrt[x*(a + b*x^2
 + c*x^4)])
```

Maple [A] time = 0.015, size = 72, normalized size = 1.4

$$-\frac{1}{2}\sqrt{x}\sqrt{cx^4+bx^2+a}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x\left(cx^4+bx^2+a\right)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x)
```

```
[Out] -1/2*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1
/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296396, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\sqrt{cx^5+bx^3+ax}\left(abx^2+2\,a^2\right)\sqrt{x}-\left(\left(b^2+4\,ac\right)x^5+8\,abx^3+8\,a^2x\right)\sqrt{a}}{x^5}\right)}{4\,\sqrt{a}}, -\frac{\arctan\left(\frac{\left(bx^3+2\,ax\right)\sqrt{-a}}{2\,\sqrt{cx^5+bx^3+ax}a\sqrt{x}}\right)}{2\,\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] [1/4*log((4*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*x^2 + 2*a^2)*sqrt(x) -
 ((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x)*sqrt(a))/x^5)/sqrt(a),
 -1/2*arctan(1/2*(b*x^3 + 2*a*x)*sqrt(-a)/(sqrt(c*x^5 + b*x^3 + a
*x)*a*sqrt(x)))/sqrt(-a)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(cx^4 + bx^2 + a)x}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)
```

3.136 $\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}} \, dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 +
c*x^7])]/(2*Sqrt[a])
```

Rubi [A] time = 0.120266, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 +
c*x^7])]/(2*Sqrt[a])
```

Rubi in Sympy [A] time = 22.7174, size = 78, normalized size = 1.47

$$-\frac{x^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] -x**(3/2)*sqrt(a + b*x**2 + c*x**4)*atanh((2*a + b*x**2)/(2*sqrt(
a)*sqrt(a + b*x**2 + c*x**4)))/(2*sqrt(a)*sqrt(a*x**3 + b*x**5 +
c*x**7))
```

Mathematica [A] time = 0.0846057, size = 91, normalized size = 1.72

$$\frac{x^{3/2}\sqrt{a+bx^2+cx^4}\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)\right)}{2\sqrt{a}\sqrt{x^3\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] (x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*(Log[x^2] - Log[2*a + b*x^2 + 2*
Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]]))/(2*Sqrt[a]*Sqrt[x^3*(a + b*x
^2 + c*x^4)])
```

Maple [A] time = 0.012, size = 74, normalized size = 1.4

$$-\frac{1}{2}x^{\frac{3}{2}}\sqrt{cx^4+bx^2+a}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{x^3\left(cx^4+bx^2+a\right)}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x)
```

```
[Out] -1/2/(x^3*(c*x^4+b*x^2+a))^(1/2)*x^(3/2)*(c*x^4+b*x^2+a)^(1/2)/a^
(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282487, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\sqrt{cx^7+bx^5+ax^3}\left(abx^2+2\,a^2\right)\sqrt{x}-\left(\left(b^2+4\,ac\right)x^6+8\,abx^4+8\,a^2x^2\right)\sqrt{a}}{x^6}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\left(bx^4+2\,ax^2\right)\sqrt{-a}}{2\sqrt{cx^7+bx^5+ax^3}a\sqrt{x}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*log((4*sqrt(c*x^7 + b*x^5 + a*x^3)*(a*b*x^2 + 2*a^2)*sqrt(x)
 - ((b^2 + 4*a*c)*x^6 + 8*a*b*x^4 + 8*a^2*x^2)*sqrt(a))/x^6)/sqrt
(a), 1/2*sqrt(-a)*arctan(1/2*(b*x^4 + 2*a*x^2)*sqrt(-a)/(sqrt(c*x
^7 + b*x^5 + a*x^3)*a*sqrt(x)))/a]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{(cx^4 + bx^2 + a)x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)
```

3.137 $\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3
])
```

Rubi [A] time = 0.057932, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]

[Out] -ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/(2*Sqrt[3
])
```

Rubi in Sympy [A] time = 8.57512, size = 36, normalized size = 0.9

$$-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(-3x^2+6)}{6\sqrt{x^4-3x^2+3}}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(x**4-3*x**2+3)**(1/2),x)

[Out] -sqrt(3)*atanh(sqrt(3)*(-3*x**2 + 6)/(6*sqrt(x**4 - 3*x**2 + 3)))
/6
```

Mathematica [A] time = 0.0514968, size = 45, normalized size = 1.12

$$\frac{\log\left(x^2\right)-\log\left(-3x^2+2\sqrt{3}\sqrt{x^4-3x^2+3}+6\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

```
[Out] (Log[x^2] - Log[6 - 3*x^2 + 2*Sqrt[3]*Sqrt[3 - 3*x^2 + x^4]])/(2*
Sqrt[3])
```

Maple [A] time = 0.005, size = 31, normalized size = 0.8

$$-\frac{\sqrt{3}}{6}Artanh\left(\frac{(-3x^2+6)\sqrt{3}}{6}\frac{1}{\sqrt{x^4-3x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(x^4-3*x^2+3)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*arctanh(1/6*(-3*x^2+6)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [A] time = 0.851726, size = 27, normalized size = 0.68

$$-\frac{1}{6}\sqrt{3}\,\text{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 - 3*x^2 + 3)*x),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

Fricas [A] time = 0.287976, size = 111, normalized size = 2.78

$$\frac{1}{6}\sqrt{3}\log\left(\frac{6x^2+\sqrt{3}(2x^4-3x^2+6)-2\sqrt{x^4-3x^2+3}\left(\sqrt{3}x^2+3\right)}{2x^4-2\sqrt{x^4-3x^2+3}x^2-3x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 - 3*x^2 + 3)*x),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((6*x^2 + sqrt(3)*(2*x^4 - 3*x^2 + 6) - 2*sqrt(x^4
 - 3*x^2 + 3)*(sqrt(3)*x^2 + 3))/(2*x^4 - 2*sqrt(x^4 - 3*x^2 + 3)
*x^2 - 3*x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^4 - 3x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**4 - 3*x**2 + 3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 - 3\,x^2 + 3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(x^4 - 3*x^2 + 3)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 - 3*x^2 + 3)*x), x)
```

3.138 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/(
2*Sqrt[3])
```

Rubi [A] time = 0.0248499, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x(2-x^2)}{2\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -ArcTanh[(Sqrt[3]*x*(2 - x^2))/(2*Sqrt[3*x^2 - 3*x^4 + x^6])]/(2*
Sqrt[3])
```

Rubi in Sympy [A] time = 18.0183, size = 68, normalized size = 1.51

$$-\frac{\sqrt{3}x\sqrt{x^4-3x^2+3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(-3x^2+6)}{6\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] -sqrt(3)*x*sqrt(x**4 - 3*x**2 + 3)*atanh(sqrt(3)*(-3*x**2 + 6)/(6
*sqrt(x**4 - 3*x**2 + 3)))/(6*sqrt(x**6 - 3*x**4 + 3*x**2))
```

Mathematica [A] time = 0.0420534, size = 78, normalized size = 1.73

$$\frac{x\sqrt{x^4-3x^2+3}\left(\log\left(x^2\right)-\log\left(-3x^2+2\sqrt{3}\sqrt{x^4-3x^2+3}+6\right)\right)}{2\sqrt{3}\sqrt{x^2\left(x^4-3x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*(Log[x^2] - Log[6 - 3*x^2 + 2*Sqrt[3]*Sq
rt[3 - 3*x^2 + x^4]]))/(2*Sqrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A] time = 0., size = 58, normalized size = 1.3

$$\frac{x\sqrt{3}}{6}\sqrt{x^4-3\,x^2+3}Artanh\left(\frac{\left(x^2-2\right)\sqrt{3}}{2}\frac{1}{\sqrt{x^4-3\,x^2+3}}\right)\frac{1}{\sqrt{x^2\left(x^4-3\,x^2+3\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2*(x^4-3*x^2+3))^(1/2),x)
```

```
[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arcta
nh(1/2*(x^2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [A] time = 0.872848, size = 27, normalized size = 0.6

$$-\frac{1}{6}\sqrt{3}\,\mathrm{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

Fricas [A] time = 0.290175, size = 124, normalized size = 2.76

$$\frac{1}{6}\sqrt{3}\log\left(\frac{6x^3+\sqrt{3}\left(2x^5-3x^3+6x\right)-2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}x^2+3\right)}{2x^5-3x^3-2\sqrt{x^6-3x^4+3x^2}x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((6*x^3 + sqrt(3)*(2*x^5 - 3*x^3 + 6*x) - 2*sqrt(x
^6 - 3*x^4 + 3*x^2)*(sqrt(3)*x^2 + 3))/(2*x^5 - 3*x^3 - 2*sqrt(x^
6 - 3*x^4 + 3*x^2)*x^2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(x^4-3x^2+3)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

3.139 $\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/
Sqrt[3])
```

Rubi [A] time = 0.0733449, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/
Sqrt[3])
```

Rubi in Sympy [A] time = 14.1813, size = 65, normalized size = 1.51

$$-\frac{\sqrt{3}\sqrt{x}\sqrt{x^2-3x+3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(-3x+6)}{6\sqrt{x^2-3x+3}}\right)}{3\sqrt{x^3-3x^2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)
```

```
[Out] -sqrt(3)*sqrt(x)*sqrt(x**2 - 3*x + 3)*atanh(sqrt(3)*(-3*x + 6)/(6
*sqrt(x**2 - 3*x + 3)))/(3*sqrt(x**3 - 3*x**2 + 3*x))
```

Mathematica [A] time = 0.0344567, size = 67, normalized size = 1.56

$$\frac{\sqrt{x}\sqrt{x^2-3x+3}\left(\log(x)-\log\left(2\sqrt{3}\sqrt{x^2-3x+3}-3x+6\right)\right)}{\sqrt{3}\sqrt{x\left(x^2-3x+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

```
[Out] (Sqrt[x]*Sqrt[3 - 3*x + x^2]*(Log[x] - Log[6 - 3*x + 2*Sqrt[3]*Sq
rt[3 - 3*x + x^2]]))/(Sqrt[3]*Sqrt[x*(3 - 3*x + x^2)])
```

Maple [A] time = 0.017, size = 50, normalized size = 1.2

$$\frac{\sqrt{3}}{3}\sqrt{x}\sqrt{x^2-3\,x+3}Artanh\left(\frac{(x-2)\sqrt{3}}{2}\frac{1}{\sqrt{x^2-3\,x+3}}\right)\frac{1}{\sqrt{x\,(x^2-3\,x+3)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x)
```

```
[Out] 1/3*x^(1/2)/(x*(x^2-3*x+3))^(1/2)*(x^2-3*x+3)^(1/2)*3^(1/2)*arcta
nh(1/2*(x-2)*3^(1/2)/(x^2-3*x+3)^(1/2))
```

Maxima [A] time = 0.901081, size = 34, normalized size = 0.79

$$\frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{\sqrt{3}x}{|x|}-\frac{2\sqrt{3}}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(sqrt(3)*x/abs(x) - 2*sqrt(3)/abs(x))
```

Fricas [A] time = 0.304501, size = 69, normalized size = 1.6

$$\frac{1}{6}\sqrt{3}\log\left(\frac{12\sqrt{x^3-3\,x^2+3\,x}(x-2)\sqrt{x}+\sqrt{3}\left(7\,x^3-24\,x^2+24\,x\right)}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((12*sqrt(x^3 - 3*x^2 + 3*x)*(x - 2)*sqrt(x) + sqr
t(3)*(7*x^3 - 24*x^2 + 24*x))/x^3)
```

Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A]   time = 0.293546, size = 69, normalized size = 1.6

$$-\frac{1}{3}\,\sqrt{3}\ln\left(\left|-x+\sqrt{3}+\sqrt{x^2-3\,x+3}\right|\right)+\frac{1}{3}\,\sqrt{3}\ln\left(\left|-x-\sqrt{3}+\sqrt{x^2-3\,x+3}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*ln(abs(-x + sqrt(3) + sqrt(x^2 - 3*x + 3))) + 1/3*sq
rt(3)*ln(abs(-x - sqrt(3) + sqrt(x^2 - 3*x + 3)))
```

3.140 $\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$

Optimal. Leaf size=70

$$-\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

```
[Out] -(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c
*x^(2*n - q) + a*x^q])]/(Sqrt[a]*(n - q)))
```

Rubi [A] time = 0.103628, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}(n-q)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

```
[Out] -(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c
*x^(2*n - q) + a*x^q])]/(Sqrt[a]*(n - q)))
```

Rubi in Sympy [A] time = 24.4809, size = 99, normalized size = 1.41

$$-\frac{x^{\frac{q}{2}}\sqrt{a+bx^{n-q}+cx^{2n-2q}}\,\operatorname{atanh}\left(\frac{2a+bx^{n-q}}{2\sqrt{a}\sqrt{a+bx^{n-q}+cx^{2n-2q}}}\right)}{\sqrt{a}\,(n-q)\,\sqrt{ax^q+bx^n+cx^{2n-q}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)
```

```
[Out] -x**(q/2)*sqrt(a + b*x**(n - q) + c*x**(2*n - 2*q))*atanh((2*a +
b*x**(n - q))/(2*sqrt(a)*sqrt(a + b*x**(n - q) + c*x**(2*n - 2*q)
)))/(sqrt(a)*(n - q)*sqrt(a*x**q + b*x**n + c*x**(2*n - q)))
```

Mathematica [A] time = 0.346561, size = 0, normalized size = 0.

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n + cx^{2n-q} + ax^q}}\,dx$$

Verification is Not applicable to the result.

```
[In]  Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

```
[Out] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]
```

Maple [F] time = 0.2, size = 0, normalized size = 0.

$$\int 1x^{-1+\frac{q}{2}} \frac{1}{\sqrt{bx^n + cx^{2\,n-q} + ax^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

```
[Out] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}\,q-1}}{\sqrt{cx^{2\,n-q} + bx^n + ax^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q),x, algorithm="maxima"
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q),x, algorithm="fricas"
```

```
[Out] Exception raised: TypeError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{1}{2}q-1}}{\sqrt{cx^{2n-q} + bx^n + ax^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
          # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```